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ELECTRICAL ENGINEERING

VOLUME II.

BY
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PREFACE

THIS second volume of Electrical Engineering is founded on the author's previous book, *A.C. Electrical Engineering*, omitting those parts which have been utilised for Volume I.

The space set free for this transference has permitted the inclusion of new chapters on symbolic notation, mercury arc rectifiers, and symmetrical components. The rest of the work has been revised throughout, and new matter added, particularly on thermionic valves and on protection, together with a new treatment of the 1-phase induction motor.

These alterations make it fulfil more completely the object of its predecessor. This was "to provide a textbook which will cover the fundamentals of all branches (of A.C. work)," and to "prove useful . . . for those connected with the running of A.C. power and sub-stations, and for the staffs of A.C. test-rooms."

It is intended to cover the fundamentals of all branches of Electrical Engineering dealt with in the usual three years' Day Courses. For evening classes, it is suitable for Higher National Certificate courses (A2), together with much of the Post-Certificate classes. These latter correspond with the Final Grade of the City and Guilds of London Institute Examination in Electrical Engineering Practice.

The symbols and abbreviations used are in agreement with the recommendations of the British Standard Institution as embodied in their publication No. 560. Similarly the nomenclature is in accordance with the B.S.I.'s "Glossary of terms used in Electrical Engineering" (publication No. 205).

W. TOLMÉ MACCALL.

ELECTRICAL ENGINEERING

IN TWO VOLUMES

By W. T. Maccall, M.Sc., M.I.E.E.

For the benefit of the reader of this second volume of the above work, a brief summary of the contents of the first volume is given below.

CONTENTS—VOLUME I

I. Introductory
II. Units and Standards
III. Conductors and Insulators
IV. Electromagnetism
V. Alternating Currents
VI. Polyphase Systems
VII. Measuring Instruments
VIII. Direct Current Generators
IX. Armature Reaction and Commutation
X. Characteristics and Construction
XI. Direct Current Motors
XII. Dynamo Efficiencies
XIII. Accumulators
XIV. Lighting and Illumination
XV. Incandescent Lamps
XVI. Special D.C. Machines
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XVIII. Transmission and Distribution
Appendix, Answers, Index

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LIST OF SYMBOLS

A = cross-sectional area.
B = flux-density.
C = capacitance.
D, d = diameter.
E = electromotive force.
f = frequency in cycles per sec.
H = magnetic force.
I, i = current.
j = 90° rotation operator.
K = dielectric field intensity.
L = inductance.
M = mutual inductance.
N_1, N_2 = numbers of stator and rotor conductors.
n = revolutions per minute.
p = number of pole-pairs.
Q, q = quantity of electricity.
R = resistance.
r = radius.
S = speed as fraction of synchronous speed.
S_1, S_2 = numbers of stator and rotor slots.
s = slip; spacing between centres of overhead wires.
T = torque.
t = time.
V, v = voltage, potential difference.
W = watts.
X = reactance.
Y = admittance.
Z = impedance.
δ = air-gap radical length.
λ = permeance per cm.
120° rotation operator.
μ = permeability.
σ = leakage (dispersion) coefficient.
τ = pole-pitch.
ϕ = angle of lag or lead.
Φ = magnetic flux.
Ψ = dielectric flux.
℧ = number of turns.

ABBREVIATIONS

A.C. = alternating current.
B.S.I. = British Standards Institution.
B.S.S. = British Standard Specification.
C.G.S. = centimetre-gram-second.
D.C. = direct current.
e.m.f. = electromotive force.
E.M.U. = electromagnetic units.
E.S.U. = electrostatic units.
h.p. = horse-power.
I.E.E. = The Institution of Electrical Engineers.
m.m.f. = magnetomotive force.
p.d. = potential difference.
p.f. = power factor.
r.p.m. = revolutions per minute.
R.M.S. = Root-mean-square.

ABBREVIATIONS AFTER NUMBERS

A = amperes.
b.h.p = brake horse-power.
cyc. = cycles per sec.
kV = kilovolts.
kVA = kilovolt-amperes.
kVAR = reactive kilovolt-amperes.
kW = kilowatts.
kWh kilowatt-hours.
MVA megavolt-amperes.
V volts.
μF microfarads.
ampere-turns.

ABBREVIATIONS AFTER QUESTIONS AND EXAMPLES

C. & G., II. = City and Guilds of London Institute examination in Electrical Engineering Practice, Grade II.
C. & G., Fin. = Ditto, Final Grade, 1st paper.
C. & G., A = Ditto, Final Grade, 2nd paper, Section A, *Theory, Performance and Design of Electrical Machinery and Transformers.*
C. & G., B = Ditto, Section B, *Generation and Transmission.*
C. & G., C = Ditto, Section C, *Distribution and Utilization.*
C. & G., E = Ditto, Section E, *Electrical Instruments and Measurements.*

ELECTRICAL ENGINEERING

VOLUME II

CHAPTER I

SYMBOLIC NOTATION

1. General Idea

The use of vectors in alternating current problems has been shown in Chapter V. of Volume I. In most cases vector diagrams are used to indicate what calculations are to be made, rather than being drawn to scale to give the desired result. When the former method is used, a considerable reduction in the amount of writing needed can be effected by using symbolic notation.

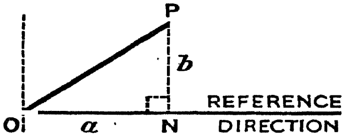


Fig. 1.01.—Symbolic Notation.

This consists in specifying vectors in the following way. A reference direction is taken and a vector (OP, Fig. 1.01) is specified by its components, a in this direction, and b at right angles to the reference direction. The vector is then written in the form $a + jb$; the symbol j indicating that b has been rotated anticlockwise through a right angle from the reference direction.

For any vector in the first quadrant a and b are both positive. For vectors in other quadrants, one or both of these quantities are negative. Thus in Fig. 1.02; OQ is written $-c + jd$, whereas OR is written $-c - jd$. And a vector in the fourth quadrant, such as OS, is written $e - jf$.

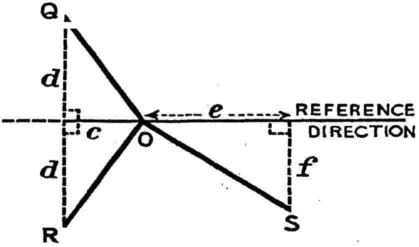


Fig. 1.02.—Vectors Not in First Quadrants.

Since the projection of the sum of two vectors on any line is the sum of the projections of the separate vectors (cf. Volume I., Chapter V., Art. 6), the sum of two vectors written in symbolic notation can be obtained at once.

Thus if OP $= a + jb$ and OQ $= c + jd$, their vector sum, OR, is $a + c + j(b + d)$. For in Fig. 1.03, OL = ON + NL = ON + OM $= a + c$: and RL = KL + RK = PN + QM $= b + d$.

This still holds good when any of the quantities a, b, c, d are negative, provided the algebraic sums are taken.

2. The Value of *j*

Since the symbol j denotes that the quantity to which it is attached is to be turned through a right angle from the reference line, the symbol jj, or j^2, denotes that the quantity is to be turned through two right angles from the reference line. But this operation exactly reverses it, and so is equivalent to multiplying it by -1. Thus $j^2 = -1$; and $j = \sqrt{-1}$. In other words, j is a vector operator which turns a vector anticlockwise through 90°; and its algebraic interpretation is $\sqrt{-1}$.

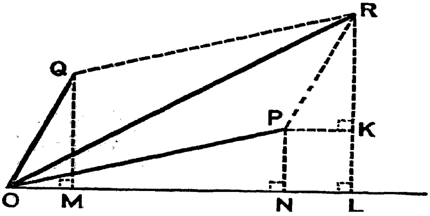


Fig. 1.03.—SUM OF TWO VECTORS.

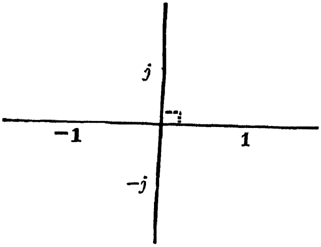


Fig. 1.04.—FOURTH ROOT OF UNITY.

Since no real numerical value can be given to $\sqrt{-1}$ the j components of vectors are often called the *unreal* or *imaginary* parts. But this must be understood as merely a convenient mathematical expression, for these components are just as real as those in the reference direction.

Another interpretation of j is to consider it as one of the fourth roots of unity. For $j^4a = a$, since applying j four times to any quantity means turning it through four right angles which brings it back to its original position.

$$\text{Now if } x^4 = 1,\ x^4 - 1 = 0;$$
$$\therefore (x^2 - 1)(x^2 + 1) = 0, \text{ i.e. } x^2 = 1, \text{ or } x^2 = -1;$$
$$\therefore x = \pm 1, \text{ or } \pm\sqrt{-1}, \textit{ i.e. } \pm 1, \text{ or } \pm j.$$

Thus the four fourth roots of unity are four lines of unit length, drawn in four directions at right angles (see Fig. 1.04).

3. Application to Impedance

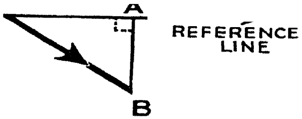


Fig. 1.05.—VOLTAGE AND CURRENT.

The impedance (Z) of a circuit to alternating currents is equal to the voltage to produce unit current, so it can be split up into two components at right angles; viz. one equal to the resistance R in phase with the current, and the other equal to the reactance X leading the current by 90°. Hence it can be expressed in the form $R + jX$.

Then, if a voltage (V) in the reference position is applied to the impedance, the current $(\dot{I}) = \dot{V}/\dot{Z} = V/(R + jX)$. The symbols $\dot{I}$, $\dot{V}$ and $\dot{Z}$ represent the *vector*, values of current, voltage and impedance.

To interpret the last expression it must be *rationalised*, i.e. j must be eliminated from the denominator. This is done by multiplying both numerator and denominator by $R - jx$.

Then:—

$$\begin{aligned}\dot{I} &= V\,(R - jX)/(R + jX)\,(R - jX)\\ &= (VR - jVX)/(R^2 - j^2X^2)\\ &= (VR - jVX)/(R^2 + X^2)\\ &= VR/(R^2 + X^2) - jVX/(R^2 + X^2)\end{aligned}$$

that is, the current (see Fig. 1.05) consists of:—

$$OA = VR/(R^2 + X^2), \text{ along reference line;}$$
$$\text{and } AB = VX/(R^2 + X^2), \text{ 90° } \textit{behind} \text{ reference line.}$$

The total current, OB, $= \sqrt{OA^2 + AB^2} = \dfrac{V}{R^2 + X^2}\sqrt{R^2 + X^2}$

$= \dfrac{V}{\sqrt{R^2 + X^2}}$; and if ϕ = angle of lag of current, $\tan\phi = AB/OA = X/R$; and $\cos\phi = OA/OB = R/Z$. These agree with the expressions found in the usual way (Volume I., Chapter V., Art. 8), and thus the correctness of the symbolic notation in such cases is confirmed.

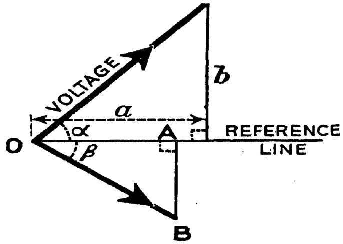


Fig. 1.06.—CURRENT AND VOLTAGE.

Again if the voltage is ahead of the reference line by an angle a and is equal to it can be expressed as $a + jb$; and $\tan a = b/a$ (see Fig. 1.06). Then, using symbolic notation:—

$$\dot{I} = \dot{V}/\dot{Z} = (a + jb)/(R + jX)$$
$$= (a + jb)\,(R - jX)/(R + jX)\,(R - jX)$$
$$= \{AR + bX - j\,(aX - bR)\}/(R^2 + X^2) = \sqrt{OA^2 + AB^2}.$$

Thus:—

$$\text{Total current} = \sqrt{\{(aR + bX)^2 + (aX - bR)^2\}/(R^2 + X^2)}$$
$$= \sqrt{\{(a^2 + b^2)\,(R^2 + X^2)\}/(R^2 + X^2)}$$
$$= \sqrt{(a^2 + b^2)}/\sqrt{R^2 + X^2} = V/Z$$

and if $\angle\, AOB = \beta$, $\tan\beta = AB/OA = (aX - bR)/(aR + bX)$.

The angle of lag of current $(\phi) = \alpha + \beta$. Therefore:—

$$\tan\phi = \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$= \frac{b/a + (aX - bR)/(aR + bX)}{1 - b\,(aX - bR)/a\,(aR + bX)}$$
$$= \frac{b\,(aR + bX) + a\,(aX - bR)}{a\,(aR + bX) - b\,(aX - bR)}$$
$$= \frac{(b^2 + a^2)\,X}{(a^2 + b^2)\,R} = \frac{X}{R}.$$

This proves the accuracy of the symbolic method in the more general case. For a voltage in one of the other quadrants the above still holds provided the correct signs are given to a and b. If the impedance has capacitive reactance the sign of X is made negative. The value of tan ϕ thus becomes negative, indicating that the current leads the voltage.

In 1-phase circuits the voltage can be taken along the reference line, thus simplifying the working. This is not always possible in polyphase circuits, nor in 1-phase circuits when dealing with voltages across parts of the circuit.

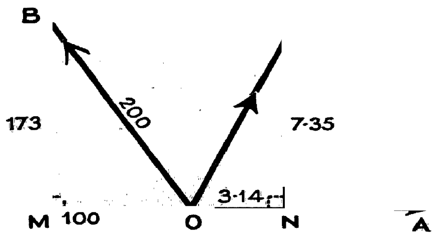


Fig. 1.07.—Current in Leading Phase.

Example 1. *Three-phase voltages of 200 V., R.M.S., to neutral are applied to a balanced Y-connected load, each leg of which consists of 15 ohms resistance and 20 ohms inductive reactance. Find the current in the leading phase in magnitude, and in phase relation to the reference voltage.*

The voltage, OB, applied to the leading phase, leads the reference voltage, OA, by 120°. Hence:—

$$OB = -100 + j.173;$$

$\therefore$ Current in leading phase $= (-100 + j.\,173)/(15 + j.\,20)$
$= (-100 + j.173)\,(15 - j.20)/(15 + j.20)\,(15 - j.20)$
$= \{-1500 + 3460 + j\,(2595 + 2000)\}/(225 + 400)$
$= \{1960 + j.4595\}/625$
$= \{78{\cdot}4 + j.183{\cdot}8\}/25$ (This makes the denominator = Z.)
$= 3{\cdot}14 + j.7{\cdot}35$, which could be reached without the previous intermediate step;

$$\therefore \text{ current} = \sqrt{(3{\cdot}14^2 + 7{\cdot}35^2)} = \sqrt{(9{\cdot}86 + 54{\cdot}02)}$$
$$= \sqrt{63{\cdot}88} = 8{\cdot}0 \text{ A.} = \text{OC}.$$

Current leads OA by $\tan^{-1}(7{\cdot}35/3{\cdot}14) = \tan^{-1} 2{\cdot}34 = 67° = \angle$ COA.

These results can be checked as follows:—Current $= V/Z = 200/25$ (see above) $= 8{\cdot}0$ A. Lag of current behind OB $= \angle$ BOC $= \tan^{-1} 20/15 = \tan^{-1} 1{\cdot}33 = 53°$; $\therefore$ lead of current ahead of OA $= 120° - 53° = 67°$.

4. Application to Admittance

Since admittance (Y) is the current produced by unit p.d., it is the reciprocal of impedance, and so its symbolic expression is $1/(R + jX)$, which on being rationalised becomes
$(R - jX)/\,(R^2 + X^2) = R/(R^2 + X^2) - jX/(R^2 + X^2) = G - jB$;
where G = the conductance, *i.e.* the component in phase with the voltage; and B = the susceptance, *i.e.* the component lagging the voltage, by 90° (see Volume I., Chapter V., Art. 20). Hence the combined admittance, $\overline{Y}$, of two admittances Y_1, Y_2, in parallel $= Y_1 + Y_2 = G_1 + G_2 - j\,(B_1 + B_2)$.

If two impedances, Z_1 and Z_2, are connected in parallel their combined impedance may be expressed in symbolic notation as $Z_1Z_2/(Z_1 + Z_2)$, just as for two resistances in parallel. For the total current produced by applying a voltage V is the vector sum of V/Z_1 and V/Z_2; and this sum is $V\,(Z_1 + Z_2)/Z_1Z_2$. Hence the combined impedance = V/(total current) $= Z_1Z_2\,(Z_1 + Z_2)$.

Similar methods can be applied for three or more impedances in parallel.

Example 2. *Two coils A and B are connected in parallel and a p.d. of* 300 *V.* 50 *cyc. applied to their terminals. Given that at this frequency their impedances are* 8 *ohms and* 11 *ohms respectively, and their resistances* 7 *ohms and* 4 *ohms respectively,* (a) *find the total current and the total power-factor;* (b) *find the total current and power-factor if a condenser of* 120μ*F. capacitance is connected in parallel.*

This is Example 11, Volume I., Chapter V., Art. 20, modified.

$$(a)\quad X_A = \sqrt{\{8^2 - 7^2\}} = 3{\cdot}873 \text{ ohms};$$
$$\therefore\ G_A = 7/8^2 = {\cdot}109$$
$$\text{and } B_A = 3{\cdot}873/8^2 = {\cdot}061$$
$$X_B = \sqrt{\{11^2 - 4^2\}} = 10{\cdot}25;$$
$$\therefore\ G_B = 4/11^2 = {\cdot}033$$
$$B_B = 10{\cdot}25/11^2 = {\cdot}085;$$

$$\text{Total admittance} = \cdot 142 - j.\cdot 146;$$
$$\text{Current} = 300\,\bar{Y} = 42\cdot 6 - j\,43\cdot 8.$$
$$\text{Magnitude} = \sqrt{(42\cdot 6)^2 + (43\cdot 8)^2} = 61\cdot 1 \text{ A.}$$
$$\cos\phi_T = \frac{42\cdot 6}{61\cdot 1} = \cdot 70.$$

Alternatively:—Total impedance $= Z_1Z_2/(Z_1 + Z_2)$

$$\begin{aligned}
&= (7 + j\,3\cdot 87)\,(4 + j\,10\cdot 25)/(11 + j\,14\cdot 12)\\
&= (-\,11\cdot 7 + j\,87\cdot 2)\,(11 - j\,14\cdot 12)/(11^2 + 14\cdot 12^2)\\
&= \{-\,129 + 1230 + j\,(959 + 165)\}/(121 + 199)\\
&= (1101 + j\,1124)/320 = 3\cdot 44 + j\,3\cdot 51;
\end{aligned}$$

$$\begin{aligned}
\therefore\ \text{Current} &= 300/(3\cdot 44 + j.3\cdot 51)\\
&= 300\,(3\cdot 44 - j\,3\cdot 51)/(3\cdot 44^2 + 3\cdot 51^2)\\
&= (1032 - j\,1053)/24\cdot 15 = 42\cdot 7 - j\,43\cdot 6.
\end{aligned}$$

$$\text{Magnitude of current} = \sqrt{(42\cdot 7)^2 + (43\cdot 6)^2} = 61\cdot 1 \text{ A.}$$
$$\cos\phi_T = \frac{42\cdot 7}{61\cdot 1} = \cdot 70.$$

This method is longer in such cases, but is given to show the correctness of the expression for total impedance. The method is useful for series-parallel circuits, and in dealing with general problems.

(*b*) Condenser admittance $= I/V = \omega C = \cdot 038$.

This is all susceptance; and must be written $+\,j\,\cdot 038$, since its effect is opposite to the susceptance of an inductive circuit;

$$\therefore\ \text{Total admittance} = \cdot 142 - j\,\cdot 146 + j\,\cdot 038 = \cdot 142 - j\,\cdot 108;$$
$$\therefore\ \text{Current} = 42\cdot 6 - j\,32\cdot 4$$
$$\text{Magnitude} = \sqrt{42\cdot 6^2 + 32\cdot 4^2} = 53\cdot 5$$
$$\cos\phi_T = \frac{42\cdot 6}{53\cdot 5} = 0\cdot 80 \text{ (lagging).}$$

5. Application to Power

If the applied voltage, V, is expressed in the symbolic form, $a + jb$, and makes an angle α with the reference line (see Fig. 1.08), $\cos\alpha = a/\sqrt{a^2 + b^2}$, and $\sin\alpha = b/\sqrt{a^2 + b^2}$. Similarly if the current (I) $= c + jd$, and makes an angle β with the reference line, $\cos\beta = c/\sqrt{c^2 + d^2}$, and $\sin\beta = d/\sqrt{c^2 + d^2}$. If ϕ is the lag of the current behind the voltage

$$\phi = \alpha - \beta; \quad \therefore\ \cos\phi = \cos\alpha\cos\beta + \sin\alpha\sin\beta;$$
$$\therefore\ \cos\phi = (ac + bd)/\sqrt{(a^2 + b^2)\,(c^2 + d^2)}.$$

The magnitude of the voltage is $\sqrt{a^2 + b^2}$, and that of the current is Hence:—

$$\text{Power} = VI\cos\phi = ac + bd:$$

that is, the power is obtained by multiplying separately the "real" and the "imaginary" components of voltage and current and adding the products.

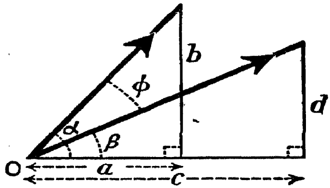


Fig. 1.08.

The physical significance of this is that since a current $\frac{1}{4}$-period out of phase with a voltage gives zero average power, the "imaginary" part of the current (d) gives zero power with the "real" part of the voltage (a); and the same applies to c and b.

If any of the quantities a, b, c, d are negative this must be taken into account. *E.g.* if d is negative and equal to $-d_1$, the power is $ac - bd_1$.

The correctness of this can be seen by noting that this signifies that the current vector is below the reference line at an angle β. The angle of lag is thus increased and the power reduced. The value of sin β becomes $-d_1/\sqrt{c^2 + d_1^2}$, and so the power becomes $ac - bd_1$ as stated.

Example 3. *Find the power in the leading phase in Example* 1 (page 4).

$$V = -100 + j.173 \text{ volts}$$
$$I = 3{\cdot}14 + j.7{\cdot}35 \text{ amperes;}$$
$$\therefore \text{ Power} = -314 + 1272 = 958 \text{ watts.}$$

This can be checked by calculating the value of I^2R, which is 63·88 × 15 = 958 watts.

6. Polar Form

For some purposes it is more convenient to express a vector by means of its length, r, and the angle, θ, that it makes with the reference line. If this is done in symbolic notation the following shows the result:—

$$\begin{aligned} a + jb &= r \cos \theta + j.r \sin \theta \\ &= r.(e^{j\theta} + e^{-j\theta})/2 + j.r\,(e^{j\theta} - e^{-j\theta})/2j \\ &= r.e^{j\theta}. \end{aligned}$$

The final expression is the symbolic polar form of the vector.

It has the advantage that the phase difference between two vectors is given by the difference between their respective values of θ. Thus if an alternating voltage, expressed as $Ve^{j\theta}$, is applied to an impedance Z, the resulting current is $(V/Z)e^{j(\theta-\phi)}$.

It follows that the power due to a voltage $Ve^{j\theta}$ and a current $Ie^{j\theta'}$ is VI cos $(\theta - \theta')$. This can, however, be obtained by calculation as follows:—

$$Ve^{j\theta} = V \cos \theta + j.V \sin \theta$$
$$Ie^{j\theta'} = I \cos \theta' + j.I \sin \theta'.$$

Hence (see Art. 5) the power is given by:—

$$\text{Power} = VI \cos\theta \cos\theta' + VI \sin\theta \sin\theta' = VI \cos(\theta - \theta').$$

For the addition of vectors the $a + jb$ form is more convenient, and when this is required with vectors in the radial form it is simplest to convert them into the former before adding. When the opposite conversion is required, viz. from $a + jb$ into the radial form it is done by using the relationships $r = \sqrt{a^2 + b^2}$; and $\tan\theta = b/a$.

7. Product and Quotient of Vectors

If a vector $a + jb$ has been expressed in the polar form $re^{j\theta}$, often it is written as $r\,\underline{|\theta}$.

Thus the vector $a - jb$ may be written $re^{-j\theta}$, or $r\,\underline{|-\theta}$, or $r\,\overline{|\theta}$.

Then if $\dot{r}_1 = r_1\,\underline{|\theta_1}$, and $\dot{r}_2 = r_2\,\underline{|\theta_2}$, the product $\dot{r}_1 \times \dot{r}_2 = r_1e^{j\theta_1} \times r_2e^{j\theta_2} = r_1r_2e^{j(\theta_1+\theta_2)} = r_1r_2\,\underline{|(\theta_1+\theta_2)}$, *i.e.* the product is formed by multiplying the magnitudes, and adding the arguments (*i.e.* the angles with the reference line).

A fuller proof of the truth of this is obtained by writing:—

$$\dot{r}_1 = r_1 (\cos\theta_1 + j \sin\theta_1)$$

$$\text{and } \dot{r}_2 = r_2 (\cos\theta_2 + j \sin\theta_2)$$

$$\begin{aligned}\therefore \dot{r}_1 \times \dot{r}_2 &= r_1r_2 \{\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \\ &\qquad + j (\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2)\} \\ &= r_1r_2 \{\cos(\theta_1 + \theta_2) + j \sin(\theta_1 + \theta_2)\} \\ &= r_1r_2\,\underline{|(\theta_1+\theta_2)}.\end{aligned}$$

This product is sometimes called the "cross" product, to distinguish it from the product used in obtaining the power, viz. $r_1r_2 \cos(\theta_1 - \theta_2)$ (see Art. 6). The latter is called the "dot" product. It should be noted that the cross product is itself a vector, whereas the dot product is merely a magnitude.

The rule for the quotient obtained by dividing one vector by another can be proved from the cross product rule as follows.

$$\begin{aligned} r_2\,\underline{|\theta_2} \times (r_1/r_2)\,\underline{|\theta_1-\theta_2} &= (r_2 \times r_1/r_2)\,\underline{|\theta_2+\theta_1-\theta_2} \\ &= r_1\,\underline{|\theta_1}; \end{aligned}$$

$$\therefore \dot{r}_1/\dot{r}_2 = r_1\,\underline{|\theta_1}/r_2\,\underline{|\theta_2} = (r_1/r_2)\,\underline{|\theta_1-\theta_2},$$

i.e. divide the magnitudes, and subtract the argument of the divisor from that of the dividend.

Example 4. *Work out Example 2 (a) (page 5) by the use of the polar form.*

$$\dot{Z}_A = 7 + j.3{\cdot}87$$

$$\dot{Z}_B = 4 + j.10{\cdot}25;$$

$$\therefore\ \dot{Z}_A + \dot{Z}_B = 11 + j.14{\cdot}12 = \sqrt{11^2 + (14{\cdot}12)^2}\,\underline{|\tan^{-1}(14{\cdot}12/11)}$$

$$= 17{\cdot}89\,\underline{|52{\cdot}1^\circ}$$

$$\text{Again } \dot{Z}_A = 8\,\underline{|\cos^{-1}(7/8)} = 8\,\underline{|29^\circ{\cdot}0}$$

$$\text{and } \dot{Z}_B = 11\,\underline{|\cos^{-1}(4/11)} = 11\,\underline{|68^\circ{\cdot}7};$$

$$\therefore\ \dot{Z}_A \times \dot{Z}_B = 88\,\underline{|97^\circ{\cdot}7};$$

$$\therefore\ \text{Joint impedance} = \frac{\dot{Z}_A \times \dot{Z}_B}{\dot{Z}_A + \dot{Z}_B} = \frac{88\,\underline{|97^\circ{\cdot}7}}{17{\cdot}89\,\underline{|52{\cdot}1^\circ}}$$

$$= \frac{88}{17{\cdot}89}\,\underline{|97^\circ{\cdot}7 - 52^\circ{\cdot}1} = 4{\cdot}92\,\underline{|45^\circ{\cdot}6}$$

Hence if the voltage is taken as the reference line:—

$$I = \frac{300\,\underline{|0}}{4{\cdot}92\,\underline{|45^\circ{\cdot}6}} = 61{\cdot}0\,\underline{|-45^\circ{\cdot}6} = 61{\cdot}0\,\overline{|\cos^{-1}0{\cdot}70};$$

$$\therefore\ \text{Total current} = 61{\cdot}0 \text{ A.}$$

$$\text{Combined power-factor} = \cos 45^\circ{\cdot}6 = 0{\cdot}70 \text{ lagging.}$$

8. Application to Transformers in Parallel.

If two transformers with the same turns ratio are connected in parallel both on the primary side and on the secondary side, the problem of how they will share a given load can be solved readily by the use of symbolic notation.

Since their behaviour can be represented by means of equivalent resistances and reactances (see Vol. I., Fig. 17.09 *d*) the problem is reduced to one of parallel circuits. The drop in transformer A must be equal to that in transformer B, hence $\dot{I}_A\dot{Z}_A = \dot{I}_B\dot{Z}_B$, where $\dot{Z}_A$, $\dot{Z}_B$ are the respective vectorial equivalent impedances. Moreover $(\dot{I}_A + \dot{I}_B)$ must equal the total load current. From these two equations the values of the transformer currents can be found. And if the secondary terminal voltage is taken as the reference vector the power components of these currents are obtained incidentally (see Example 5*a*).

Moreover if the turns ratios of the two transformers differ slightly the problem is still soluble by making the following additional calculation. On no load the circulating current is $(V_B - V_A)/(\dot{Z}_A + \dot{Z}_B)$; where V_A and V_B are the no-load secondary voltages of the two transformers when not parallelled on the secondary side. This circulating current is then added vectorially to the load current of B as determined by the previous method. Similarly it is subtracted from the load current of A (see Example 5 *b*).

Example 5. *Two 400 kVA transformers have the following equivalents :—A, 2·0% resistance, 4·7% reactance ; B, 1·6% resistance, 7·0% reactance. How will they share a load of 600 kVA, 0·8 lagging power factor, (a) if their open circuit secondary voltages are equal ; (b) if that of B is 1% higher than that of A ? Find the regulation in each case.*

Note that currents are expressed in equivalent *k*VA, not in amperes.

$$(a)\ \dot{I}_A \times (2{\cdot}0 + j.4{\cdot}7) = \dot{I}_B \times (1{\cdot}6 + j.7{\cdot}0);$$
$$\therefore\ \dot{I}_A = \dot{I}_B\,(1{\cdot}6 + j.7{\cdot}0)/(2{\cdot}0 + j.4{\cdot}7)$$
$$= \dot{I}_B\,(1{\cdot}38 + j.0{\cdot}248);$$
$$\therefore\ \dot{I}_A + \dot{I}_B = \dot{I}_B\,(2{\cdot}38 + j.0{\cdot}248) = 600 \times 0{\cdot}8 - j.600 \times 0{\cdot}6;$$
$$\therefore\ \dot{I}_B = (480 - j.360)/(2{\cdot}38 + j.0{\cdot}248)$$
$$= 184 - j.170,$$

*i.e. k*VA of B $= \sqrt{\{(184)^2 + (170)^2\}} = 250;$

*k*W of B = 184;

power factor of B = 184/250 = 0·736

$\phi_B = 42{\cdot}6°.$

$\dot{I}_A$ can be found from $(184 - j.170) \times (1{\cdot}38 + j.0{\cdot}248)$,

or from $480 - j.360 - (184 - j.170)$,

either of which gives $I_A = 296 - j.190$,

*i.e. k*VA of A $= \sqrt{\{(296)^2 + (190)/^2\}} = 352;$

*k*W of A = 296;

power factor of A = 296/352 = 0·841,

$\phi_A = 32{\cdot}8°.$

Regulation of A $= \tfrac{352}{400}\,(2{\cdot}0 \times 0{\cdot}841 + 4{\cdot}7 \times 0{\cdot}542)\%;$

$= (2{\cdot}0 \times 296 + 4{\cdot}7 \times 190)/400 = 3{\cdot}71\%$ (see Vol. I., Chap. XVII., Art. 13).

Regulation of B $= \tfrac{250}{400}\,(1{\cdot}6 \times 0{\cdot}736 + 7{\cdot}0 \times 0{\cdot}679)\%;$

$= (1{\cdot}6 \times 184 + 7{\cdot}0 \times 170)/400 = 3{\cdot}71\%.$

(*b*) Circulating current in *k*VA $= \dfrac{0{\cdot}01 \times 400}{3{\cdot}6 + j.11{\cdot}7} \times 100 = \dfrac{14{\cdot}4 - j.46{\cdot}8}{1{\cdot}50}$

$= 9{\cdot}6 - j\,31{\cdot}2;$

$\therefore\ \dot{I}_A \doteqdot 286 - j.159,$

*i.e. k*VA of A = 327;

power factor of A = 286/327 = 0·874;

$\phi_A = 29{\cdot}1°;$

and $\dot{I}_B \doteqdot 194 - j.201$

i.e. kVA of B = 279;

power factor of B = 194/279 = 0·695;

$\phi_B = 46{\cdot}0°.$

Regulation of A $= (2{\cdot}0 \times 286 + 4{\cdot}7 \times 159)/400\%$

$= 3{\cdot}30\%.$

Regulation of B $= (1{\cdot}6 \times 194 + 7{\cdot}0 \times 201)/400\%$

$= 4{\cdot}29\%.$

These differ by nearly the 1% by which the open-circuit voltage of B exceeds that of A (the slight error being due to approximation).

When the two are paralleled on no-load the regulation of

B $= (1{\cdot}6 \times 9{\cdot}6 + 7{\cdot}0 \times 31{\cdot}2)/400\%$

$= 0{\cdot}58\%.$

Hence the regulation due to the 600 *k*VA load

$$= 4{\cdot}29 - 0{\cdot}58 = 3{\cdot}71\%,$$

i.e. the same value as in (*a*).

QUESTIONS ON CHAPTER I.

1. When $100 + j.50$ volts are applied to a circuit a current $20 - j.30$ amperes flows. Find the volt-amperes, the power, and the power-factor.

2. A circuit consisting of two impedances in series takes 15 A. at a leading power-factor of 0·8 when connected to a supply of 200 V., 60 cycles per sec. If one impedance is $8 + j.15$ find the values of the resistance and capacitance (in series) forming the other impedance.

3. A resistance of 20 ohms, and a condenser of 120 microfarads capacitance are in parallel with each other and with an unknown impedance. When supplied at 200 V., 50 cycles per sec. the total current is 11·6 A. at a leading power-factor of 0·975. Find by the symbolic method the resistance and inductance of the unknown impedance.

4. Two impedances, $5 + j.2$, and $6 - j.4$, are connected in parallel; and an impedance $1 + j.3$ is connected in series with the combination. If an alternating supply at 200 V. is connected to the arrangement find the total current and the currents in the parallel branches; and the power-factor of the total current.

5. A circuit consists of an impedance $10 + j.15$ in series with an unknown impedance. If an A.C. supply at 200 V. 50 cyc. sends a current of 10 A. through them, with a leading power-factor of 0·8, find the unknown impedance completely.

If the same two impedances were connected in parallel to the same supply, what would be the magnitude and power-factor of the total current.

6. Find the resultant of the following four e.m.f.s acting in series:— $10\,\underline{|0}$; $15\,\underline{|60^\circ}$; $12\,\underline{|90^\circ}$; $12\,\underline{|-45^\circ}$: expressing the result in the same form.

Find the new value of the resultant when the third of these e.m.f.s is reversed.

7. Two transformers, A and B, connected in parallel, supply a load of 230 volts, 450 amperes, 0·6 power factor. Find the current and power factor of each transformer and the regulation, (*a*) if their open circuit voltages are equal; (*b*) if that of A is 2 volts higher.

The equivalent impedances referred to secondaries are made up of:—For A, 0·011 ohm resistance, 0·027 ohm reactance; for B, 0·021 ohm resistance, 0·016 ohm reactance.

CHAPTER II

HARMONIC ANALYSIS

1. Fourier's Series

A quantity y is called a **periodic function** of another x if the values assumed by y are repeated in the same order every time the value of x increases by a fixed quantity (P) called the period (cf. Volume I., Chapter V., Art. 1). Expressed in symbols the above becomes—let y_1 be the value of y when $x = x_1$, where x_1 may have *any* value between zero and P. Then

$$y = y_1, \quad \text{when} \quad x = x_1 + \text{P}, \quad x_1 + 2P, \quad x_1 + 3\text{P}, \text{ etc.}$$

Thus any alternating current or voltage of constant R.M.S. value and frequency and wave-form is a periodic function of the time. The power corresponding to such a voltage and current is likewise a periodic function of the time. The difference between this and the case of a current or a voltage is that the latter have zero mean value when averaged over a period, whereas the former nearly always has a mean value differing from zero.

Fourier showed that any periodic function could be expressed as the sum of a number of sine and cosine terms together with a constant, *i.e.* in the form

$$\begin{aligned} y = a_0 &+ a_1 \sin\theta + a_2 \sin 2\theta + a_3 \sin 3\theta + \text{etc.}, \\ &+ b_1 \cos\theta + b_2 \cos 2\theta + b_3 \cos 3\theta + \text{etc.}, \end{aligned}$$

where

$$\theta = 2\pi x/\text{P}.$$

This is known as **Fourier's Series.** It satisfies the condition for a periodic function stated above. And it can be shown that by taking sufficient terms the values of the original function can be represented within any desired degree of approximation (see further Example 1).

a_0 is the average value of y taken over a period (or a number of whole periods). Consequently its value is zero for an alternating current or voltage, and is equal to the mean power (*i.e.* the reading of an accurate wattmeter) in an A.C. power wave.

The terms $(a_1 \sin\theta + b_1 \cos\theta)$ form the **fundamental** of the periodic function. This name is given to them because their period is the same as that of the original function (y). The terms $(a_2 \sin 2\theta + b_2 \cos 2\theta)$ form the second **harmonic,** the fundamental being the first harmonic. Similarly $(a_3 \sin 3\theta + b_3 \cos 3\theta)$ is the third harmonic, etc.

The two terms of any harmonic can be combined into a single term since—

$$a_n \sin n\theta + b_n \cos n\theta = c_n \sin (n\theta + \phi_n),$$

where $$c_n = \sqrt{a_n^2 + b_n^2},$$

and $$\tan \phi_n = b_n/a_n.$$

The truth of this can be proved readily by using the crank diagram (Volume I., Chapter V., Art. 5) to combine the two terms.

The combined form shows more clearly how the function (y) is built up, for c_n is the amplitude of the nth harmonic, and ϕ_n is its phase angle (lead). The lead of the nth harmonic ahead of the fundamental is $(\phi_n - \phi_1)$. But the separated form is more suitable for purposes of analysis.

The second harmonic goes through two complete cycles in the period of the function, *i.e.* its frequency is double that of the function.

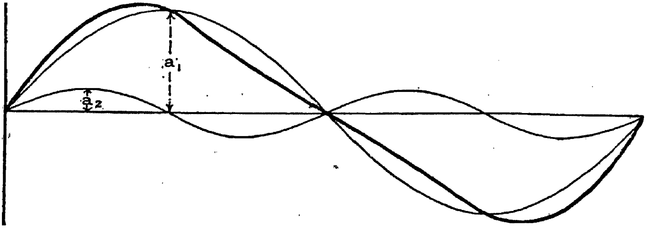


Fig. 2.01.—WAVE WITH SECOND HARMONIC.

Similarly the nth harmonic has a frequency n times the original frequency, and so n times that of the fundamental.

2. Absence of Even Harmonics in A.C. Waves

The negative half of an alternating current or voltage wave is the same as the positive half reversed, the negative values occurring in the same order as the positive ones (see Volume I., Chapter V., Art. 1). Thus the values at any two instants half a period apart are equal but opposite. It follows that no even harmonics can be present since these give equal values of *the same sign* at instants half the main period apart. This is illustrated in Fig. 2.01 which shows a wave composed of a fundamental, $a_1 \sin \theta$, and a second harmonic, $a_2 \sin 2\theta$. It will be seen that the values in the negative half wave occur in the *reverse order* to those in the positive half. The same thing happens when the fourth, or any other even harmonic, or a combination of several such, is present.

On the other hand, any odd harmonic gives the required reversed values at half-period intervals, and so any of these may be present in an alternating current or voltage wave. An example is given in Fig. 2.02 in which the wave consists of a fundamental, $a_1 \sin \theta$, and a third harmonic, $b_3 \cos 3\theta$. The values in the negative half now occur in the same order as in the positive half.

The proof of the above general statements lies in the identities:—

$$\sin 2\,(\theta + \pi) = \sin 2\theta; \text{ and } \cos 2\,(\theta + \pi) = \cos 2\theta,$$

$$\sin 2n\,(\theta + \pi) = \sin 2n\theta; \text{ and } \cos 2n\,(\theta + \pi) = \cos 2n\theta,$$

i.e. no change of sign for even harmonics.

$$\sin 3\,(\theta + \pi) = -\sin 3\theta; \text{ and } \cos 3\,(\theta + \pi) = -\cos 3\theta,$$

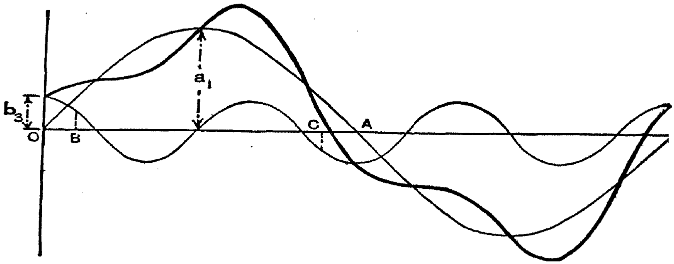


Fig. 2.02.—WAVE WITH THIRD HARMONIC.

$$\sin (2n + 1)\,(\theta + \pi) = -\sin (2n + 1)\,\theta; \text{ and } \cos (2n + 1)\,(\theta + \pi) = -\cos (2n + 1)\,\theta,$$

i.e. reversal of sign for odd harmonics.

3. Symmetrical Waves

In many cases alternating current and voltage waves have each half symmetrical. In such cases the Fourier series can be simplified further. If the time-zero is taken at the instant when the current or voltage is zero, all the cosine terms can be omitted and only the sine terms retained. For instance, the waves in Fig. 2.03 are made up of a fundamental, $a_1 \sin \theta$, and a third harmonic, $\pm a_3 \sin 3\theta$; the positive sign results in a flat-topped wave, and the negative sign in a peaked wave. A comparison of these with Fig. 2.02 shows how the cosine term gives an unsymmetrical half-wave, and the sine terms symmetrical ones. The negative halves are of the same shapes as the positive halves.

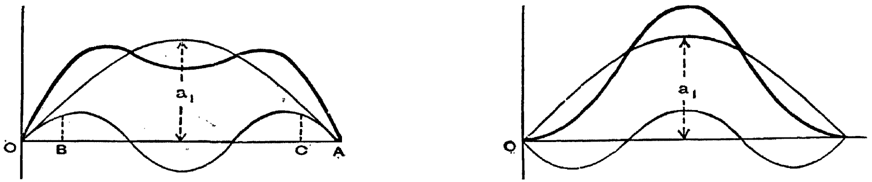
The same result occurs with any odd harmonic since

$$\sin (2n + 1)(\pi - \theta) = \sin (2n + 1)\theta,$$
$$\cos (2n + 1)(\pi - \theta) = -\cos (2n + 1)\theta,$$

i.e. if in the half period (OA, Fig. 2.03*a*) any instant such as B is taken, and another instant, C, as much before the half-period as B is after the zero (so that CA = OB), the ordinates at B and C will be equal for any odd-numbered sine term. But for any odd-numbered cosine term these ordinates will be equal but opposite, as in the example shown in Fig. 2.02.

Further examples of symmetrical half-waves are shown in Fig. 2.04: the negative halves are of the same shapes.

It will be seen (Fig. 2.04 *a* and *b*) that the effect of a fifth harmonic is opposite to that of a third harmonic as regards the flat-topped or peaked nature of the resultant wave. The effect of a seventh harmonic is similar to that of a third harmonic in this respect. Consequently a positive fifth and a negative seventh combine to make the resultant wave peaked.



(*a*) POSITIVE 3RD HARMONIC. (*b*) NEGATIVE 3RD HARMONIC.

Fig. 2.03.—SYMMETICAL HALF-WAVES.

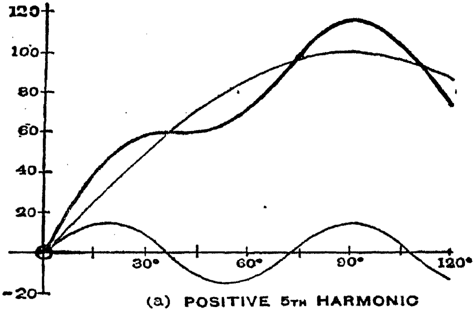
Further examples of such waves are given in Fig. 2.05 at the end of Example 1 in Art. 4.

4. Analysis by Integration

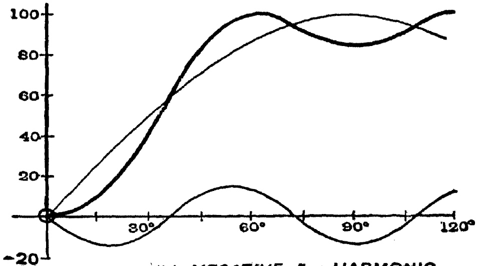
When the wave form can be expressed by a simple equation, the best method of finding the amplitudes and phases of the harmonics is by integration in the following manner. The value of a_0 is found by integrating the function y over a period and dividing by the period,

i.e.
$$a_o = \frac{1}{P}\int_0^P y\,dx.$$

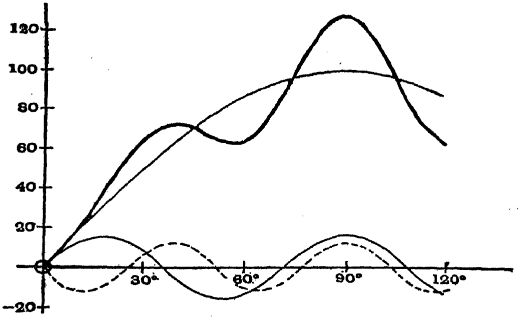
This follows from the fact that the integral of a sine or a cosine over any number of complete periods is zero.



(a) POSITIVE 5TH HARMONIC



(b) NEGATIVE 5TH HARMONIC



(*c*) POSITIVE 5TH AND NEGATIVE 7TH HARMONICS.

Fig. 2.04.—WAVES WITH 5TH AND 7TH HARMONICS.

- Fundamental and 5th harmonics. - - - - 7th harmonic. —— Resultants.

To find the value of any other term use is made of the following definite integrals:—

(*a*) $\int_0^{2\pi} \sin n\theta \sin m\theta d\theta$

$$= \int_0^{2\pi} \tfrac{1}{2} (\cos \overline{n - m}\theta - \cos \overline{n + m}\theta)\, d\theta$$

$$= \frac{1}{2}\left[\frac{\sin \overline{n - m}\theta}{n - m} - \frac{\sin \overline{n + m}\theta}{n + m}\right]_0^{2\pi} = 0.$$

(*b*) $\int_0^{2\pi} \cos n\theta \sin m\theta d\theta$

$$= \int_0^{2\pi} \tfrac{1}{2} (\sin \overline{n + m}\theta - \sin \overline{n - m}\theta)\, d\theta$$

$$= \frac{1}{2}\left[- \frac{\cos \overline{n + m}\theta}{n + m} + \frac{\cos \overline{n - m}\theta}{n - m}\right]_0^{2\pi} = 0.$$

(*c*) $\int_0^{2\pi} \sin^2 m\theta d\theta = \int_0^{2\pi} \frac{1 - \cos 2m\theta}{2} d\theta$

$$= \frac{1}{2}\left[\theta - \frac{\sin 2m\theta}{2m}\right]_0^{2\pi} = \tfrac{1}{2}\,[2\pi - 0] = \pi.$$

(*d*) $\int_0^{2\pi} \cos^2 m\theta . d\theta = \int_0^{2\pi} \frac{1 + \cos 2m\theta}{2} d\theta$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2m\theta}{2m}\right]_0^{2\pi} = \pi.$$

(*e*) $\int_0^{2\pi} \sin m\theta . \cos m\theta . d\theta = \int_0^{2\pi} \frac{\sin 2m\theta}{2} d\theta$

$$= \frac{1}{2}\left[\frac{- \cos 2m\theta}{2m}\right]_0^{2\pi} = 0.$$

If every term in Fourier's series (see Art. 1) is multiplied by $\sin m\theta$ and integrated over a period the result is, therefore, zero, except in the case of the term $a_m \sin m\theta$, which gives πa_m. Hence:—

$$a_m = \frac{1}{\pi}\int_0^{2\pi} y \sin m\theta d\theta.$$

And the coefficients of all the sine terms can be found in the same way if the integral, can be evaluated.

Similarly, by multiplying each term by $\cos m\theta$ and integrating it is found that

$$b_m = \frac{1}{\pi}\int_0^{2\pi} y \cos m\theta d\theta.$$

It may be noted that for current and voltage waves the integral may be taken between the limits o and π and multiplied by $2/\pi$, since this gives the same result as the above when m is odd. The same is true for a power wave, since its period is half that of the corresponding voltage and current.

Example 1. *Analyse a triangular wave of voltage, of maximum value* V. The equation of this wave may be written:—

$$y = 2V\theta/\pi, \qquad \text{from } \theta = 0 \text{ to } \theta = \pi/2,$$

and

$$y = 2V(\pi - \theta)/\pi, \text{ from } \theta = \frac{\pi}{2} \text{ to } \theta = 3\pi/2.$$

(The equation for the fourth quadrant is unnecessary.)

The value of the coefficient (a_1) of the fundamental is, therefore, given by:

$$\pi a_1 : \qquad (2V\theta \sin \theta/\pi)\, d\theta + \int_{\pi}^{\pi} (2V\overline{\pi - \theta} \sin \theta/\pi)\, d\theta$$

The 2 being introduced because the integration extends over only half a period.

$$\therefore \pi a_1/2 = (2V/\pi)\left\{\Big[- \theta \cos \theta + \sin \theta\Big]_{0}^{\frac{\pi}{2}} + \Big[(\pi - \theta) \cos \pi - \sin \theta\Big]_{\frac{\pi}{2}}^{\pi}\right\}$$

$$= (2V/\pi)\{1 + [0 - (-1)]\}$$
$$= 4V/\pi;$$
$$\therefore a_1 = V.(8/\pi^2) = 0{\cdot}811V.$$

The second half of the integration gives the same result as the first half, which considerations of symmetry show is necessarily true. The same applies to any odd harmonic with this wave, and with any wave having symmetrical half-waves. Hence:—

$$\pi a_{2n+1} = 4\int_{0}^{\frac{\pi}{2}} (2V\theta \sin (2n + 1)\,\theta/\pi)\, d\theta$$

$$(8\pi V/)\left[- \theta \frac{\cos (2n + 1)\,\theta}{2n + 1} + \frac{\sin (2n + 1)\,\theta}{(2n + 1)^2}\right]$$

$$= (-1)^n\, 8V/\pi\, (2n + 1)^2;$$

$$\therefore a_{2n+1} = (-1)^n\, 8V/\pi^2\, (2n + 1)^2$$

Whence $a_3 = - V\,(8/9\pi^2) = - 0{\cdot}090V,$

$a_5 = + V\,(8/25\pi^2) = 0{\cdot}0324V,$

$a_7 = - V\,(8/49\pi^2) = - 0{\cdot}0165V,$ etc.,

i.e. the wave form is given by the equation

$$y = 8V/\pi^2\, \{\sin \theta - \tfrac{1}{9} \sin 3\theta + \tfrac{1}{25} \sin 5\theta - \tfrac{1}{49} \sin 7\theta + \text{etc.}\}.$$

In Fig. 2.05 are shown the original triangular wave, the fundamental, 3rd, 5th, and 7th harmonics, and the sums of the first two, the first three, and all four of these harmonics. It will be seen that each additional harmonic brings the resultant closer to the original wave.

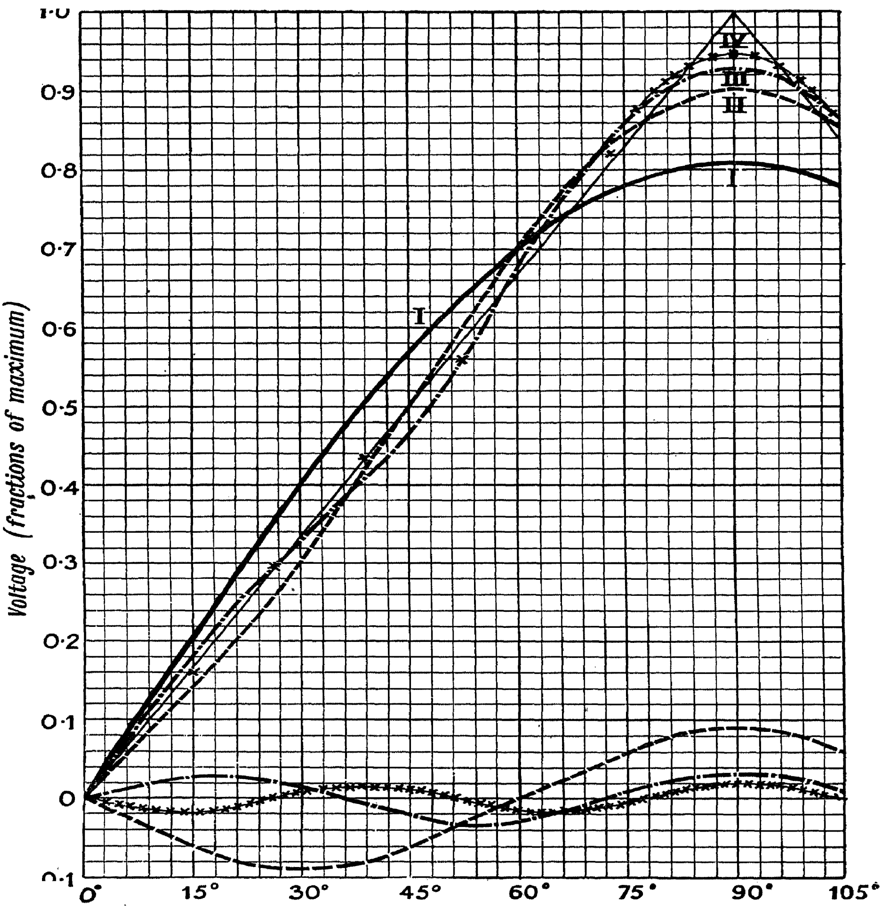


Fig. 2.05.—HARMONICS OF TRIANGULAR WAVE.

—— Original wave and fundamental (I).
- - - Third harmonic and sum (II) of I and 3rd harmonic.
— • — • — Fifth harmonic and sum (III) of 1st, 3rd and 5th harmonics.
x-x-x-x-x Seventh harmonic and sum (IV of 1st, 3rd, 5th and 7th harmonics.

5. Perry's Method

When a wave is given by a curve of which the equation is not known the integrations cannot be effected by the exact mathematical method.

Perry's method consists in performing the integrations by an approximate method. Ordinates are measured at a number of equidistant values of θ over half a period. To obtain any particular harmonic's coefficient the ordinates are multiplied by the corresponding values of the sine (or cosine) of $n\theta$ (see Example 2). The mean value of this product is obtained, and *twice* this gives the required coefficient.

The reason for doubling the mean value is that, as shown in Art. 4, the coefficient is obtained by dividing the integral ($\int y \sin m\theta d\theta$ or $\int y \cos m\theta d\theta$) by *half* the range over which the integral is taken. It depends on the fact that the mean square of either a sine or a cosine taken over any number of complete periods is $\frac{1}{2}$.

Example 2. *Find the fundamental, and the third and fifth harmonics of the following alternating voltage :—*

$\theta = 0°, \quad 15°, \quad 30°, \quad 45°, \quad 60°, \quad 75°, \quad 90°, \quad 105°, \quad 120°, \quad 135°,$

$v = \quad 0, \quad 46, \quad 114, \quad 169, \quad 173, \quad 174, \quad 183, \quad 174, \quad 173, \quad 169,$ etc.

he half-wave being symmetrical.

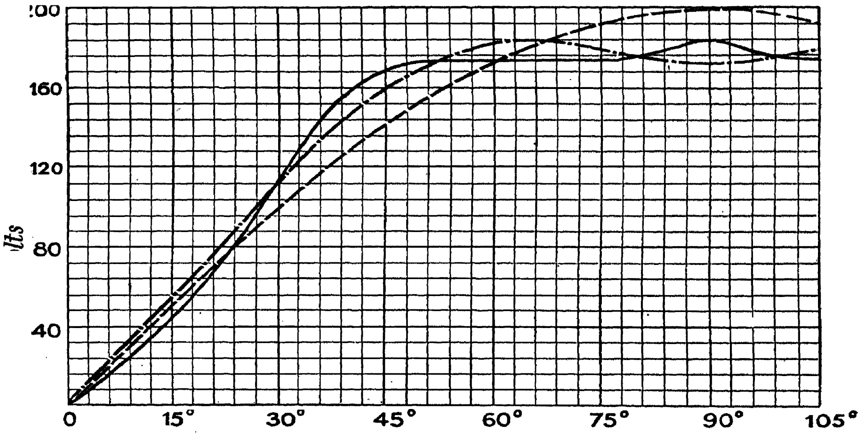


Fig. 2.06.—ANALYSIS BY PERRY'S METHOD.

· Original. —·—— Fundamental. —•—•— Fundamental 3rd and 5th harmonics.

The wave is plotted in Fig. 2.06. Owing to symmetry the integration need extend over only quarter of a period, the end ordinate (90°) being halved; and only sine terms occur.

To find the fundamental:

$$\sin\theta = 0,\ \cdot 259,\ \cdot 500,\ \cdot 707,\ \cdot 866,\ \cdot 966,\ 1\ (\times \tfrac{1}{2}).$$
$$v\sin\theta = 0,\ 11\cdot 6,\ 57\cdot 0,\ 119\cdot 5,\ 149\cdot 8,\ 168\cdot 1,\ 91\cdot 5.$$

Mean value of $v \sin\theta = (597\cdot 5)/6 = 99\cdot 6$;

$$\therefore \text{ amplitude of the fundamental } = 99\cdot 6 \times 2 = 199\cdot 2.$$

This is drawn in Fig. 2.06, the difference between it and the original wave showing the extent of the higher harmonics.

$$\sin 3\theta = 0,\ \cdot 707,\ 1\cdot 000,\ \cdot 707,\ 0,\ -\cdot 707,\ -1\ (\times \tfrac{1}{2}).$$

The arithmetic can be simplified in this case by adding together the appropriate values of v before multiplying.

Thus:—$6 \times$ mean value of $v \sin 3\theta = \cdot 707\,(46 + 169 - 174) + 114 - 91\cdot 5$
$$= 29\cdot 0 + 114 - 91\cdot 5 = 51\cdot 5;$$
$$\therefore \text{ Amplitude of the 3rd harmonic } = (51\cdot 5/6) \times 2 = 17\cdot 2.$$

To find the fifth harmonic:

$$\sin 5\theta = 0,\ \cdot 966,\ \cdot 500,\ -\cdot 707,\ -\cdot 866,\ \cdot 259,\ 1\ (\times \tfrac{1}{2}),$$
$$v \sin 5\theta = 0,\ 44\cdot 4,\ 57\cdot 0,\ -119\cdot 3,\ -149\cdot 8,\ 45\cdot 1,\ 91\cdot 5.$$
$$\therefore \text{ Amplitude of the 5th harmonic } = (238\cdot 0 - 269\cdot 1) \times 2/6 = -10\cdot 4.$$

The sum of the fundamental, third, and fifth harmonics is drawn in Fig. 2.06. It will be seen that the original wave contains still higher harmonics of considerable amplitude.

If the above method is applied to determine the seventh harmonic it will be found that its amplitude is $-9\cdot 5$, and that the residual harmonics are small.

The above wave thus is represented closely by—

$$v = 199\cdot 2 \sin\theta + 17\cdot 2 \sin 3\theta - 10\cdot 4 \sin 5\theta - 9\cdot 5 \sin 7\theta.$$

When $\theta = 90°$ this gives $v = 181\cdot 1$ instead of 183, and at other points the discrepancy is less.

When cosine terms occur they can be found by the same method. An alternative is to separate the cosine terms from the sine terms and then to determine the harmonics of each set separately.

The separation is effected by placing the ordinates of the second quadrant in the reverse order against those of the first quadrant. Then half the sum gives the sine terms, and half the difference gives the cosine terms.

Example 3. *Separate the sine and cosine terms in the alternating voltage wave given by :—*

$\theta =$ 0, 15°, 30°, 40°, 60°, 75°, 90°, 105°, 120°, 135°, 150°, 165°, 180°
$v =$ 60, 128, 198, 255, 283, 277, 240, 187, 133, 85, 42, − 4, − 60 volts.

Reversing the second quadrant and placing it under the first gives:—

$\theta =$	0,	15°,	30°,	45°,	60°,	75°,	90°
	60,	128,	198,	255,	283,	277,	240 volts
	−60,	−4,	42,	85,	133,	187,	volts
½ Sum =	0,	62,	120,	170,	208,	232,	240 volts.
½ Difference =	60,	66,	78,	85,	75,	45,	0 volts.

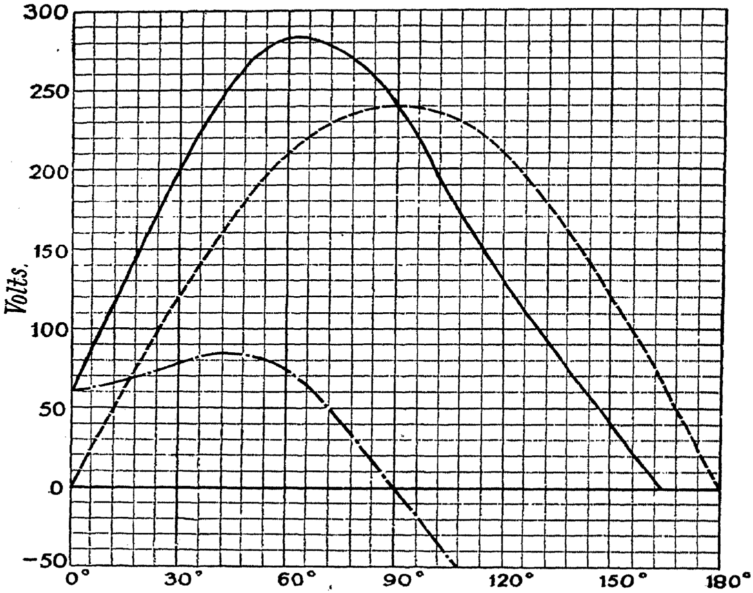


Fig. 2.07.—SEPARATION OF SINE AND COSINE TERMS.

—— Complete half-wave. — — — Sine terms. — · — · — Cosine terms.

The original wave and the sine and cosine components are plotted in Fig. 2·07. It will be seen that the sine portion is almost purely a sine wave of amplitude 240, whereas the cosine part has a strong third harmonic.

The complete wave is given very closely by :—

$$v = 240 \sin \theta + 90 \cos \theta - 30 \cos 3\theta \quad \text{(cf. Example 4)}$$
$$= 256 \sin (\theta + \phi) - 30 \cos 3\theta;$$
$$\text{where} \quad \phi = \tan^{-1} (90/240) = 20{\cdot}6^\circ \text{ (see Art. 1)}.$$

The proof of the above method of separation lies in the identities:—

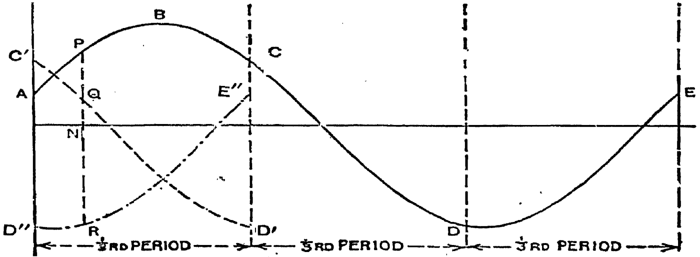
$$\sin (180^\circ - \theta) = \sin \theta$$
$$\cos (180^\circ - \theta) = -\cos \theta$$
$$\sin (2n + 1) (180^\circ - \theta) = \sin (2n + 1) \theta.$$
$$\cos (2n + 1) (180^\circ - \theta) = -\cos (2n + 1) \theta.$$

6. Method of Superposition

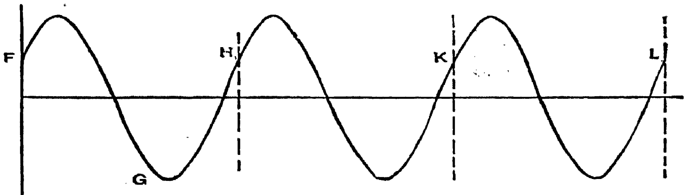
If a sinusoidal wave has its period divided into any number of equal parts and the parts are superposed the sum of the ordinates at any point is zero. For example in Fig. 2.08*a*, the sinusoidal wave ABCDE has its period divided into thirds. The second part of the wave (CD) is then re-plotted at C′D′ between the end ordinates of the first part, and the third part (DE) is re-plotted at D″E′ between these same ordinates. It will then be found that the sum of the three ordinates at any point is zero: *e.g.* PN + QN = NR.

It will be seen that this is equivalent to the addition of the separate voltages in a three-phase supply, and the proof that the sum of the ordinates is always zero can be made in the same way (see Volume I., Chapter VI., Art. 4). An alternative proof lies in the fact that the sum of three equal vectors 120° apart is zero.

If the same process is applied to a third harmonic, *e.g.* FGHKL in Fig. 2.08*b*, the superposed parts coincide; *i.e.* HK and KL coincide with FGH. Therefore the sum of the ordinates at any point is three times the original ordinate at that point. Vectorially



(a) FUNDAMENTAL



(b) 3RD HARMONIC

Fig. 2.08.—ANALYSIS BY SUPERPOSITION.

this may be expressed by the statement that, since $\frac{1}{3}$rd of the fundamental period is equal to a complete period of the third harmonic, the three vectors representing the three superposed portions coincide, therefore their sum is three times any one vector.

Applying the process to a curve made up of a fundamental and a third harmonic results in the fundamental being eliminated and three times the third harmonic being left.

Example 4. *Analyse the following wave which is symmetrical about the 90° ordinate.*

θ = 0, 15°, 30°, 45°, 60°, 75°, 90°, 105°.
v = 0, 45, 75, 85, 78, 66, 60, 66, etc., volts.

N.B.—This is the cosine portion of the wave given in Example 3, but shifted $\frac{1}{4}$-period earlier.

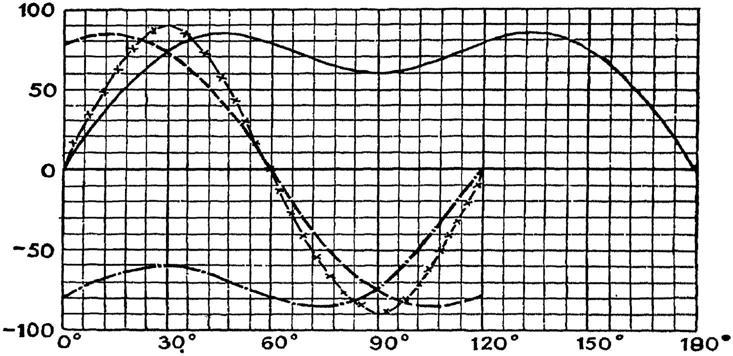


Fig. 2.09.—ESTIMATION OF TRIPLE HARMONICS.

—— Original half-wave. — — — Second third of complete wave re-plotted.
— · — · — Last third of complete wave re-plotted.
x—x—x Triple harmonics on scale enlarged three times.

The graphical analysis is shown in Fig. 2.09 (compare Fig. 2.08), but the following numerical method is equivalent.

θ =	0,	15°,	30°,	45°,	60°,	75°,	90°,	105°,	120°	
v =	0,	45,	75,	85,	78,	66,	60,	66,	78	Original
	78,	85,	75,	45,	0,	− 45,	− 75,	− 85,	− 78	2nd third
	− 78,	− 66,	− 60,	− 66,	− 78,	− 85,	− 75,	− 45,	0	3rd third
	0,	64,	90,	64,	0,	− 64,	− 90,	− 64,	0	Sum
	0,	21,	30,	21,	0,	− 21,	− 30,	− 21,	0	$\frac{1}{3}$ of sum.

Since $21 = 30 \sin (3 \times 15°)$, the last line is given by $30 \sin 3\theta$.
Subtracting the last line from the original leaves:—

0, 24, 45, 64, 78, 87, 90, 87, 78, etc.

which is given very nearly by 90 sin θ. Therefore the original wave is given by:—

$$90 \sin \theta + 30 \sin 3\theta.$$

If the same process of dividing into three and adding corresponding ordinates is applied to a fifth harmonic the total is again zero at every point. For the second portion is five-thirds of its own period out of phase with the first, *i.e.* a complete period and two-thirds: and the third portion is ten-thirds, *i.e.* three and one-third periods out of phase with the first. The three vectors are therefore at 120°, as for the fundamental, and so their sum is zero.

The same result is produced in the case of a seventh harmonic or one of any order not divisible by 3. But for the 9th, 15th, and others divisible by 3 the result is the same as for the third harmonic, *i.e.* the harmonic is increased three-fold.

Thus the result in the case of a wave containing various harmonics is to separate all those of orders divisible by 3 from the fundamental and the harmonics of orders indivisible by 3.

In the same way if the original wave is divided into five and the corresponding ordinates of the five portions added the result gives the 5th, 15th, 25th, etc., harmonics increased five-fold, and with the fundamental and the other harmonics eliminated. The fact that the fundamental is eliminated can be seen by noting that its five portions are given by five vectors of equal length making equal angles of 72° with one another. The sum of these vectors is zero. Similar arguments apply to the third, seventh, and other harmonics not divisible by 5.

This method is tedious if more than a few harmonics are required. It is useful when some one particular harmonic is required without a complete analysis.

7. Method of Selected Ordinates

The principle of superposition can be used without drawing anything more than the original half wave, and measuring certain ordinates, selected to fit the particular harmonic which is being found. For instance if the sine term of the fifth harmonic, $a_5 \sin 5\theta$, is required it is evident from Art. 6 that $5a_5$ = sum of ordinates at 18°, 90°, 162°, 234°, and 306°, since $\cos 5\theta$ is zero at each of these points. (The effects of 15th, 25th, etc., harmonics are included as in Art. 6, but only as regards their sine parts.)

Moreover, since two ordinates 180° apart are equal in magnitude but opposite in sign, the last two ordinates may be replaced by those at 54° and 126° with their signs reversed. Thus $5a_5$ = (sum of ordinates at 18°, 90°, and 162°) — (sum of ordinates at 54° and 126°).

Reference to the end of Art. 5 will show that this method eliminates all cosine terms from the value obtained for the amplitude of any sine term. To find the cosine terms another set of ordinates is taken. Thus the amplitude of cos 5θ = {(sum of ordinates at 0°, 72°, and 144°) — (sum of ordinates at 36° and 108°)} ÷ 5.

For any other particular harmonic the correct ordinates can be obtained in a similar way.

The general rules for the mth harmonic (m odd) are:—for sine term, first ordinate at $\pi/2m$, intervals π/m, signs alternately plus and minus, divide result by m; that is:—

$$a_m = \{(\text{sum of ordinates at } \pi/m,\ 5\pi/m, \text{ etc.}) - (\text{sum of ordinates at } 3\pi/m,\ 7\pi/m, \text{ etc.})\} \div m:$$

and similarly for the cosine term:—

$$b_m = \{(\text{sum of ordinates at } 0,\ 4\pi/m,\ 8\pi/m, \text{ etc.}) - (\text{sum of ordinates at } 2\pi/m,\ 6\pi/m, \text{ etc.})\} \div m.$$

8. Thompson-Runge Method

When a considerable number of waves has to be analysed, particularly if the analysis is to be carried to the 11th harmonic or beyond, a schedule of the operations saves time. In this the multipliers for the various ordinates for each harmonic are entered. The analysis then consists in carrying out the necessary multiplications, etc., and becomes purely arithmetical.

The straightforward way of doing this is to separate the sine and cosine terms and use Perry's method (Art. 5) but to divide the multipliers once for all by half the number of ordinates per quadrant; *e.g.* with ordinates every 15° divide by 3. Thus the multipliers for the fundamental sine would be (see Example 2):—

0, ·086, ·167, ·236, ·289, ·322, ·167.

When the ordinates have been multiplied by these factors the sum of the products gives the amplitude of the fundamental sine.

Similarly the multipliers for the 3rd and 5th sine harmonics are:—

3rd.. 0, ·236, ·333, ·236, 0, — ·236, — ·167.
5th.. 0, ·322, ·167, — ·236, — ·289, ·086, ·167.

P. Kemp* has drawn up such a schedule for ordinates every 10° (but starting at 5°) for harmonics up to and including the 17th. His multipliers can be obtained by the above method, though he finds them in a different way.

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* *Journal I.E.E.*, Suppl. to Vol. 57, p. 88, and *ibid.*, Vol. 58, p. 892.

Clayton* has investigated the relative accuracy of various methods and has drawn up a schedule for analysis up to the 25th harmonic inclusive, taking 26 ordinates in the half period.

The method devised by Prof. Runge and adapted for A.C. work by S. P. Thompson is similar to the above but reduces the number of multiplications by grouping. The following schedule applies when harmonics up to the 11th are required. The zero point of the curve is taken at 0°, and ordinates, $y_1, y_2, \ldots y_{11}$, are taken every 15°. The sine and cosine terms are then separated as in Art. 5. The sums (not halved), *i.e.* twice the sine terms, are denoted by s_1 . s_6; and the differences, *i.e.* twice the cosine terms, by d_1, d_2, etc. Thus $s_1 = y_1 + y_{11}$ and $d_1 = y_1 - y_{11}$, $s_2 = y_2 + y_{10}$, etc., and $s_6 = y_6$ (not $2y_6$).

For the 3rd and 9th harmonics these are grouped as follows:—

$$s_1 + s_3 - s_5 = r_1;\ s_2 - s_6 = r_2;\ \text{and}\ d_1 - d_3 - d_5 = e_1.$$

SCHEDULE

MULTIPLIERS	SINE TERMS						COSINE TERMS					
	1st & 11th.		3rd & 9th.		5th & 7th.		1st & 11th.		3rd & 9th.		5th & 7th	
sin 15° = ·259 . .	s_1				s_5			d_5				d_1
sin 30° = ·500 . .		s_2				s_2	d_4				d_4	
sin 45° = ·707 . .	s_3		r_1		$-s_3$			d_3		e_1		$-d_3$
sin 60° = ·866 . .		s_4				$-s_4$	d_2				$-d_2$	
sin 75° = ·966 . .	s_5				s_1			d_1				d_5
sin 90° = 1 . .		s_6		r_2		s_6			$-d_4$			
Sums . .												
Sum of two . .	$6a_1$		$6a_3$		$6a_5$		$6b_1$		$6b_3$		$6b_5$	
Difference of two	$6a_{11}$		$6a_9$		$6a_7$		$6b_{11}$		$6b_9$		$6b_7$	

In using the schedule the various terms shown are multiplied by the factor given in the column headed " Multipliers " and then set down as shown. The columns are added separately, and then the sums and differences of the various pairs are taken. These give six times the amplitudes of the various harmonics.

Example 5. *Analyse by the above method the following alternating voltage wave :—*

$\theta =$ 0°,	15°,	30°,	45°,	60°,	75°,	90°,	105°,	120°,	135°,	150°,	165°,	180°.
$v =$ 0,	42,	94,	156,	230,	253,	251,	225,	194,	140,	84,	35,	0.

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* *Journal I.E.E.*, Vol. 59, p. 491.

Reversing the second quadrant gives:—

	0	42	94	156	230	253	251
		35	84	140	194	225	
Sums	0	77	178	296	424	478	251
Differences		7	10	16	36	28	
	No.	1	2	3	4	5	6

Grouping these gives:—

$$r_1 = 77 + 296 - 478 = -105,$$
$$r_2 = 178 - 251 = -73,$$
$$e_1 = 7 - 16 - 28 = -37.$$

Multiplying by the sines (multipliers) in the schedule and entering the results in a similar table gives the following:—

	SINE TERMS						COSINE TERMS					
	1st & 11th.		3rd & 9th.		5th & 7th.		1st & 11th.		3rd & 9th.		5th & 7th.	
	19·9				123·5			7·3				1·8
		89·0				89·0	18·0				18·0	
	209·3		−74·2		−209·3			11·3		−26·2		−11·3
		367·2				−367·2	8·7				−8·7	
	461·7				74·4			6·8				27·0
		251		−73		251			−36			
Sums	690·9	707·2	−74·2	−73	−11·4	−27·2	26·7	25·4	−36	−26·2	9·3	17·5
Sum of two	1398·1		−147·2		−38·6		52·1		−62·2		26·8	
Difference	−16·3		−1·2		+15·8		1·3		−9·8		−8·2	

The first term is obtained by multiplying s_1 (77) by $\sin 15^\circ = \cdot259$, and so on.

Dividing the numbers in the last two rows by 6 gives:—

$a_1 = 233\cdot0$, $a_3 = -24\cdot5$, $a_5 = -6\cdot4$, $a_7 = 2\cdot6$,
$a_9 = -0\cdot2$, $a_{11} = -2\cdot7$, $b_1 = 8\cdot7$, $b_3 = -10\cdot4$,
$b_5 = 4\cdot5$, $b_7 = -1\cdot4$, $b_9 = -1\cdot6$, and $b_{11} = 0\cdot2$.

Combining corresponding sine and cosine terms (see Art. 1) gives:—

$$c_1 = \sqrt{a_1^2 + b_1^2} = 233\cdot2, \qquad \phi_1 = \tan^{-1}\frac{8\cdot7}{233} = 2\cdot1^\circ \text{ lead},$$

$$c_3 = -\sqrt{\{24\cdot5^2 + 10\cdot4^2\}} = -26\cdot6, \qquad \phi_3 = \tan^{-1}\frac{10\cdot4}{24\cdot5} = 23\cdot0^\circ \text{ lead},$$

$$c_5 = -\sqrt{\{6\cdot4^2 + 4\cdot5^2\}} = -7\cdot8, \qquad \phi_5 = -\tan^{-1}\frac{4\cdot5}{6\cdot4} = 35\cdot0^\circ \text{ lag}.$$

$$c_7 = \sqrt{\{2\cdot6^2 + 1\cdot4^2\}} = 3\cdot0, \qquad \phi_7 = -\tan^{-1}(2\cdot6/1\cdot4) = 61\cdot7^\circ \text{ lag},$$

$$c_9 = -\sqrt{\{0\cdot2^2 + 1\cdot6^2\}} = 1\cdot6, \qquad \phi_9 = \tan^{-1}(1\cdot6/0\cdot2) = 82\cdot9^\circ \text{ lead},$$

$$c_{11} = -\sqrt{\{2\cdot7^2 + 0\cdot2^2\}} = -2\cdot7, \qquad \phi_{11} = -\tan^{-1}(0\cdot2/2\cdot7) = 4\cdot2^\circ \text{ lag},$$

i.e. the equation of the wave is approximately:—

$$v = 233\cdot2 \sin(\theta + 2\cdot1^\circ) - 26\cdot6 \sin(3\theta + 23^\circ) - 7\cdot8 \sin(5\theta - 35^\circ)$$
$$+ 3\cdot0 \sin(7\theta - 61\cdot7^\circ) + 1\cdot6 \sin(9\theta + 82\cdot9^\circ) - 2\cdot7 \sin(11\theta - 4\cdot2^\circ).$$

Fig. 2.10 shows the original wave and the first three components. Their sum is nearly equal to the original wave.

9. Simple Method

For symmetrical half-waves no cosine terms occur (see Art. 3), and so only the first half of the Thompson-Runge schedule is needed. If the third and fifth harmonics alone are required this method becomes very simple. A schedule may be used, but is of little advantage in this case.

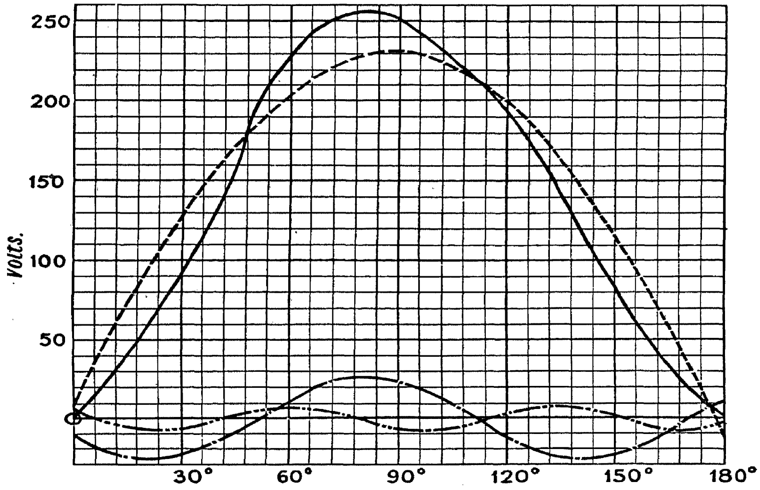


Fig. 2.10.—ANALYSIS BY THOMPSON-RUNGE METHOD.

—— Original — — — — Fundamental. — · — · — Third harmonic. —· · —· · —· · Fifth harmonic.

The method may be put as follows:—

Let the ordinates be denoted by—

$$\begin{array}{cccccc} & y_1, & y_2, & y_3, & y_4, & y_5, \\ \text{at} & 30^\circ, & 60^\circ, & 90^\circ, & 120^\circ, & 150^\circ. \end{array}$$

[N.B.—$y_2 = y_4$, and $y_1 = y_5$, but it is better to measure both, so as to obtain an average value if there is a small departure from symmetry.]

Then if the equation of the wave is:—

$$y = a_1 \sin \theta + a_3 \sin 3\theta + a_5 \sin 5\theta,$$

$$a_3 = \frac{y_1 - y_3 + y_5}{3}. \qquad \text{(Compare with Art. 7.)}$$

Again $$a_1 + a_5 = y_3 + a_3 = \frac{y_1 + 2y_3 + y_5}{3}$$

and $$a_1 - a_5 = \frac{y_2 + y_4}{\sqrt{3}},$$

whence a_1 and a_5 can be determined.

These formulae still hold good for the sine terms when applied to unsymmetrical waves.

Thus, if $y_1 = y_5 = 114$, $y_2 = y_4 = 173$, and $y_3 = 183$ (*i.e.* the wave of Example 2 and Fig. 2.06),

$$a_3 = \frac{114 - 183 + 114}{3} = 15,$$

$$\left.\begin{aligned} a_1 + a_5 &= 183 + 15 = 198 \\ a_1 - a_5 &= \frac{173 + 173}{\sqrt{3}} = 200 \end{aligned}\right\} \text{whence } \begin{aligned} a_1 &= 199, \\ a_5 &= -1, \end{aligned}$$

i.e. the wave is given approximately by

$$y = 199 \sin \theta + 15 \sin 3\theta - \sin 5\theta.$$

The difference between this and the more accurate analysis of Example 2 is due to the neglect of higher harmonics. The result obtained is a more accurate representation of the wave than if the analysis is carried out exactly and then harmonics of higher order than the fifth are neglected.

The truth of the formulae can be proved by expressing the ordinates (y) in terms of the amplitudes (a), *e.g.* $y_1 = \tfrac{1}{2}a_1 + a_3 + \tfrac{1}{2}a_5$, and then combining these in the ways shown by the formulae.

10. R.M.S. and Mean Values

The R.M.S. value of a wave with harmonics is the square root of half the sum of the squares of the amplitudes of the separate harmonics: *i.e.* if

$$y = a_1 \sin \theta + a_3 \sin 3\theta + a_5 \sin 5\theta + \text{etc.} + b_1 \cos \theta + b_3 \cos 3\theta + \text{etc.},$$

then R.M.S. value of $y = \sqrt{\tfrac{1}{2}(a_1^2 + a_3^2 + a_5^2 + \ldots + b_1^2 + b_3^2 + \ldots)}$.

Thus it is independent of the relative phases of the harmonics, and depends only on their amplitudes.

The truth of the above can be proved by utilising the integrations effected in Art. 4.

The mean value of the half-wave, on the other hand, *is* affected by the phases of the harmonics. *E.g.* the three waves in Fig. 2.11,

$$\text{(i) } 100\sin\theta + 30\sin 3\theta, \qquad \text{(ii) } 100\sin\theta - 30\sin 3\theta,$$
$$\text{(iii) } 100\sin\theta - 30\cos 3\theta,$$

all have an R.M.S. value of $\sqrt{\tfrac{1}{2}\{100^2 + 30^2\}}$, *i.e.* 74. But the mean value of (i) is $(2/\pi)\,(100 + \tfrac{1}{3} \times 30)$, *i.e.* 70, while that of (ii) is $(2/\pi)\,(100 - \tfrac{1}{3} \times 30)$, *i.e.* 57 (see Fig. 2.11 *a*).

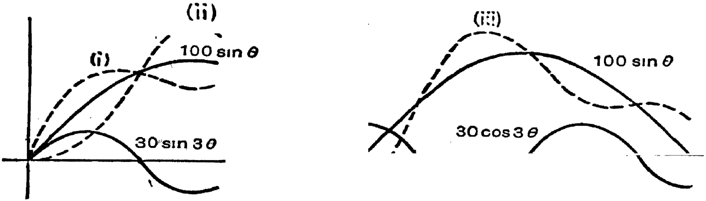
Wave (iii) is zero when $\theta = 13\tfrac{1}{2}^\circ$, thus the mean value is

$$\frac{1}{\pi}\int_{13\frac{1}{2}^\circ}^{193\frac{1}{2}^\circ} (100\sin\theta - 30\cos 3\theta)\, d\theta,$$

$$\text{i.e. } (1/\pi)\Big[-100\cos\theta - 10\sin 3\theta\Big]_{13\frac{1}{2}^\circ}^{193\frac{1}{2}^\circ}$$

i.e. $(1/\pi)\,(200\cos 13\tfrac{1}{2}^\circ + 20\sin 40\tfrac{1}{2}^\circ)$, *i.e.* $(2/\pi)\,(97{\cdot}2 + 6{\cdot}5)$, *i.e.* 66.

The three waves, therefore, have form factors as follows:—



(*a*) (*b*)

Fig. 2.11.—EFFECTS OF RELATIVE PHASE OF 3RD HARMONIC.

(i) 74/70, *i.e.* 1·06; (ii) 74/57, *i.e.* 1·30; (iii) 74/66, *i.e.* 1·12; compared with 1·11 for a sinusoidal wave.

The figures show that the maximum values also depend on the phase relationship, and the peak factors are affected similarly.

11. Inductance and Capacitance Effects

If a current with harmonics in its wave-form is flowing in an inductance the self-induced e.m.f. has similar but more pronounced harmonics in its wave-form. For if the current is given by:—

$$i = a_1\sin\omega t + a_3\sin 3\omega t + a_5\sin 5\omega t + \text{etc.},$$

and the inductance is L, then the self-induced e.m.f. equals $-\dfrac{di}{dt}$,

i.e. $\quad -\omega L\,(a_1\cos\omega t + 3a_3\cos 3\omega t + 5a_5\cos 5\omega t + \text{etc.})$.

Or, in other words, the effect of applying to an inductance a p.d. not of sinusoidal form is to produce a current including all the harmonics of the p.d. but *reduced* in comparison with the fundamental. The higher the order of the harmonic the greater its relative reduction.

A result of this is that the reactance of a coil at a given frequency is greater when harmonics are present than with sinusoidal waves. *E.g.* if the applied p.d. is $\omega L\,(100 \cos \omega t + 30 \cos 3\omega t)$, the current is $100 \sin \omega t + 10 \sin 3\omega t$. The ratio (R.M.S. p.d./R.M.S. current) is then $\omega L \sqrt{\frac{1}{2}(100^2 + 30^2)}/\sqrt{\frac{1}{2}(100^2 + 10^2)}$ (see Art. 10), *i.e.* $1{\cdot}04\omega L$. The increase in reactance is thus 4 per cent. with this p.d. wave, which departs widely from the sinusoidal; consequently the change will be negligible for any p.d. whose wave-form is roughly sinusoidal.

When resistance alone is in circuit the current wave is of the same shape as the p.d. wave. Hence for a circuit possessing both resistance and inductance, the reduction of the higher harmonics will be less marked than with inductance alone.

With capacitance the current is given by $i = C\frac{dv}{dt}$, and so the relation of current and p.d. is the reverse of their relation with inductance. Hence the harmonics in the p.d. are reproduced in the capacitance current in *increased* ratio to the fundamental, and the higher the harmonic the greater the relative increase.

In this case, too, there is a change in the reactance. The apparent capacitance is increased, since the R.M.S. value of the current is increased by the harmonics more than is the R.M.S. value of the p.d. Thus the capacitance reactance is diminished in magnitude, *i.e.* its susceptance is increased. *E.g.* if

$$v = 100 \sin \omega t + 20 \sin 3\omega t + 10 \sin 5\omega t;$$

then

$$i = \omega C\,(100 \cos \omega t + 60 \cos 3\omega t + 50 \cos 5\omega t);$$

$$\therefore\ I/V = \omega C \sqrt{\tfrac{1}{2}(100^2 + 60^2 + 50^2)}/\sqrt{\tfrac{1}{2}(100^2 + 20^2 + 10^2)}$$

$$= \omega C \times 1{\cdot}24,$$

i.e. the apparent capacitance and the susceptance are increased in this case by 24 per cent.; or the capacitance reactance is reduced in the ratio (100/124), *i.e.* by 19 per cent.

With circuits possessing both inductance and capacitance the most important additional point to note is that resonance may occur with one of the higher harmonics when there is no danger of this with the fundamental. With a fixed inductance, the capacitance to give resonance with the third harmonic is only $\frac{1}{9}$ of the amount required for resonance with the fundamental, and $\frac{1}{25}$ of the latter gives resonance with the fifth harmonic, and so on.

12. Power

Since the mean power is $\int vidt$ taken over a complete period, its value can be found by dealing with the harmonics separately and then adding algebraically the values found. For the mean power due to the product of any term in the voltage expression by any term *of different order* in the current series is zero (cf. Art. 4).

The power may, therefore, be expressed as:—

$$V_1 I_1 \cos \phi_1 + V_3 I_3 \cos \phi_3 + \text{etc.}$$

or

$$\tfrac{1}{2}(\hat{V}_1 \hat{I}_1 \cos \phi_1 + \hat{V}_3 \hat{I}_3 \cos \phi_3 + \text{etc.})$$

where $\hat{V}_1, \hat{V}_3, \ldots =$ amplitudes of voltage components,
$\hat{I}_1, \hat{I}_3, \ldots =$ amplitudes of current components,
and V_1, I_1, etc., are the R.M.S. values *of the components*.

Example 6. *Find the values of the power and power-factor when the p.d. given by:—*

$$v = 100 \sin \theta + 30 \sin 3\theta$$

sends through an inductive circuit a current given by:—

$$i = 20 \sin (\theta - 30^\circ) + 3{\cdot}5 \sin (3\theta - 60^\circ).$$

[Note that the third harmonic has been reduced relatively in the current, but not down to $\frac{1}{3}$ of the proportional value (see Art. 11); and that its angle of lag is greater than that of the fundamental.]

The power due to the fundamentals is $\frac{1}{2} \times 100 \times 20 \cos 30^\circ$
$= 866$ watts.

The power due to the third harmonics is $\frac{1}{2} \times 30 \times 3{\cdot}5 \cos 60^\circ$
$= 26$ watts.

$\therefore$ Total power $= 866 + 26 = 892$ watts.

R.M.S. p.d. $= \sqrt{\tfrac{1}{2}(100^2 + 30^2)} = 74$ volts.

R.M.S. current $= \sqrt{\tfrac{1}{2}(20^2 + 3{\cdot}5^2)} = 14{\cdot}4$ amperes.

$\therefore$ Power-factor $= 892/(74 \times 14{\cdot}4) = 0{\cdot}84$,

which is lower than the power-factor (0·866) of the fundamentals.

The above results can be checked by taking a number of ordinates of the two waves, and then applying the method of Volume I., Chapter V., Example 3, page 132. Further:—

Impedance $= 74/14{\cdot}4 = 5{\cdot}14$ ohms,
Resistance $= 892/(14{\cdot}4)^2 = 4{\cdot}33$ ohms,
Equivalent reactance $= \sqrt{\{5{\cdot}14^2 - 4{\cdot}33^2\}} = 2{\cdot}77$ ohms.

Whereas with the fundamentals alone:

Impedance $= 5$ ohms,
Resistance $= 5 \times 0{\cdot}866 = 4{\cdot}33$ ohms (as before),
Reactance $= \sqrt{5^2 - (4{\cdot}33)^2} = 2{\cdot}50$ ohms.

An interesting example of power with non-sinusoidal waves is furnished by the alternating current arc between carbon electrodes. The usual shapes of the p.d. and current curves are shown in Fig. 2.12. Until the p.d. has risen to a high value the current is very small. It then rises rapidly with a *drop* in the p.d.

The result of this difference of shape is that although the current and p.d. pass through zero at the same instant (and so are in phase in this sense) the power-factor is considerably below unity, being of the order of 0·6 in some cases.

If the two waves are analysed it will be found that the relative amplitudes of the harmonics are very different in the two, and that harmonics of the same order are not in phase. This explains the apparent anomaly of a power-factor of 0·6 with zero angle of lag.

But even if the current component of each frequency is in phase with the voltage component of the same frequency the power-factor may be less than unity; *e.g.* if the p.d. is a sine wave, and the current is the sum of several sine waves of different frequency. As the formula for power shows, the power is then VI_1, where I_1 is the fundamental of the current wave. But I_1 is less than the R.M.S. current, and so the watts are less than the volt-amperes.

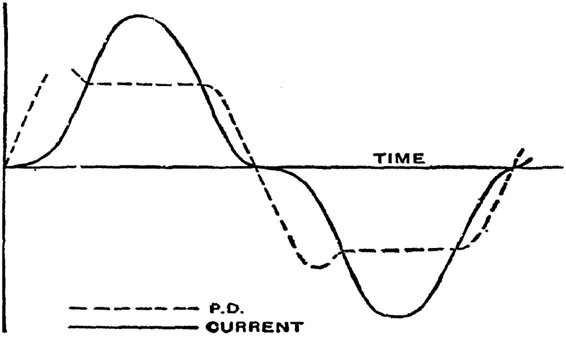


Fig. 2.12.—VOLTAGE AND CURRENT WAVES OF A.C. CARBON ARC.

13. Two-Phase Circuits

When two similar waves, each containing a third harmonic, differing in phase by quarter of a period are subtracted (as in obtaining the voltage across the outers of a two-phase 3-wire supply) the resultant differs in shape from the original waves.

For instance (see Fig. 2.13) let the third harmonics of the two waves be in phase with their fundamentals, giving hollow-topped waves. The result of subtracting the lagging fundamental (Phase II) from the leading one is a sinusoidal wave of $\sqrt{2}$ (*i.e.* 1·41) times the amplitude of the fundamentals, leading Phase I by $\frac{1}{8}$th of a period (45°). But the subtraction of Phase II's third harmonic from that of Phase I, while again $\sqrt{2}$ times the amplitude of these two *lags* behind the third harmonic of Phase I by $\frac{1}{8}$th of its period, *i.e.* $\frac{1}{24}$th of the fundamental period, *i.e.* 15°. The resultant third harmonic therefore reaches its maximum at (30° + 15°), *i.e.* 45°, which is the same instant as the resultant fundamental reaches its maximum. The resultant wave is consequently a peaked one, its third harmonic being opposite in phase to its fundamental.

This result can be proved vectorially. Fig. 2.13B gives the vectors for the fundamentals, showing that the resultant is $\sqrt{2}$

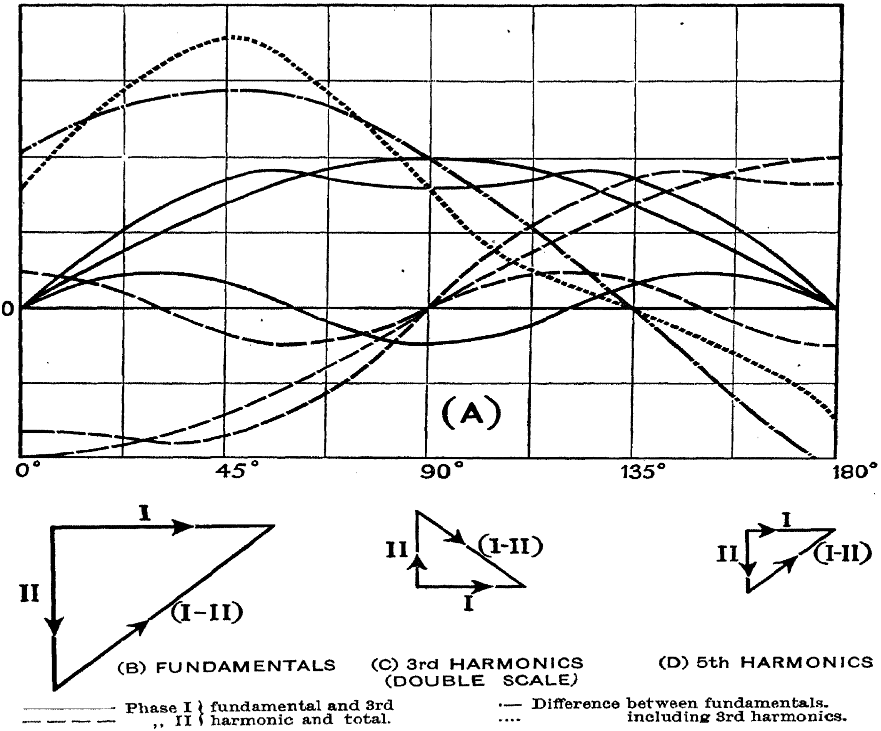


Fig. 2.13.—TWO-PHASE WAVES WITH HARMONICS.

times as large and leads Phase I by 45°. In Fig. 2.13C the vectors for the third harmonics are given. They must be drawn as shown because they rotate three times as fast as the fundamental vectors. Hence when IIB has reached the zero position to the right, IIC will have done the same, so that they are in phase. In the positions drawn they show that the wave is hollow topped since the total voltage is their arithmetical difference. The resultant third harmonic is thus seen to lag 45° behind that of Phase I as stated above.

The peaked nature of the resultant wave can be seen by rotating the fundamental through 45°, bringing it to its maximum. During this movement the third harmonic will have turned 135°, and so reaches its positive maximum at the same instant.

By applying either of the above methods to a fifth harmonic it can be shown that, since Phase II lags 450°, *i.e.* 360° + 90°, the resultant fifth harmonic *leads* that of Phase I (Fig. 2.13D). Consequently the resultant fifth harmonic is opposite in phase to the resultant fundamental if the two are in phase in the original wave.

Investigation on the same lines shows that the 7th and 9th harmonics in the resultant retain their original phase relation to the fundamental. Higher harmonics taken in pairs are alternately reversed in phase (11th, 13th, 19th, 21st, etc.) and unchanged (15th, 17th, 23rd, etc.). Hence for most non-sinusoidal waves the resultant differs in shape from the waves of the phases. The resultant of any harmonics of the same order is $\sqrt{2}$ (*i.e.* 1·41) times the magnitude of its two components. Hence, since the R.M.S. values depend only on the amplitudes of the components (see Art. 10), the R.M.S. value of the resultant is $\sqrt{2}$ times that of each of the components for *all* wave forms.

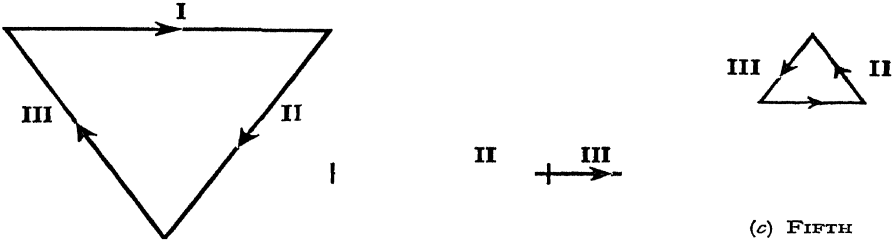
For unequal voltages or currents no general statement can be made, but any particular case can be worked out by the above methods.

14. Three-Phase Circuits

If an alternator is Δ-connected and each phase generates an equal sinusoidal e.m.f. there is no resultant e.m.f. round the mesh. But with other wave-forms there may be a resultant (see Volume I., Chapter VI., Art. 4, Example 3). If the harmonics of the phase waves are known it is an easy matter to obtain the resultant e.m.f.

The fundamentals (Fig. 2.14*a*) of Phase I, Phase II lagging $\frac{1}{3}$ period, and Phase III lagging $\frac{2}{3}$ period, give zero resultant. For the fifth harmonics Phase II lags behind Phase I by $\frac{5}{3}$ periods, *i.e.* a complete period and $\frac{2}{3}$rds period; Phase III lags behind Phase I by $\frac{10}{3}$, *i.e.* $3\frac{1}{3}$ periods. Hence these three likewise give zero resultant (Fig. 2.14*c*). The same applies to any harmonics of an order not divisible by 3.

On the other hand the third harmonics (Fig. 2.14*b*) will be out of phase by 3 × $\frac{1}{3}$ period, and 3 × $\frac{2}{3}$ period, *i.e.* by complete periods.



(*a*) FUNDAMENTALS. (*b*) THIRD HARMONICS. (*c*) FIFTH HARMONICS

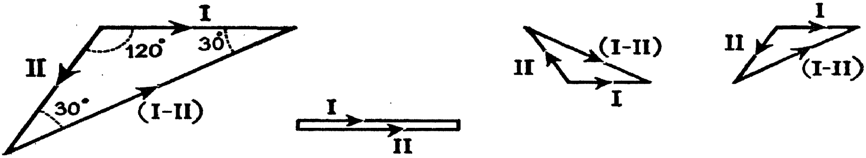
Fig. 2.14.—VECTOR DIAGRAMS FOR THREE-PHASE DELTA.

Hence the resultant is three times the third harmonic of each phase. Similarly the ninth harmonics will give a resultant three times the ninth harmonic of each phase, and the same applies to all harmonics of orders divisible by three.

Thus the resultant e.m.f. will have a frequency three times that of the phase e.m.f.s as was seen in Volume I., Chapter VI., Example 3. If it is due to the third harmonics alone it will be sinusoidal, but if the ninth harmonic, or other higher harmonic divisible by three, is present its shape will be modified accordingly.

In a Y-connected system the voltage between any two lines is $\sqrt{3}$ (*i.e.* 1·73) times the phase voltage with sinusoidal waves. With other shapes this ratio may be altered, and the two voltages will have different wave-forms.

In Fig. 2.15*a* the vector diagram shows that if the voltage of Phase II lags 120° behind that of Phase I the voltage of terminal I above terminal II is $\sqrt{3}$ times the phase voltage, and *leads* that of Phase I by 30°. With the fifth harmonics (Fig. 2.15*c*), the ratio is again $\sqrt{3}$, but the resultant lags behind Phase I by 30°. If the fundamental vectors are turned back through 30° to bring the resultant to zero the fifth harmonic vectors must be turned back through 150°. This brings their resultant to zero but opposite in



(*a*) FUNDAMENTALS. (*b*) THIRD. (*c*) FIFTH. (*d*) SEVENTH.

Fig. 2.15.—VECTOR DIAGRAMS FOR THREE-PHASE STAR.

phase to the fundamental. This shows that the fifth harmonic in the resultant has the same amplitude ratio to the fundamental but is reversed in phase.

The vectors of the seventh harmonics are shown in Fig. 2-15*d*, the resultant leading Phase I by 30°. But when the fundamental has been turned back to zero, the resultant seventh harmonic will have turned back through 210° and so is opposite in phase, *i.e.* it behaves like the fifth harmonic. A similar investigation shows that the 11th and 13th harmonics on the other hand retain their original phases in the resultant, and so on in alternate pairs.

But with the third harmonics the phase difference becomes 360°, *i.e.* they are in phase. Consequently the terminal voltage contains no third harmonic. The same applies to the ninth and 15th harmonics, and to all of orders divisible by three.

When no such harmonics occur in the phase voltages the R.M.S. value of the terminal voltage is $\sqrt{3}$ (*i.e.* 1·73) times the R.M.S. value of each phase voltage as with sinusoidal waves. But when triple harmonics (including ninth, 15th, etc., in this term) occur in the phase voltage, their absence from the terminal voltage reduces its relative value. *E.g.* if the amplitudes in the phase voltages are:—fundamental 130, third harmonic 26, fifth harmonic 4, the R.M.S. value of the phase voltage is $\sqrt{\{\frac{1}{2}(130^2 + 26^2 + 4^2)\}}$, *i.e.* 93·8 volts; and that of the terminal voltage is $\sqrt{\{3 \times \frac{1}{2}(130^2 + 4^2)\}}$, *i.e.* 159 volts. The ratio is therefore 1·70 instead of 1·73, *i.e.* about 2 per cent. less. This shows that the sinusoidal ratio is very nearly correct even for waves containing harmonics, unless the third harmonics are unusually large.

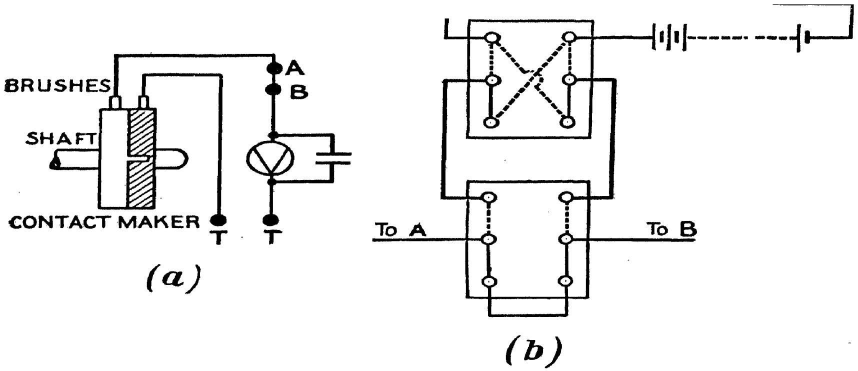
15. Wave-Tracing with Contact-Maker

One method of obtaining the shape of an alternating wave is by means of a synchronously-rotating contact-maker. If the wave is to be traced at the place where the supplying alternator is, the contact-maker may be attached directly to this. In other cases it requires a small synchronous motor to drive it.

The contact-maker consists of two stationary brushes which are momentarily connected together once in every period. This is effected by a rotating metal ring with narrow projecting pieces, one for every pole-pair of the driving machine. One brush rests on the ring, and the other is touched by the projecting pieces as they pass it; in the intervals it rests on insulating material.

The terminals between which the p.d. wave is required are connected through the contact-maker (see Fig. 2.16*a*) to an *electrostatic* voltmeter. As this requires no current it deflects almost at once to the voltage occurring at the instant in the period at which contact is made. If the R.M.S. voltage is constant the p.d. at the instant of contact is the same every time. By shifting the position of the brushes the contacts occur at a different point in the period. Hence if the brush holder is provided with a scale of electrical degrees any desired number of instantaneous voltages can be measured, from which the wave-form can be plotted.

In order to read voltages below the lowest scale reading of the voltmeter a battery of small cells is used, giving a voltage higher than this lowest reading. By connecting this in series with the contact-maker when required (*e.g.* between A and B in Fig. 2.16), low deflections can be brought up onto the scale. The known voltage of the battery is then subtracted from the reading to obtain the true value of the alternating wave at this point.



(*a*) Voltmeter Connexions. (*b*) Battery Connexions.

Fig. 2.16.—Wave-tracing with Rotating Contact Maker.

Similarly if the voltage at any point exceeds the maximum reading of the voltmeter the battery is connected so as to oppose this voltage. The true value is then obtained by adding the battery voltage to the reading on the voltmeter scale. By means of two double-pole two-way switches (Fig. 2.16*b*) the battery can be connected in either way, or left out of the circuit.

It is usual to connect a condenser in parallel with the voltmeter to minimise the possibility of inaccuracy due to leakage during the intervals when the brushes of the contact-maker are not connected.

Hospitalier's **ondograph** works on this principle. It includes a small synchronous motor driving a contact-maker of the above type. The voltmeter is of the moving coil recording type with a pen instead of a pointer. The contact-maker is moved round by the synchronous motor through a reduction gear with a ratio of 1000 to 999. Thus the moment of contact is gradually changed relative to the voltage, and the voltage is varied through one complete cycle in a thousand cycles. The recording drum is turned once in the same time, and so the pen traces out the wave-form. A condenser of large capacitance is used so that the voltmeter current in the interval between two successive contacts does not reduce the pressure appreciably.

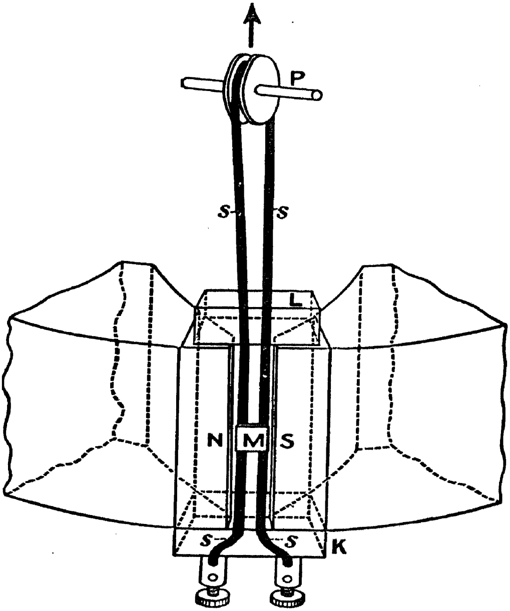


Fig. 2.17.—MOVING-COIL OSCILLOGRAPH.

16. Oscillographs

The ondograph (Art. 15) requires the wave-form to remain constant for 1000 cycles in order to draw it, but the **oscillograph** gives the shapes even when successive waves differ in magnitude of form or both. The latter is therefore of much wider application, particularly for the transitory effects of switching off or on, and of alterations of load.

The moving-coil form was devised by Blondel and improved by Duddell and has been modified as the result of experience by the Cambridge Instrument Co. The essential feature, as shown in Fig. 2.17, is a loop of fine phosphor-bronze strip (*s*, *s*, *s*, *s*) passing through narrow gaps between the poles (N, S) of a magnet. The loop passes over an ivory pulley (P), which is pulled upwards by a spring and so keeps the loop in tension. A small mirror (M) is attached to the two sides of the loop. When a current is passed through the loop one side is pulled back and the other forward, and the mirror is twisted through a small angle proportional to the current. If the current is reversed the direction of twist is reversed.

Owing to the very small mass of the moving parts and the relatively large tension the natural frequency is high (1 800 oscillations to 10 000 oscillations per second in different patterns). They are in consequence able to follow the changes of an alternating current or voltage of the usual frequencies.

If a beam of light from a fixed source is thrown on to the small mirror the reflected beam will therefore move with the changes of the current (or voltage). If received on a fixed surface it will trace out a line the whole of which appears bright owing to the rapidity of the motion. By moving the surface at right angles to this line it is drawn out into an undulating curve which gives the required wave-form.

Several different methods are used for obtaining this motion which represents the time. In the drum-camera pattern a strip of photographic film or of sensitised paper is rotated by a small motor. An automatic arrangement, operated by the same motor, opens a shutter-slot to permit the oscillating reflected beam to fall on the film. The shutter is closed again after one revolution of the drum, or after a partial revolution. In the latter case two to four different records can be made on the same strip. A similar method is to use a roll of photographic film which is wound past the slot by a small electric or clockwork motor at a definite rate. This enables a large number of successive waves to be recorded, and so is better than the drum camera when conditions are changing.

Another class of method uses mirrors rotating about an axis parallel to the movement of the reflected beam. This is used for viewing the wave-forms before photographing them by one of the above methods. When the shutter is closed the back of it reflects the beam on to a semi-transparent celluloid screen. This is viewed in the rotating mirrors. If these are driven at synchronous speed the curve traced out by the spot of light appears stationary.

In the visual type a similar arrangement is used but the beam falls on the mirror (driven by a small synchronous motor) before reaching the screen. This permits the use of a large screen (10 cm. square) so that a number of people can see the curve simultaneously.

By making this screen translucent it becomes possible to record by hand the form of any wave which remains of constant shape and size for a time. Tracing paper is clipped to the screen on the opposite side to that from which the light comes. The bright line traced out by the reflected light remains fixed and so can be followed with pen or pencil and transferred to the tracing paper. By the use of a dark slide a photographic record can be obtained.

For high pressure work an electrostatic type of oscillograph is preferable. One of this type is shown in Figs. 2.18, 2.19. This was devised by Prof. Ho* of Tokio and made by the Cambridge Instrument Co., Ltd. The mirror, *m*, is attached to two vibrating strips as in Duddell's pattern, but the loop over the straining pulley, *p*, is completed by a silk insulating thread, *t*. The other ends of the strips are connected to the ends of a battery of about 400 volts. The high pressure to be investigated is connected between two metal plates, F_1 and F_2, equidistant from the strips. The latter move in opposite directions owing to the electrostatic forces and so cause the mirror to vibrate. A window, W_1, is cut in the front plate to allow a beam of light to be thrown onto and reflected from the mirror. An exactly similar window, W_2, is cut in the back plate to maintain the symmetry of the electric field.

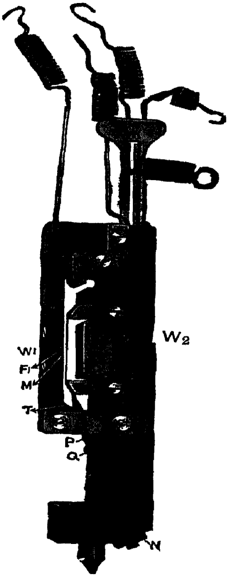


Fig. 2.18.—ELECTROSTATIC OSCILLOGRAPH.

The centre point of the battery, *k*, is joined to a point between two approximately equal condensers, C_1, C_2, which are in series across the terminals, *a*, *b*, of the oscillograph. When an alternating voltage is applied across *a*, *b* the strips are deflected by equal but opposite amounts proportional to the product of the alternating voltage and the battery voltage. The condenser C_2 is made adjustable to facilitate zero adjustment.

When p.d.s greater than 2 500 V. R.M.S. are to be measured a condenser, C, is connected in series with the oscillograph circuit and the p.d. is applied across a_0, b_0 (see Fig. 2.19). By choosing a suitable condenser the voltage across *a*, *b*, can be made such a fraction of the total voltage that it does not exceed 2 500 V. The amplitude of the half-wave with a scale distance of 60 cm. is about 6 mm. per 1000 V. maximum.

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* See "An Electrostatic Oscillograph" (H. Ho and S. Koto), *Proc. Phys. Soc.*, 15th Dec., 1913.

18. The Current Wave-Form

This depends partly on that of the applied voltage, and partly on the properties of the circuit. Certain cases have been dealt with in Arts. 11 and 12. Another cause of difference between p.d. and current wave-forms is the varying permeability of the iron in the magnetic circuit of an inductive coil, *e.g.* a transformer winding.

If the p.d. wave is sinusoidal the current wave will be peaked, owing to its having to increase faster than the flux as the iron becomes saturated. This can be seen by considering such a coil with negligible resistance. If the applied p.d. is a cosine wave, the rate of change of linkages is the same multiplied by a constant, and so the linkage-time graph is a sine-wave.

Hence if the p.d. wave (Fig. 2.20) is obtained by projection from a rotating line OA, the linkage wave is given to some scale by the projection of an equal line, OB, lagging 90°. *E.g.* when the p.d. is PN, the number of linkages is QN. But the projection of OB on the vertical is equal to the projection (OM) of OA on the horizontal. Let the curve ACDE give the relation between linkages and current, turned through a right angle from its usual position for convenience. From this curve the current, CM, corresponding with the linkages OM, is obtained, and RN is made equal to this.

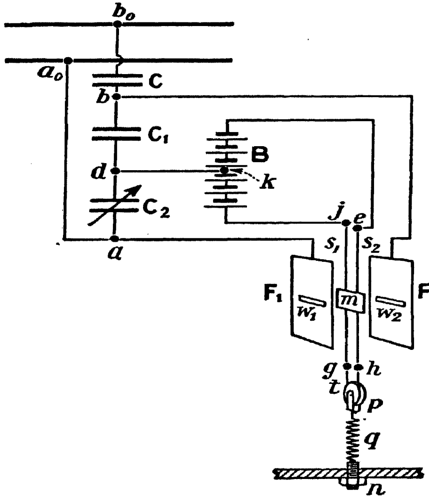


Fig. 2.19.—ELECTROSTATIC OSCILLOGRAPH CIRCUITS.

By repeating this construction for a number of points the current graph is obtained. It will be seen that it is more peaked than a sinusoidal wave, but is symmetrical. It usually contains a third harmonic opposite in phase to its fundamental.

The same method can be applied when hysteresis is present. The difference is that two different curves have to be used according to the direction of change of the linkages. The result is shown in Fig. 2.21.

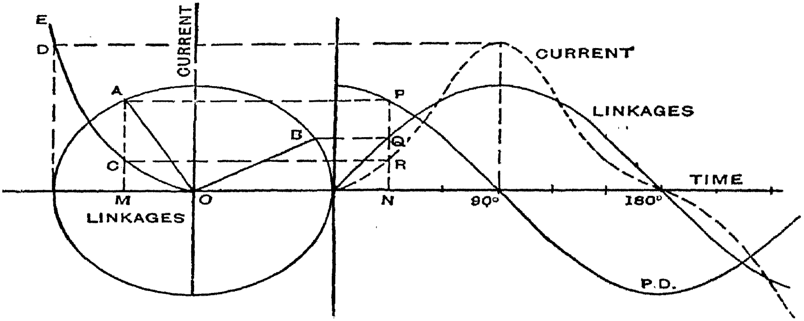


Fig. 2.20.—CURRENT WAVE OF IRON-CORED COIL.

For two equal values of the linkages (OM) corresponding with equal and opposite values of the p.d. (PN, P′N′) there are two different values of the current, viz. MC or RN for increasing linkages, and MC′ or R′N′ for decreasing linkages.

Hence the current half-wave is unsymmetrical. For the maximum is unchanged, but the zero values occur earlier. It is, therefore, no longer 90° out of phase with the p.d. as in Fig. 2.20, and so has a power component. This is called the hysteresis current. It is in phase with the p.d., and the power represented by the product of these two is that lost owing to hysteresis in the iron.

When there is no hysteresis the energy expended during one-quarter of a period is restored during the next quarter, and so the average power is zero. This can be seen from the symmetry of the curves in Fig. 2.20. With hysteresis more energy is expended in

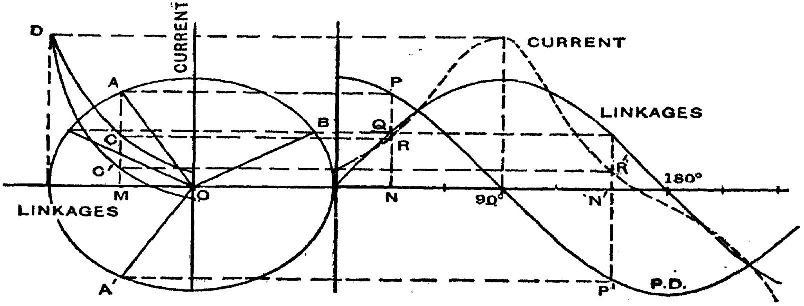


Fig. 2.21.—CURRENT WAVE WITH HYSTERESIS.

magnetising the iron (which process occupies more than quarter of a period) than is returned during demagnetisation (which lasts less than a quarter of a period). Hence there is a net expenditure of energy, and the average power expended has a positive value.

QUESTIONS ON CHAPTER II

1. Analyse by integration an A.C. wave which increases uniformly for a quarter of the half-period, remains constant for the next half of this, and then diminishes uniformly for the last quarter.

2. The following instantaneous values of an alternating voltage were obtained:—

Angle	0°	15°	30°	45°	60°	75°	90°	105°	etc.
Volts	0	44	64	73	80	85	87	85	etc.

each half of the wave being symmetrical.

(*a*) Plot the positive half-wave.

(*b*) Analyse it approximately into a fundamental and third and fifth harmonics. Plot the sum of these three.

(*c*) Find the amplitudes of the seventh and ninth harmonics.

Do these affect the values found in (*b*) ?

3. Plot an e.m.f. wave whose values at intervals of $\frac{1}{24}$th period are: 0; 46; 114; 169; 173; 174; 185; 174; 173; 169; 114; 46; 0; − 46; etc.
Find the fifth and seventh harmonics of this wave. State whether the method employed does or does not include in the results the harmonics whose orders are multiples of five and seven respectively.
Plot the original wave with the fifth and seventh harmonics removed. From an inspection of the result estimate the amplitudes of the fundamental and of the third harmonic.

4. A current alternating at 50 cycles per sec. with a strong third harmonic is given by the expression

$$30 \sin 100\pi t + 6 \sin 300\pi t.$$

If this flows in a coil with a resistance of 12 ohms and an inductance of 0·04 henry, find the R.M.S. values of the current and of the applied voltage, the power absorbed, and the power-factor.

5. An alternating-current voltage given by

$$100 \sin 314t + 10 \sin 1\,570t$$

is applied to the terminals of a condenser having a capacitance of 1 microfarad. Find the expression for the current flowing through the condenser.
[C. & G., II.

6. An alternating voltage is given by:—

$$v = 200\sqrt{2} \sin 300t + 40\sqrt{2} \sin 900t.$$

What reading would a moving-iron voltmeter give when connected across this supply ?
Find the magnitude of the current produced by this voltage if connected (*a*) to an inductance of 0·04 henry; (*b*) a capacitance of 40 microfarads.
Sketch the wave forms of the voltage and of the two currents, and comment on the differences between the two latter.

7. The e.m.f. of an alternator contains a fundamental wave of 500 volts, a third harmonic of 50 volts, and a fifth harmonic of 50 volts (all peak values). The frequency is 50 cycles per second. Show how to calculate the current which an ammeter would read when joined up in the main circuit, and traversed by the currents flowing through a condenser of 10 μF. capacitance in parallel

with a coil of negligible resistance having an inductance of 0·2 henry. Why is it unnecessary to know the phase relationship of the various harmonics in order to calculate the result?

8. A circuit contains four microfarads capacitance in series with 80 ohms resistance. To the terminals of the circuit a potential difference of frequency 50 is supplied. What must be the inductance in the circuit to give rise to resonance with the third harmonic? The maximum values of the first, third, and fifth harmonics are 1 200, 240, and 120 respectively What are the R.M.S. values of the current and of the p.d. across the condenser?

9. Explain why, with currents of irregular wave shape, it is possible to have a power factor less than unity, even when the time of zero current and zero potential difference is the same. [C. & G., II.

10. Calculate the power-factor of an alternating current arc from the curves given in Fig. 2.12.

11. What is the "power-factor" in an alternating current circuit? Give the approximate value of the "power-factor" with a sine-wave p.d. in the following cases:—an incandescent lamp, the primary of an unloaded transformer, an electric accumulator, and an arc lamp.

12. Explain why with non-sinusoidal waves in a two-phase three-wire system the ratio of the voltage across the outers to the voltage per phase is always $\sqrt{2}$ whatever the wave-form, and in spite of the differences between the two wave-forms.

Illustrate your answer by taking the phase voltage of the form given in Question 1, and with a maximum value of 100 volts.

13. Explain why in a Y-connected three-phase alternator the terminal voltage may be less but cannot be more than $\sqrt{3}$ times the phase voltage.

Find the ratio of the voltages if the phase voltage increases uniformly for $\frac{1}{12}$th period, remains constant for $\frac{1}{3}$rd period and then diminishes uniformly.

14. Prove that the 11th and 13th harmonics in the phase voltages of a Y-connected three-phase alternator reappear in the terminal voltage unchanged in phase and in magnitude relative to the fundamental.

15. Describe a method of obtaining the wave-forms of an alternating voltage and current.

16. A transformer is supplied from a generator with a pure sine wave p.d., and then from a generator with a triangular wave form. In each case the frequency and R.M.S. voltage is the same. What will be the ratio of the hysteresis loss in the two cases?

17. Analyse the two wave-forms of the resultant m.m.f. of a 3-phase stator given in Fig. 6.24, taking the maximum m.m.f. of one phase as unity.

18. A sinusoidal e.m.f. of R.M.S. value 280 volts and 50 cyc. is applied to a coil of 60 turns on an iron core of mean magnetic length 80 cm., and 150 cm.^{2} cross-section. The magnetic properties of the iron are as follows:—

Ampere-turns per cm. ..	2	4	6	8	10	12
Kilolines per cm.^{2}	5·5	10·1	12·2	13·1	13·9	14·5

Plot the wave-form of the magnetising current neglecting hysteresis.

19. The magnetising current and the flux in a transformer core are as given below:—

Flux (kilolines)	0	75	150	180	195	190	180	95	75	0
Magnetising current (amp.)	2·4	3·4	8·5	15·0	24·0	15·0	10·0	0	—1·2	—2·4

Assuming that the flux is a simple sine wave with a maximum value of 195 kilolines determine the wave-form of the magnetising current.

CHAPTER III

INSTRUMENTS FOR A.C. ONLY

1. Instrument Transformers

In many cases A.C. instruments are connected to the circuits in which measurements are required, not directly but through the medium of small transformers, called instrument transformers. The methods of connexion for ammeters and voltmeters are shown in Fig. 3.01. It will be seen that the primary winding of each transformer is connected as the instrument would be in the case of direct connexion, while the secondary winding is connected directly to the corresponding instrument.

The reasons for using instrument transformers are:—

(*a*) With high pressures the pressure applied to the instrument can be made as low as desired; hence its insulation does not have to withstand a high pressure, and moreover the instrument can be handled without any possibility of a dangerous shock: this applies to both ammeters and voltmeters.

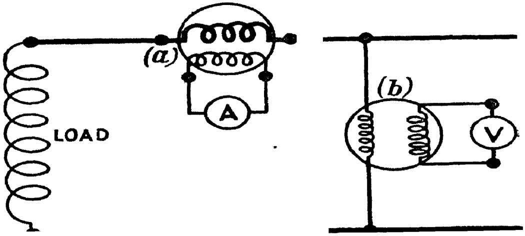


Fig. 3.01.—CONNEXIONS OF INSTRUMENT TRANSFORMERS.

(*a*) "Current" or "Series" transformer.
(*b*) "Voltage" or "Potential" transformer.

(*b*) With large currents the transformer can be placed so as to keep the large-sized conductors short, and only light leads carried to the ammeter, *e.g.* in a central station part of the bus bar itself may be used as the primary, and the ammeter for measuring the current in it can be mounted wherever is most convenient.

(*c*) The same instrument can be used with a number of different transformers so as to cover a wide range, *e.g.* for testing purposes, in the same way as a D.C. ammeter can be used with a number of different shunts, or a voltmeter with a number of different series resistances.

When an instrument is to be used always with the same transformer its scale is marked so as to give the current flowing in the main circuit, or the p.d. applied to the primary of the potential transformer, as the case may be, and not the actual current in (or p.d. across) the instrument itself.

If a voltmeter is to be disconnected without interfering with the rest of the circuit, either the primary or the secondary winding of the potential transformer may be opened by a suitable switch. But in the case of an ammeter one of the two windings of the current transformer must be *short-circuited* before the instrument is disconnected. If the secondary of a current transformer is open-circuited and the primary is left in circuit, the result may be a dangerous increase in the p.d.s across *both* windings. In addition the greatly increased iron losses will cause so large a rise of temperature that the quality of the iron may be altered permanently, and the accuracy of the transformation ratio spoiled.

The value of the transformation ratio depends chiefly on the ratio of the turns in the two windings, but is affected to some extent by the resistances of these two windings, and by the current required to produce the alternating flux upon which the action of the transformer depends. The resistance error is readily kept negligible, and to reduce the latter error the iron of an instrument transformer is preferably made without joints, or with imbricated joints. A well-designed potential transformer will give within $\frac{1}{2}$ per cent. of its nominal ratio from full pressure to $\frac{1}{10}$ of this at any ordinary frequency, the instrument reading being slightly low at high pressures (particularly if the frequency is low), unless the scale has been adjusted to allow for this.

Similarly, a good current transformer will give the correct ratio at unity power-factor within $\frac{1}{2}$ per cent. (or $\frac{1}{4}$ per cent. with a suitable ammeter) from full current down to $\frac{1}{5}$ of the full current. With lagging currents the ammeter reading becomes too low, the error as before being greatest at low frequencies (see Figs. 3.02, 03).

The reason for the increase of error at low power-factors is that the combined magnetising and iron-loss current has a very low power-factor. Hence the lower the power-factor of the secondary load the greater the ratio error (cf. Volume I., Chapter XVII., Art. 3).

Similarly lowering the frequency increases the flux, and so necessitates greater magnetising and iron-loss currents. The ratio error at any particular load and power-factor is increased in consequence.

The standard secondary voltage for potential transformers is 110 V. (B.S.S., No. 81), and the standard ratings 10 (for precision work), 15, 50, 100, and 200 volt-amperes. Thus the total current supplied by a 15 V.-A. potential transformer to a number of instruments (or other apparatus) connected in parallel should not exceed 136 mA., otherwise the accuracy is liable to fall below standard. The actual volt-amperes supplied by the secondary is called the "burden," to distinguish it from the load on the system in which the instrument transformer is connected.

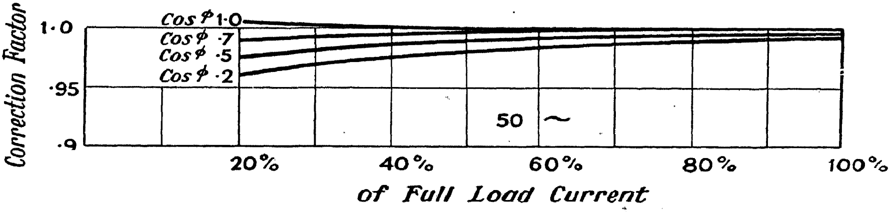


Fig. 3.02.—CURRENT TRANSFORMER ERRORS AT 50 CYC.

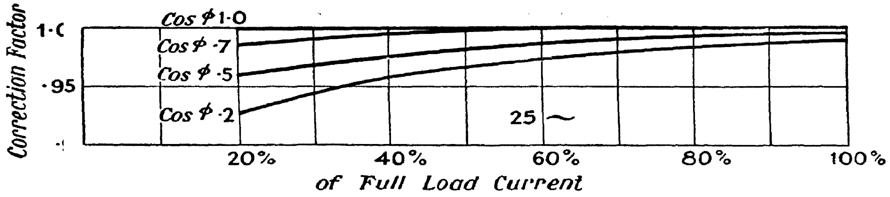


Fig. 3.03.—CURRENT TRANSFORMER ERRORS AT 25 CYC.

The standard secondary currents for current transformers are 5 A. and 1 A. And the standard rated burdens 7½ V.-A. for precision work, and 5, 15, and 40 V.-A. for other uses. The total impedance of a number of ammeters (or other apparatus) connected in series to a 15 V.-A. current transformer should not exceed 0·6 ohm if the full load secondary current is 5 A.

For wattmeters (see Volume I., Chapter VII., Art. 13) the amount of phase difference between the currents in the primary and secondary windings is of importance, but this does not affect the accuracy of the readings of ammeters and voltmeters.

2. Frequency Indicators

These indicators are generally of the **tuned reed** pattern. In this a number of steel strips are adjusted so that each vibrates at a particular frequency. The strips are all acted on by an electromagnet the winding of which is connected to the supply in the same way as a voltmeter. The result is that each strip is attracted *twice* in every cycle the force falling to zero in each interval between successive attractions. Consequently the strip whose natural time of vibration corresponds with the frequency of the attractive force vibrates to a large extent, whereas the other strips vibrate little or not at all.

To facilitate the reading of the frequency each strip has attached to its free end a small rectangular piece of metal painted white.

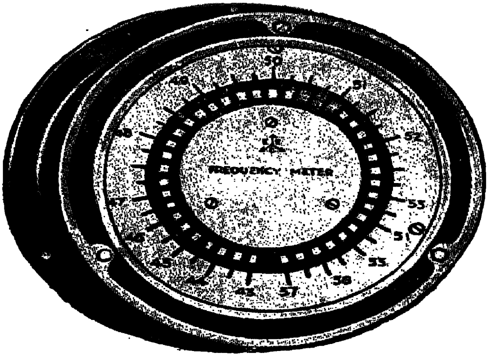


Fig. 3.04.—FREQUENCY METER.
(*Everett, Edgcumbe & Co.*)

The strips are arranged in order of frequency in a circle (see Fig. 3.04) or in one or two lines with a scale of frequencies near their tips. The frequency is then obtained by noting on the scale the number opposite the one or two reeds which are thrown into vibration.

If the reeds are polarised, *i.e.* permanently magnetised, they will be attracted once and repelled once in each cycle. The frequency of electrical supply to cause vibration is then equal to the reed frequency, instead of half this as with unmagnetised reeds.

The same effect is produced by using a polarised magnet core of sufficient strength. For then the pull of the magnet varies between a maximum when the coil current assists the permanent magnetism and a minimum when the two are opposed. For experimental purposes the frequency indicator may be provided with two electromagnets, one polarised and the other not. The polarised one will thus read frequencies twice those of the non-polarised one with the same set of reeds.

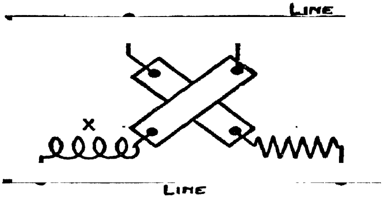


Fig. 3.05.—CONNEXIONS OF FREQUENCY INDICATOR.

3. Switchboard Frequency Indicators

These frequency indicators may be of the tuned reed type (see previous Art.) or may employ the variation of reactance with frequency for indicating the frequency.

A simple arrangement of this sort is to use two coils at right angles one with a non-inductive resistance (R) in series and the other with a highly inductive resistance (X) in series. The two circuits are connected in parallel across the mains like voltmeters (see Fig. 3.05). The *direction* of the resultant alternating field depends on the ratio of the currents in the two coils and so is unaffected by a change of voltage (cf. the case of the ohmmeter, Volume I., Chapter III., Art. 20).

But a change of frequency will alter the current in X while not affecting that in R. Consequently the resultant field changes in direction, becoming more nearly perpendicular to the plane of the coil in series with R the higher the frequency.

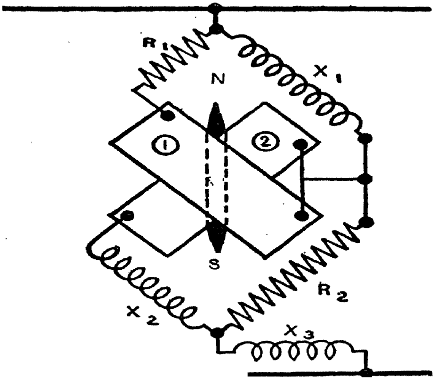
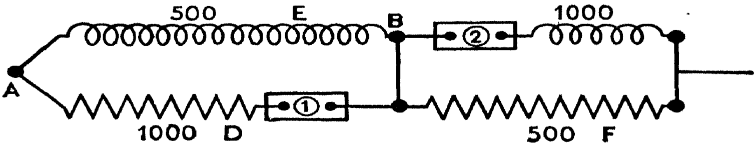


Fig. 3.06.—CONNEXIONS OF WESTON FREQUENCY INDICATOR.

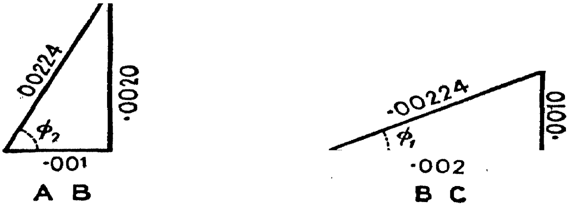
A soft iron needle (in Fig. 3.06, NS) is pivoted at the common centre of the two coils, and being balanced and uncontrolled takes up a position along the resultant magnetic field. The pointer attached to this needle moves over a scale marked in cycles per second. A damping device is fitted so that the pointer gives a steady reading.

The instrument is calibrated by trial, and has the disadvantage compared with the tuned reed type of being affected to some extent by the wave form (see Chap. II., § 11).

A more elaborate arrangement using the same principle is employed by the Weston Co. In this (see Fig. 3.06) two resistances and two reactances are connected to the coils in a manner analogous to Wheatstone's Bridge. The result is that with suitable resistances and reactances an increase of frequency not only reduces the current in coil No. 2 (as in the simpler arrangement) but increases the current in coil No. 1 at the same time (see Example No. 1). The result is that a very open scale is obtained.



(*a*) CIRCUIT DIAGRAM.



(*b*) ADMITTANCE TRIANGLES.

Fig. 3.07.—FREQUENCY INDICATOR AT 50 CYCLES PER SEC.

The series reactance (X_3, Fig. 3.06) is used to reduce the effect of the higher harmonics in the voltage wave, and so diminishes the effect on the indications of the instrument of any departure from the sinusoidal shape.

Example 1. *If in Fig.* 3.06, R_1 = 1 000 *ohms*, R_2 = 500 *ohms, both non-inductive ;* X_1 = 500 *ohms.* X_2 = 1 000 *ohms, both at* 50 *cycles per sec. and both of negligible resistance ; find the currents in the coils with* X_3 *omitted and* 200 *volts applied to the terminals,* (*a*) *at* 50 *cycles per sec.,* (*b*) *at* 60 *cycles per sec.*

The circuits are as shown in Fig. 3.07*a*.

(*a*) Admittance of ADB = ·001 (entirely conductance)
Admittance of AEB = ·002 (entirely susceptance) ;

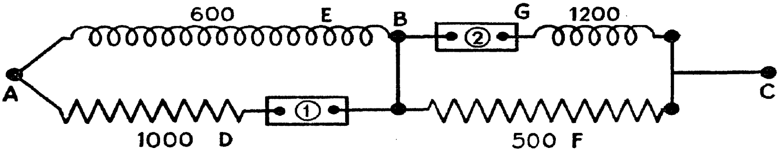
$\therefore$ Total admittance of AB $= \sqrt{\{(\cdot 001)^2 + (\cdot 002)^2\}} = \cdot 00224$;
$\therefore$ Equivalent impedance of AB $= 1/\cdot 00224 = 447$ ohms,
Resistance component of this impedance $= (\cdot 001/\cdot 00244) \times 447 = 200$ ohms,
and reactance component $= (\cdot 002/\cdot 00244) \times 447 = 400$ ohms.
Similarly for BC; admittance $= \sqrt{\{(\cdot 002)^2 + (\cdot 001)^2\}}$
$= \cdot 00224$;
Impedance $= 447$ ohms; resistance $= 400$ ohms; reactance $= 200$ ohms.

Hence (see Volume I., Chapter V., Art. 20):—
Total impedance of ABC $= \sqrt{\{(400 + 200)^2 + (200 + 400)^2\}} = 848$ ohms;
$\therefore$ Total current $= 200 \div 848 = 0\cdot 236$ amp.

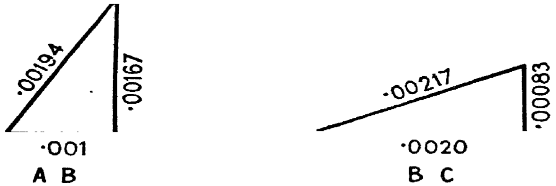
This current divides itself between the parallel portions in the ratio of the admittances;

$$\therefore \text{ Current in coil No. 1 } (\textit{i.e.} \text{ in ADB}) = \cdot 236 \times \frac{\cdot 001}{\cdot 00224} = 0\cdot 105 \text{ amp.}$$

and the current in coil No. 2 has the same value.



(a) CIRCUIT DIAGRAM.



(b) ADMITTANCE TRIANGLES.

Fig. 3.08.—FREQUENCY INDICATOR AT 60 CYCLES PER SEC.

Note that currents in AEB and BFC are each $\cdot 236 \times (\cdot 002/\cdot 00224) = 0\cdot 210$ amp.

(b) The reactances vary with the frequency and so become 1 200 ohms, and 600 ohms respectively at 60 cyc. (see Fig. 3.08*a*).

Admittance of ADB $= \cdot 001$ (entirely conductance) as before.
Admittance of AEB $= 1/600 = \cdot 00167$ (entirely susceptance);
$\therefore$ Total admittance of AB $= \sqrt{\{(\cdot 001)^2 + (\cdot 00167)^2\}} = \cdot 00194$;
$\therefore$ Impedance of AB $= 1/\cdot 00194 = 515$ ohms,
Resistance component $= (\cdot 001/\cdot 00194) \times 515 = 265$ ohms,
and reactance component $= (\cdot 00167/\cdot 00194) \times 515 = 443$ ohms.
Admittance of BFC $= \cdot 002$; and of BGC $= 1/1200 = \cdot 00083$;
$\therefore$ Total admittance of BC $= \sqrt{\{(\cdot 002)^2 + (\cdot 00083)^2\}} = \cdot 00217$;

$$\therefore \text{ Impedance of BC} = 1/{\cdot}00217 = 461 \text{ ohms,}$$

$$\text{Resistance component} = ({\cdot}002/{\cdot}00217) \times 461 = 426 \text{ ohms,}$$

$$\text{and reactance component} = ({\cdot}00083/{\cdot}00217) \times 461 = 177 \text{ ohms;}$$

$$\therefore \text{ Total impedance of ABC} = \sqrt{\{(426 + 265)^2 + (177 + 433)^2\}} = 928 \text{ ohms;}$$

$$\therefore \text{ Total current} = 200 \div 928 = 0{\cdot}216 \text{ amp;}$$

$$\therefore \text{ Current in coil No. 1} = {\cdot}216 \times ({\cdot}001/{\cdot}00194) = 0{\cdot}111 \text{ amp.,}$$

i.e. an increase of 6 per cent. over its value at 50 cyc.

$$\text{and current in coil No. 2} = {\cdot}216 \times ({\cdot}00083/{\cdot}00217) = 0{\cdot}083 \text{ amp.,}$$

i.e. a decrease of 21 per cent. from its value at 50 cyc.

[Note that with the simpler arrangement of Fig. 3.05 the current in coil No. 1 would be unchanged and that in coil No. 2 decreased by $\frac{1}{6}$th = 17 per cent. only.]

4. Power-Factor Meters

Power-factor meters may be used to indicate directly the power factor of a circuit instead of obtaining this by a wattmeter (see Volume I., Chapter VII., Art. 13), an ammeter and a voltmeter.

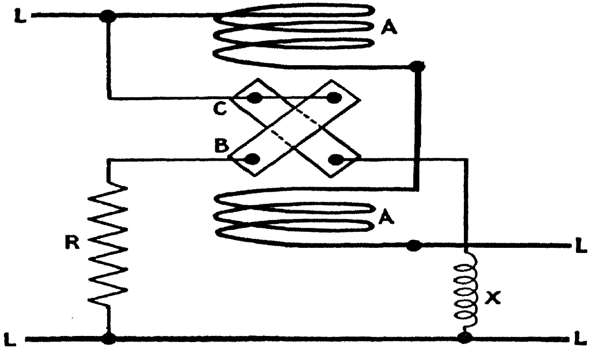


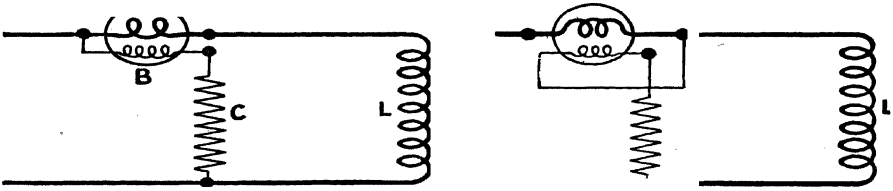
Fig. 3.09.—CIRCUITS OF POWER-FACTOR METER.

The method of action of such an instrument is shown in Fig. 3.09. A fixed coil divided into two halves (A, A) carries the main current. Pivoted between the two portions is a pair of moving coils B, C. These are fixed at right angles to each other and move together, with no controlling force. One of them (B) has a resistance (R) in series and the other (C) a reactance (X) in series, and the two circuits are connected in parallel across the mains. Consequently B carries a current proportional to and in phase with the voltage, whereas the current in C lags behind the voltage by a large angle.

When the main current is in phase with the voltage B sets itself parallel to A, A. If the main current lags by the same amount as the current in C the latter coil will set itself parallel to A, A. For intermediate values of lag of the main current the moving coils (B, C) will take up an intermediate position, such that their resultant field is in line with the field of AA at the instant when the latter is at its maximum value.

With leading main current the same is true, but this necessitates a movement of BC in the opposite direction to that caused by a lagging current. Consequently a pointer attached to the moving coils can be used to indicate the power factor of the circuit on a scale, and to show whether the main current is lagging or leading.

On polyphase circuits the reactance (X) is replaced by a resistance and the ends of the circuits of coils BC are connected to points between which are two p.d.s differing in phase by a fixed amount; *e.g.* in a 3-phase system circuit B may be connected across one pair of mains and circuit C across a different pair. With such connexions the indications of the instrument are independent of frequency, but in the monophase pattern the instrument is accurate only at the frequency for which it has been calibrated.



(a) *(b)*

Fig. 3.10.—CONNEXIONS OF WATTMETER.

A. Current coil. B. Pressure coil. C. Series resistance L. Load.

For polyphase circuits with unbalanced loads the average power factor can be found by the reading of a single instrument. This has one fixed current coil for each phase placed at angles corresponding with the phase differences, *e.g.* for 3-phase 3 coils at 120° apart. It has the same number of moving pressure coils held at the same angles apart and joined to corresponding phases (usually Y-connected for 3-phase). The position taken up depends on the average power factor. By using short-circuiting switches in the current-coil circuits the same instrument may be used to give the power factors of the separate phases.

5. Wattmeter Errors due to Resistance

With a wattmeter connected as shown in Fig. 3.10 (*a*), the p.d. across the pressure circuit is greater than that across the load

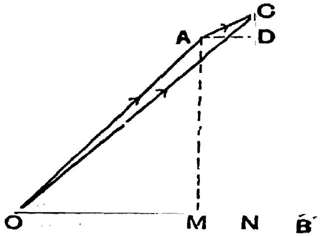


Fig. 3.11.—VECTOR DIAGRAM FOR WATTMETER.

owing to the drop of voltage over the current coil. This drop amounts to IZ_c, having a component in phase with the current equal to IR_c, where Z_c, R_c = impedance and resistance of the current coil, and I = main current. Hence if OA in Fig. 3.11 represents the p.d. on the load (V), OB the current in it (I), and AC the drop over the current coil (IZ_c), the p.d. across the voltage coil is given by OC, the vector sum of OA and AC.

Draw AM and CDN perpendicular to OB, and AD parallel to OB. Then—

$$AD = IR_c$$

$$\begin{aligned}\text{The wattmeter reading} &= OC.OB \cos \angle BOC \\ &= OB.ON = OB\,(OM + MN) \\ &= OB\,(OA \cos \angle MOA + AD) \\ &= I\,(V \cos \phi + IR_c) \\ &= VI \cos \phi + I^2R_c,\end{aligned}$$

i.e. the reading is greater than the true power by I^2R_c, the watts absorbed by the current coil.

The truth of this result can be seen by noting that with the connexions of Fig. 3.10 (*a*) the wattmeter is really measuring the power absorbed by the load and the current coil in series.

The above source of error can be avoided by connecting the pressure coil in the alternative way shown in Fig. 3.10 (*b*). But this introduces a fresh error, for the current coil now carries the vector sum of the currents in the load and in the pressure coil.

By means of a vector diagram, or by considering the connexions, it can be shown that the wattmeter reading in this case is again greater than the true power and that the excess is now equal to the power absorbed by the pressure coil, *i.e.* V^2/R_v, where R_v = resistance of pressure circuit.

The two errors are equal when $R_cR_v = V^2/I^2 = Z^2$ (where Z = impedance of the load), *i.e.* when the impedance of the load is a geometric mean between the resistances of the current coil and of the pressure circuit. For higher values of the load impedance, *i.e.* for small currents, the first method of connexion is preferable; and for lower values of the impedance, *i.e.* for large currents, the second method gives the smaller error (see Example 2).

For D.C. watt-hour meters up to 50 A., B.S.S. No. 37 states that the loss in the main coil at marked current shall be not greater than 5 watts, nor the drop greater than 1 volt (the latter provision makes no difference except for currents lower than 5 A.). For currents above 100 A. more loss is permitted and the drop is allowed to reach 0·25 volt. It provides further that the loss in the shunt circuit shall be not greater than 5 watts total or 2 watts per 100 volts, whichever is the larger.

For A.C. energy meters (see § 13) the allowable losses are less, since it is easier to keep them low in that type. The figures for the current circuit are $1\frac{1}{2}$ watts up to and including the 10 A. size, 2 watts above that and up to 50 A., and the same as for D.C. above 100 A.; and for the voltage circuit 1·8 watts up to 230 V., and 1 watt per 100 v. at higher Voltages.

Example 2. *A wattmeter for 200 volts 20 amperes has the maximum coil resistances allowable for D.C. watt-hour meters.*

Find the percentage errors due to resistance with each of the methods of connexion when measuring the power taken by loads which at 200 volts take (a) 8 A., (b) 24 A.; for power factors of unity and 0·6 respectively.

$$\text{Resistance of main coil} = \frac{\text{watts}}{(\text{current})^2} = \frac{5}{20^2} = \cdot 0125 \text{ ohm.}$$

$$\text{Resistance of shunt circuit} = \frac{(\text{volts})^2}{\text{watts}} = \frac{200 \times 200}{5} = 8\,000 \text{ ohms.}$$

At unity power-factor, connected as in Fig. 3.10 (*a*):—

(*a*) $\text{Error} = 8^2 \times \cdot 0125 = 0{\cdot}8$ watt

$\text{True power} = 8 \times 200 = 1\,600$ watts;

$$\therefore \text{Percentage error} = \frac{0{\cdot}8 \times 100}{1\,600} = 0{\cdot}05 \text{ per cent.}$$

(*b*) $\text{Error} = 24^2 \times \cdot 0125 = 7{\cdot}2$ watts

$\text{True power} = 24 \times 200 = 4\,800$ watts;

$$\therefore \text{Percentage error} = \frac{7{\cdot}2 \times 100}{4\,800} = 0{\cdot}15 \text{ per cent.}$$

Connected as in Fig. 3.10*b*:—

(*a*) $\text{Current in pressure circuit} = \frac{200}{8\,000} = \cdot 025$ amp.;

$\text{Error} = 200 \times \cdot 025 = 5$ watts;

$$\text{Percentage error} = \frac{5 \times 100}{1\,600} = 0{\cdot}31 \text{ per cent.}$$

(*b*) Error = 5 watts as before;

$$\therefore \text{Percentage error} = \frac{5 \times 100}{4\,800} = 0{\cdot}10 \text{ per cent.}$$

With a power-factor of 0·6 the errors are unchanged but the true power is reduced: the percentage errors therefore become 0·08 per cent., 0·25 per cent., 0·52 per cent., and 0·17 per cent. in the same order as above.

When the wattmeter is used for measuring the output of an alternator or other source of A.C. power the effect of the coil resistances is to make the readings too *low* for both methods of connexion.

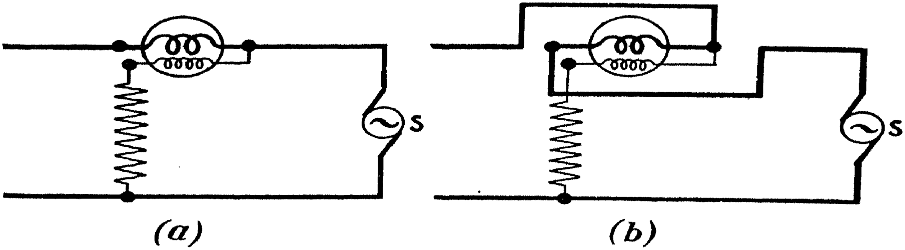


FIG. 3.12.—CONNEXIONS OF WATTMETER TO SOURCE OF POWER.

S = Source of Power, *e.g.* an alternator.

If connected as in Fig. 3.10*a* with a source of power in the position of the load in that figure the reading will be negative, and so the error is in the same direction (the positive) for both load and source. If only one of the coils has its connexions reversed without any other change being made the error becomes negative (*i.e.* reading is too low) and amounts to the loss in the main coil (I^2R_c).

On the other hand if the connexions between the wattmeter coils are unaltered and the connexions of the wattmeter to one main are reversed (see Fig. 3.12*a*), the negative error is equal to the loss in the pressure circuit (V^2/R_v).

Similarly, if the connexions of the wattmeter coils are kept as in Fig. 3.10*b*, and the connexions to one main reversed as in Fig. 3.12*b*, the negative error is again equal to the loss in current coil.

6. Compensating Coil

The resistance error can be corrected automatically by means of a compensating coil. This consists of a coil of the same number of turns as the current coil and wound on the same bobbin. It is connected in series with the pressure coil in such a way that its current flows in the opposite direction to that in the current coil (see Fig. 3.13).

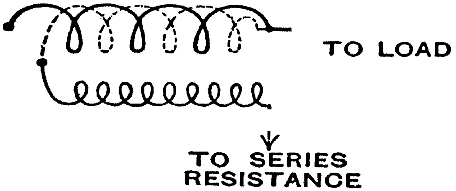


FIG. 3.13.—CONNEXIONS OF COMPENSATING COIL.

- - - Compensating coil.

Its current will be exactly opposite in phase to that in the current coil only when the load is non-inductive. But in all cases it will carry the same current as the pressure coil, and so will wipe out the additional field produced by the current coil due to its carrying the shunt current in addition to the load current. The result is that the net field is that corresponding to the load current alone, and so the wattmeter reading is made accurate in spite of the resistance of the current coil.

The drawback of this method of compensation is that it increases the inductance of the pressure circuit, and so increases the error due to this cause (see Art. 7). It is, therefore, undesirable for wattmeters which are to be used at very low power-factors.

7. Wattmeter Errors due to Inductance

Inductance in the current coil does not affect the accuracy, as the field is in phase with the current, unless modified by hysteresis or eddy currents. But inductance in the pressure circuit causes the current in that circuit to lag behind the p.d. applied to it.

The result is that the phase difference between the currents in the two coils is not the same as that between the p.d. and the main current, and the reading of the wattmeter becomes inaccurate. With a lagging main current the phase difference is reduced by the inductance, and so the wattmeter reading is made too high. With a leading main current the opposite effects are produced.

Another result is that the impedance of the pressure circuit is greater than its resistance, and so the shunt current per volt is less with A.C. than with D.C. This latter effect is so small as to be negligible in any well-designed wattmeter.

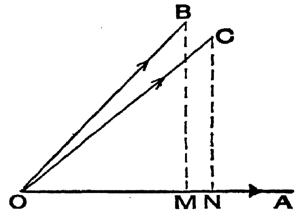


Fig. 3.14.—VECTOR DIAGRAM SHOWING EFFECT OF INDUCTANCE.

In the vector diagram shown in Fig. 3.14, let OA represent the current (I) in the main and in the current coil, and OB represent the p.d. (V). Then $\angle$ AOB $= \phi =$ angle of lag of current.

If OC represents the current in the pressure circuit, OC $= V/Z_v$; and tan $\angle$ BOC $= X_v/R_v$, where Z_v, X_v, R_v are the impedance, reactance, and resistance of the pressure circuit. Then the reading of the wattmeter is proportional to OC . OA cos $\angle$ COA, or:—

$$\text{Reading} = (\text{constant}) \times \frac{V}{Z_v} \times I \times \cos(\phi - \alpha),$$

where $\alpha =$ angle of lag of current in pressure circuit.

At unity power-factor, $\phi = 0$,

$$\therefore \text{ Reading} = (\text{constant}) \times \frac{VI \cos \alpha}{Z_v};$$

and the true power is then VI. Therefore, if the wattmeter reads accurately at unity power-factor, the constant must equal $Z_v/\cos\alpha$. Substituting this in the above expression:—

$$\text{Reading} = \frac{VI\cos(\phi-\alpha)}{\cos\alpha};$$

$$\therefore \frac{\text{Reading}}{\text{True power}} = \frac{VI\cos(\phi-\alpha)}{\cos\alpha\,.\,VI\cos\phi} = \frac{\cos\phi\cos\alpha + \sin\phi\sin\alpha}{\cos\alpha\cos\phi}$$
$$= 1 + \tan\phi\tan\alpha$$
$$= 1 + \frac{X_v}{R_v}\tan\phi.$$

Thus for all lagging currents the wattmeter will read too high, the fractional error being $X_v/R_v \tan\phi$. The error, therefore, increases as ϕ increases, and becomes relatively very large at low power-factors (see Example 3).

Moreover, for a given wattmeter the error at any particular power-factor is proportional to the frequency, since reactance varies as frequency.

If the wattmeter has been calibrated with D.C. the following modification is produced. With D.C. the angle α vanishes, and Z_v becomes equal to R_v. Therefore the constant in the expression for the reading becomes equal to R_v, and so equal to $Z_v\cos\alpha$;

$$\therefore \text{Reading} = VI\cos(\phi-\alpha)\cos\alpha.$$

This is less than in the previous case in the ratio $\cos^2\alpha/1$; and the ratio (reading/true power) is $\cos^2\alpha\,(1+\tan\phi\tan\alpha)$.

In a well-designed wattmeter α will not exceed ¼ degree up to 60 cyc., and the difference between $\cos^2\alpha$ and 1 is less than $\frac{1}{1000}$ up to an angle seven times as large. Consequently the same correcting factor may be used whether D.C. or A.C. calibration is employed.

Example 3. *If the reactance of the pressure circuit of a wattmeter is $\frac{4}{1000}$ of the resistance, calculate the percentage error at various power-factors:—*

Cos ϕ ..	1	0·8	0·6	0·5	0·4	0·3	0·2	0·1
Tan ϕ..	0	0·75	1·33	1·73	2·29	3·18	4·90	9·96
Error ..	0	0·30	0·53	0·69	0·92	1·27	1·96	3·98 *per cent.*

Tan ϕ is obtained either by using a table of cosines to find ϕ, and then a table of tangents; or else from the relation $\tan\phi = \sqrt{1-\cos^2\phi}/\cos\phi$.

The percentage error if $100\,\frac{X_v}{R_v}\tan\phi$ of true power

$$= 0{\cdot}4\tan\phi \text{ in this case.}$$

A power-factor of 0·2 corresponds with an angle of lag of 78½°, and one of 0·1 with a lag of 84¼°, so that it is only for angles of lag approaching 90° that the error becomes of importance.

The above results are plotted in Fig. 3.15. If given as percentages of the reading the errors would be slightly lower for inductive loads; and slightly higher for capacitance loads as the reading has to be increased by the error in this latter case (see Example 4).

8. Errors due to Capacitance and to Eddy Currents

If the resistance in series with the pressure coil possesses capacitance an error opposite to that due to inductance is produced. For the pressure coil will then carry a current leading the p.d., and so with lagging loads the phase difference between the currents in the two coils of the wattmeter will be greater than the true angle of lag.

The error, as in the previous case, will be $\tan\beta \tan\phi$ of the true power, where β = angle of lead of the current in the pressure coil ahead of the p.d.; but the error is negative, *i.e.* the reading is too low. With a leading main current the reading is too high by the same amount.

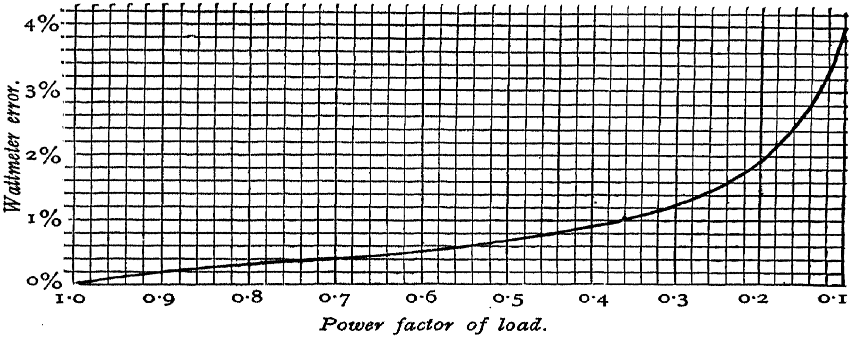


Fig. 3.15.—WATTMETER ERROR DUE TO INDUCTANCE.

Capacitance is produced in the resistance if its inductance is reduced by doubling the wire on itself before winding it into a coil. By dividing the resistance into a number of sections both capacitance and inductance are decreased inversely as the number of sections.

A small amount of capacitance is an advantage, for it corrects the error due to the necessary inductance of the pressure coil. Since both the capacitance current and the induced e.m.f. are proportional to the frequency, this correction if accurate at one frequency will be so at any other, and also for any wave form.

Eddy currents induced in solid metal in the neighbourhood of the current coil produce errors of the same sort as capacitance in the series resistance. For these currents will lag behind the main current by approximately 90°. Hence the field due to the combined effect of the main current and the eddy currents will lag behind the former. An inspection of Fig. 3.14 will show that the effect of this on the wattmeter reading is opposite to that of inductance in the series resistance, *i.e.* the same as that of capacitance in this resistance.

As, however, the angle of lag is not quite 90°, particularly if the metal is magnetic, the effect is not exactly the same. It is, therefore, advisable to avoid eddy currents by keeping solid metal away from the current coil, and by slotting the metal when this cannot be arranged.

Nevertheless, wattmeters are often enclosed in cast-iron cases to shield them from the magnetic effects of neighbouring conductors, the current coil being kept some distance from the case to diminish errors due to eddy currents.

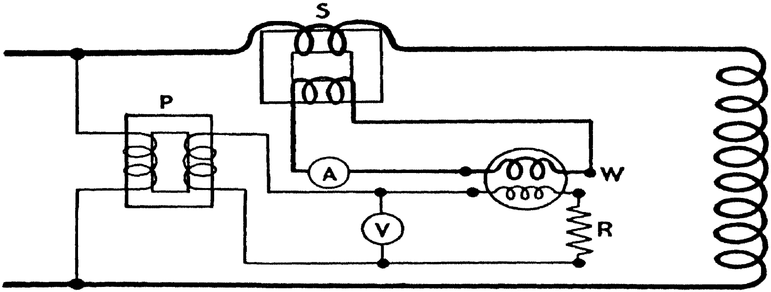


Fig. 3.16.—WATTMETER WITH INSTRUMENT TRANSFORMERS.

A. Ammeter. L. Load. P. Potential transformer. S. Current transformer.
W. Wattmeter. R. Series resistance of W. V. Voltmeter.

9. Wattmeters with Transformers

Instrument transformers are used with either the pressure circuit or the current coil of a wattmeter or with both, for the same reasons as for voltmeters and ammeters respectively (see Art. 1). When both are used the connexions are as shown in Fig. 3.16. The secondary of the potential transformer is connected to the pressure coil and its series resistance; and the secondary of the current transformer to the current coil of the wattmeter.

The same two transformers may be used for a voltmeter and an ammeter respectively, provided this does not overload the instrument transformers and so impair their accuracy.

By using a number of different current transformers with the same full load value for their secondary currents (usually 5 A. or 1 A.) the same wattmeter may be used for as wide a range of currents as desired. Similarly by the use of several pressure transformers the one instrument can be made suitable for any pressure.

When the wattmeter is to be used only with one pair of transformers its scale is marked so as to indicate the power in the main circuit directly, without the use of any multiplying factor. In other cases the ratios of the transformers are so chosen that the multiplying factors are simple ones, *e.g.* 2, 10, 50.

For use with wattmeters the phase difference between the primary and secondary pressures or currents (according to type of transformer) are of importance as well as the transformation ratio (cf. Art. 1).

The error in the case of a voltage transformer causes the current in the pressure coil to lag behind the primary p.d. by a greater angle than that due to the inductance of the pressure coil (see Art. 7), *i.e.* if α is angle of lag of current in the wattmeter's pressure circuit (as in Art. 7), and γ = phase difference of potential transformer, the ratio (reading/true power) becomes $1 + \tan\phi \tan(\alpha + \gamma)$.

Since α and γ are small angles $\tan(\alpha + \gamma)$ is very nearly equal to $\tan\alpha + \tan\gamma$, and therefore the errors due to the transformer and to the wattmeter may be measured or calculated separately and then added. This method has some advantages over the direct measurement or calculation of the total error.

The phase error of the series transformer consists in its secondary current *leading* the primary. The error thereby caused in the wattmeter reading is therefore of the same sort as that due to the pressure transformer, *i.e.* the reading is too high with inductive loads.

If δ is the angle of phase difference of the current transformer, the ratio (reading/true power)

$$= 1 + \tan\phi \tan(\alpha + \gamma + \delta)$$

$$= 1 + \tan\phi (\tan\alpha + \tan\gamma + \tan\delta) \text{ approx.},$$

i.e. the errors due to the three causes may be obtained separately and added.

In precision instrument transformers the value of γ will not exceed $\frac{1}{3}$ of a degree, and can be made less by using special alloyed iron. The value of δ varies with the load (decreasing as this increases), and with the power-factor of the circuit connected to the transformer secondary. A precision transformer should give values not exceeding 35 minutes from $\frac{1}{5}$ full load to full load when non-inductive, and not exceeding 50 min. down to $\frac{1}{10}$ of rated full load current. For good switchboard transformers γ should not exceed $\frac{1}{2}$ degree; and δ be not more than 1 degree to $\frac{1}{8}$ full load, and 2 degrees down to $\frac{1}{10}$ full load.

The resistance errors which occur in a wattmeter without transformers (see Art. 5) occur likewise with transformers, but it is then the power absorbed by the transformer which gives the correction; *i.e.* according to the method of connexion the power taken either by the pressure transformer or by the current transformer must be subtracted from the wattmeter reading to obtain the power supplied to the load.

With the connexions shown in Fig. 3.16 it is the power absorbed by the current transformer which must be thus subtracted. Similarly in measuring the output of alternators, etc., the power absorption of one of the instrument transformers must be added to the wattmeter reading (cf. Art. 5).

Example 4. *A wattmeter and its transformers have phase displacements of* 0·2° *in its pressure circuit,* 0·2° *in its pressure transformer, and in its current transformer* 0·6° *at* $\frac{1}{8}$ *load, and* 0·25° *at full load. Find the errors due to these for inductive loads with power-factors of* 0·6, 0·4, *and* 0·2 *respectively* (cf. Example 2).

$\cos\phi =$	0·6	0·4	0·2
$\tan\phi =$	1·33	2·29	4·90
Correction at $\frac{1}{8}$ load =	2·33	4·01	8·58 per cent. of true power.
	2·28	3·85	7·84 per cent. of reading.
At full load =	1·51	2·59	5·54 per cent. of true power.
	1·49	2·53	5·23 per cent. of reading.

At $\frac{1}{8}$ load, $\tan(\alpha + \gamma + \delta) = \tan 1{\cdot}0^\circ = {\cdot}0175$.

At full load, $\tan(\alpha + \gamma + \delta) = \tan 0{\cdot}65^\circ = \tan 39' = {\cdot}0113$.

$$\text{The correction} = \frac{\tan\phi \tan(\alpha + \gamma + \delta)}{1 + \tan\phi \tan(\alpha + \gamma + \delta)} \text{ of reading (inductive load)}$$

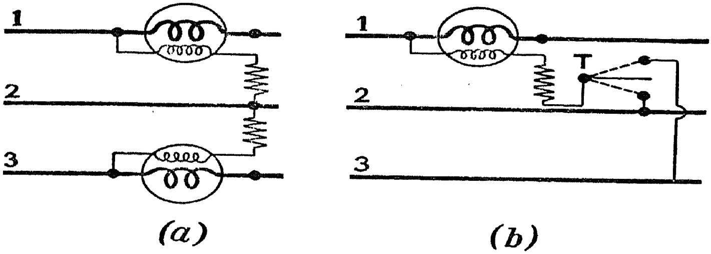
$$= \tan\phi \tan(\alpha + \gamma + \delta) \text{ of true power.}$$

10. Power Measurement in 3-Phase Circuits

As shown in Volume I., Chapter VI., Art. 6, the total power in any three-phase circuit is given by $V_1I_1 \cos\phi_1 + V_3I_3 \cos\phi_3$, and consequently can be measured by two wattmeters connected as in Fig. 3.17*a*.

The total power is the sum of the readings of the two wattmeters.

Under certain conditions the reading of one of the wattmeters will be negative: if its zero is at one end of the scale it is then necessary to reverse the connexions of the pressure coil (*or* of the main coil), and to subtract the reading from that of the other wattmeter.



(a) UNBALANCED LOAD.

(b) T = Two-way switch. BALANCED LOAD.

Fig. 3.17.—TWO-WATTMETER METHOD OF MEASURING 3-PHASE POWER.

If the load is a balanced one (*e.g.* a 3-phase motor) the two readings may be taken on the same wattmeter by connecting as in Fig. 3.17*b*. The phase relation of line 1 to line 3 is the same as that of line 3 to line 2, and so with the balanced load the readings will be the same as with the connexions of Fig. 3.17*a*.

With unbalanced loads it is still possible to use only one wattmeter but the switching arrangements are much more complicated than with a balanced load. It is necessary to short-circuit the current coil before changing it over from one line to the other, and so at least a double-pole throw-over switch and two single-pole switches are required.

11. Power-Factor in 3-Phase Circuits from Wattmeter Readings

The power-factor in a 3-phase circuit can be obtained by dividing the power by $\sqrt{3}$ VI, where V = voltage between any two lines, and I = current in any line with balanced loads, or the average of the currents in the three lines with unbalanced loads.

When the loads are balanced the power-factor can be obtained from the two wattmeter readings, with the connexions described in Art. 10, Fig. 3.17*b*, by using the formula:—

$$\cos\phi = 1/\sqrt{\left\{1 + 3\left(\frac{W_1 - W_2}{W_1 + W_2}\right)^2\right\}} = (1 + m)/2\sqrt{\{1 - m + m^2\}},$$

where W_1 = larger of the two wattmeter readings, W_2 = smaller of the two wattmeter readings, and $m = W_2/W_1$.

The advantage of this method is that the value of cos ϕ depends only on the ratio of the two wattmeter readings, whereas by the previous method the value depends on the sum of these readings and on the accuracy of the voltmeter and of the ammeter.

PROOF.—In Fig. 3.18 let OA, OB, OC represent vectorially the p.d.s across the arms of a Y-connected 3-phase load; and OD, OE, OF the corresponding currents. (R.M.S. values in all cases.)

Then with the connexions of Fig. 3.17*b* the reading (W_1) of the wattmeter when its pressure coil is connected to line No. 3 is equal to

$$\text{CA} \times \text{OD} \times \cos\,(\text{angle between CA and DA})$$
$$= \sqrt{3}.\text{OA}.\text{OD} \cos\,(\phi - 30) = \sqrt{3}\,\text{VI} \cos\,(\phi - 30^\circ).$$

Similarly when the pressure coil is connected to line No. 2, the reading (W_2) of the wattmeter gives BA.OD.cos (angle between BA and OD)

$$= \sqrt{3}\,\text{VI} \cos\,(\phi + 30^\circ);$$
$$\therefore\ W_1 + W_2 = \sqrt{3}\,\text{VI}\,\{\cos\,(\phi - 30^\circ) + \cos\,(\phi + 30^\circ)\}$$
$$= 2\sqrt{3}\,\text{VI} \cos\phi \cos 30^\circ$$
$$= \sqrt{3}\,\text{VI} \cos\phi;\ \text{since} \cos 30^\circ = \tfrac{1}{2}.$$

[This is an alternative proof, for the case of balanced loads and sinusoidal waves, of the fact that the sum of the two wattmeter readings gives the total power.]

Similarly:— $W_1 - W_2 = \sqrt{3}\,\text{VI}\,\{\cos\,(\phi - 30^\circ) - \cos\,(\phi + 30^\circ)\}$

$$= 2\sqrt{3}\,\text{VI} \sin\phi \sin 30^\circ$$
$$= 3\,\text{VI} \sin\phi;\ \text{since}\ \sin 30^\circ = \sqrt{3}/2;$$
$$\therefore\ \tan\phi = \sin\phi/\cos\phi = \sqrt{3}\,(W_1 - W_2)/(W_1 + W_2)$$

whence the above formula follows since:— $\cos\phi = 1/\sqrt{\ }(1 + \tan^2\phi)$.

For a Δ-connected load a similar proof holds, but it is then the *current* in the wattmeter which is the vector difference between the currents in two of the phases.

When $\phi = 60^\circ$, $\cos\phi = \frac{1}{2}$, and it can be seen from the formula that W_2 is then zero. The vector diagram (Fig. 3.18) likewise shows that in this case BA and OD are at right angles, and so $W_2 = 0$. When ϕ is greater than 60°, W_2 becomes negative and so the formula correctly makes $\cos\phi$ less than $\frac{1}{2}$ (see Example 4). It is then necessary (as mentioned in Art. 10) to reverse the connexions of the pressure coil.

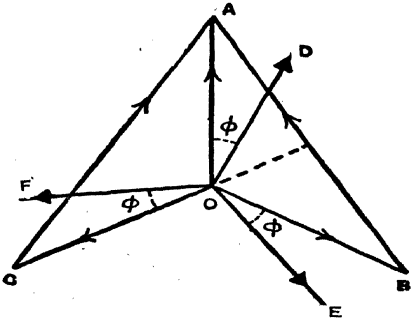


Fig. 3.18.—VECTOR DIAGRAM.

An approximate formula which is rather more convenient when a table of sines is available is:—

$$\cos\phi = \tfrac{1}{2}\,\{1 + \sin\,(m \times 90^\circ)\}.$$

This is accurate for $m = 1$, 0, and -1, and is not more than 0·015 in error except for power-factors below 0·4.

Example 5. *With the connexions of Fig.* 3.17*a the two wattmeter readings are* (i) 1 560 *and* 370 *watts;* (ii) 710 *and* — 190 *watts. Find in each case the total power, and the power-factor by the exact and approximate formulae.*

$$\begin{aligned}
\text{(i)}\qquad \text{Total power} &= 1\,560 + 370 \qquad = 1\,930 \text{ watts.}\\
\cos\phi &= 1/\sqrt{\{1 + 3\,(1\,190/1\,930)^2\}}\\
&= 1/\sqrt{\{1 + 1{\cdot}140\}}\\
&= 1/1{\cdot}463 \qquad = 0{\cdot}683.\\
\text{Approximately:—}\quad \cos\phi &= \tfrac{1}{2}\,\{1 + \sin\,(370/1\,560) \times 90^\circ\}\\
&= \tfrac{1}{2}\,\{1 + \sin 21{\cdot}3^\circ\}\\
&= \tfrac{1}{2}\,\{1 + 0{\cdot}363\} \qquad = 0{\cdot}682.\\
\text{(ii)}\qquad \text{Total power} &= 710 - 190 \qquad = 520 \text{ watts.}\\
\cos\phi &= 1/\sqrt{\{1 + 3\,(900/520)^2\}}\\
&= 1/3{\cdot}16 \qquad = 0{\cdot}316.\\
\text{Approximately:—}\quad \cos\phi &= \tfrac{1}{2}\,\{1 - \sin\,(190/710) \times 90^\circ\}\\
&= \tfrac{1}{2}\,\{1 - \sin 24{\cdot}1^\circ\}\\
&= \tfrac{1}{2}\,\{1 - 0{\cdot}408\} \qquad = 0{\cdot}296.
\end{aligned}$$

An alternative way of obtaining the power-factor from wattmeter readings is to take a reading with the current coil in one line and the pressure coil connected across the other two lines (see Fig. 3.19).

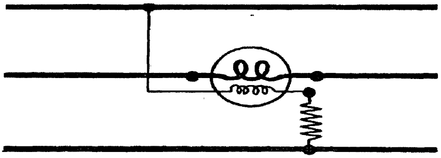


Fig. 3.19.—MEASUREMENT OF REACTIVE POWER.

Then if OE in Fig. 3.18 represents the current in the wattmeter, the p.d. across its pressure coil is the vector difference between OA and OC, *i.e.* AC;

∴ Wattmeter reading = OE.AC. cos (angle between OE and AC).

$$= \text{I}.\sqrt{3}\,\text{V}.\cos\,(90^\circ - \phi)$$

$$= \sqrt{3}\,\text{VI}\sin\phi = \text{W}_3 = (3\text{VI}\sin\phi)/\sqrt{3}.$$

[3VI sin ϕ is called the *reactive* (sometimes *wattless*) volt-amperes.]

$$\text{Hence}\quad \tan\phi = \sqrt{3}\,\text{W}_3/(\text{W}_1 + \text{W}_2).$$

12. Wattmeters with Transformers in 3-Phase Circuits

For high pressure work transformers are required. Any of the methods available without transformers (*e.g.* those of Art. 10) may be used with them, but other methods become possible in addition.

One such method using two wattmeters is shown in Fig. 3.20. The total power is given by the sum of the readings of the two wattmeters, and moreover with balanced loads the two readings are equal whatever the power factor. Therefore for the case of balanced loads either wattmeter may be used alone and the total power is twice the wattmeter reading.

The proof for the case of a Y-connected load is as follows:—

Let i_1 = instantaneous value of current in line No. 1
i_2 = ,, ,, ,, ,, No. 2
i_3 = ,, ,, ,, ,, No. 3

and v_1, v_2, v_3 = instantaneous values of p.d.s across the arms of the load connected to these lines respectively.

The current transformers A and B have their ratios (primary current/secondary current) double that for which the wattmeter has been calibrated. Owing to the cross-connexion of the secondaries the wattmeter (W_1) carries the difference between the currents in them, *i.e.* a current proportional to $\frac{1}{2}(i_1 - i_2)$; and the p.d. applied to its pressure circuit is proportional to $(v_1 - v_2)$. Therefore the reading on W_1 gives the mean value of

$$\tfrac{1}{2}(i_1 - i_2)(v_1 - v_2) = \tfrac{1}{2}(i_1v_1 + i_2v_2 - i_1v_2 - i_2v_1).$$

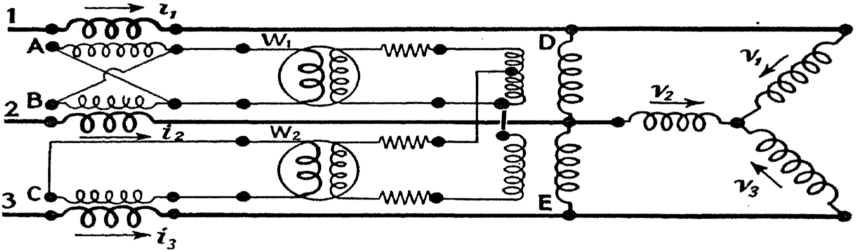


Fig. 3.20—MEASUREMENT OF 3-PHASE POWER USING TRANSFORMERS.

The pressure circuit of the other wattmeter (W_2) being connected to one end of the potential transformer E and to the middle point of the potential transformer D, with the two secondaries connected in series, has an applied p.d. proportional to

$$v_3 - v_2 + \tfrac{1}{2}(v_2 - v_1) = v_3 - \tfrac{1}{2}v_2 - \tfrac{1}{2}v_1.$$

Therefore the reading of W_2 gives the mean value of

$$i_3(v_3 - \tfrac{1}{2}v_2 - \tfrac{1}{2}v_1) = i_3v_3 + \tfrac{1}{2}(i_2 + i_1)(v_2 + v_1)$$

$$\{\text{since } i_1 + i_2 + i_3 = 0, \text{ and so } -i_3 = i_2 + i_1\}$$

$$= i_3v_3 + \tfrac{1}{2}(i_2v_2 + i_1v_1 + i_2v_1 + i_1v_2).$$

Adding the expressions for the readings of the wattmeters gives:—

$$\text{Sum of readings} = \text{mean value of } (i_3v_3 + i_1v_1 + i_2v_2)$$
$$= \text{Mean value of total power.}$$

This is true whatever the loads, and for all wave forms.

For balanced voltages of sinusoidal form the vector diagram (Fig. 3.21) shows that the p.d. on W_2 is represented by GC where G is the middle point of AB, and since GO is in line with OC and is half as long, GC = $1\frac{1}{2} \times$ OC.

Hence W_2 gives $1\frac{1}{2}$ times the power supplied to load No. 3, whether the currents are balanced or not. If the currents are balanced the total power is thrice the power for load No. 3 and so W_2 gives half the total power in this case, and consequently W_1 gives the other half of the total, *i.e.* the readings are equal.

Although balanced loads must give equal readings the converse is not always true, *i.e.* equal readings may be obtained even with unbalanced loads. Equal readings do show that load No. 3 is equal to the average of the three loads, but the other two loads may be either both equal to, or one greater and the other less than this average. Similarly, where the readings are unequal, half the difference of the readings, $(W_1 \sim W_2)/2$, equals the difference between the average of loads Nos. 1 and 2 and load No. 3.

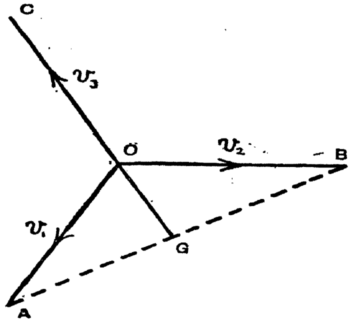


Fig. 3.21.—VECTOR DIAGRAM.

13. Energy Meters

Energy meters for A.C. purposes are necessarily of the watt-hour and not of the ampere-hour type, though the latter may be used for special tariffs (see Art. 19).

Of the methods used for D.C. meters, the electrolytic and mercury motor types (Volume I., Chapter VII., Arts. 15-18) are not used for A.C., as they would require some form of rectifier in addition, besides giving only ampere-hours. But the commutator motor meter (Volume I., Chapter VII., Art. 18) can be used, the only additional precaution being that the resistance in the pressure circuit must be wound non-inductively. They are subject to the same sources of error as the dynamometer type of wattmeter (see Arts. 5-9 of this chapter) in addition to their errors as D.C. meters.

The most usual type of A.C. energy meter is a modification of the induction type of wattmeter. In this type of instrument two electromagnets act on different parts of the same disk, which is made of copper or of aluminium. The coil of one electromagnet carries the main current (or a definite fraction of it) and so produces a magnetic flux very nearly in phase with the main current. The coil of the second electromagnet with an *inductive* resistance in series with it is connected across the mains. It therefore carries a current proportional to the p.d. but lagging by nearly 90°, and produces a corresponding flux. The torque exerted on the disk varies as the product of the two fluxes and as the sine of their phase difference, and therefore is approximately proportional to VI sin (90° — ϕ) which is equal to VI cos ϕ (see Art. 14).

A wattmeter or watt-hour meter of this type ought to have the coil in the pressure circuit as highly inductive as possible, whereas the dynamometer type requires this circuit to be as nearly as possible non-inductive. The former condition cannot be so closely approached as the latter, hence various compensating devices are used to reduce the inaccuracy due to this cause. These are dealt with in connexion with watt-hour meters (Art. 15).

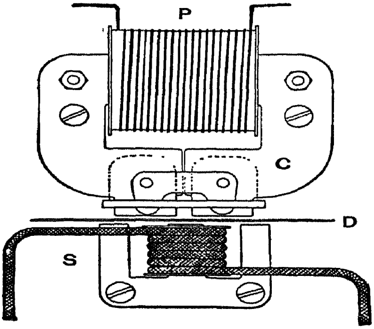


Fig. 3.22.—INDUCTION WATTMETER.

The working parts of an induction wattmeter are shown diagrammatically in plan in Fig. 3.22. The pressure coil (P) consists of a large number of turns of fine wire. Inside this is a laminated core (C) with two poles. Another laminated core, S, has a few turns of wire carrying the main current. The aluminium disk, D, comes between the two sets of poles. The torque due to the inductive action of these poles is opposed by a spiral control spring. A pointer attached to the spindle of the disk moves over a circular scale 300° in extent. In wattmeters the scale is very nearly evenly divided. A drawback of this type is that it cannot be calibrated with D.C.

The modification for use as an energy meter consists in (*a*) the replacement of the spring by a permanent magnet with its poles one on each side of the disk; (*b*) the replacement of the pointer by a worm cut on the spindle and driving, through a train of wheels, on to recording pointers or "cyclometer" figures, in the way usual for motor meters; (*c*) the placing of the operating parts with the disk horizontal.

Since the deflecting torque varies directly as the watts, and the retarding torque due to the eddy currents produced in the disk by the magnet varies directly as the speed, the total number of revolutions is proportional to the watt-hours (cf. Volume I., Chapter VII., Art. 17). Hence by using a suitable train of wheels the pointers (or figures) can be made to record the energy in kilowatt-hours.

14. Action of Induction-Type Instruments

In Fig. 3.23, A, B are two poles with windings carrying a current proportional to the p.d. but lagging $\frac{1}{4}$-period behind it, and C a pole whose winding carries the main current. The aluminium disk D moves in the gap between these poles, and the rest of the magnetic circuit is omitted in the diagram. Taking the positive direction of p.d. from the upper to the lower line and of current from left to right, these will make A and C both S. poles, and B a N. pole. A negative p.d. reverses the polarity of A and B, and a negative current reverses that of C.

The alternating flux of C, in phase with the main current, induces eddy currents in the disk proportional to the rate of change of flux, and opposing the change. The induced currents therefore lag $\frac{1}{4}$-period behind C, taking the same directions as positive, *i.e.* up through plane of diagram above A, and down through this plane above B. Since the polarities of A and B are always opposite the forces produced on the disk at any instant by their interaction with the induced currents are always in the same direction.

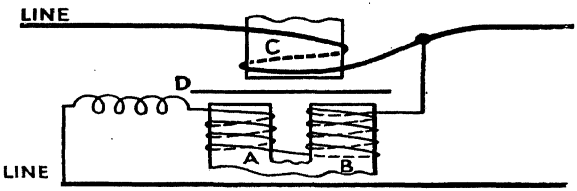


Fig. 3.23.—PRINCIPLE OF INDUCTION WATTMETER.

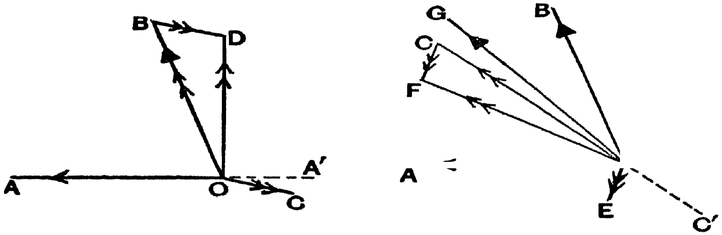
Since the current in the windings of A and B lags $\frac{1}{4}$-period behind the voltage, their flux does the same. Hence when the main current is in phase with the voltage, the induced currents in the disk are in phase with the flux of A and B. So the forces on the disk are always in the same direction.

But when the main current lags or leads, the induced currents will reverse later or earlier than the flux of A and B. So the disk forces are reversed during parts of each period, and the nett torque is reduced. Thus the driving torque depends not only on the product of volts and amperes but on their phase difference, *i.e.* it varies as the watts (cf. Chapter VI., Art. 9).

15. Correction of Errors of Induction Energy Meters

In a monophase induction type instrument the current in the pressure circuit cannot be made to lag the full 90° behind the p.d. For a voltmeter (or an ammeter) this is not of importance provided the frequency does not vary much from that for which the instrument is calibrated. But the accuracy of a wattmeter (or a watt-hour meter) depends on the phase difference between the two fluxes (due to p.d. and current respectively) being exactly ¼-period when the p.d. and the current are in phase.

There are two methods available to compensate for the lag of the pressure circuit current behind the p.d. being less than 90°. One of these is to correct the magneto-motive force due to the pressure by means of a second pressure coil. This gives a small m.m.f. approximately opposite in phase to the p.d., and is adjusted so that the resultant of this and of the m.m.f. due to the principal pressure coil is ¼-period out of phase with the p.d.



(*a*) BY SECOND PRESSURE COIL. (*b*) BY SECOND CURRENT COIL.

Fig. 3·24.—METHODS OF COMPENSATING INDUCTION METERS.

This is illustrated by the vector diagram of Fig. 3·24*a*, where OA represents the p.d.; OB the current and m.m.f. due to the principal pressure coil, lagging by an angle less than 90°; and OC the m.m.f. due to the compensating coil, lagging by a small angle behind the *reversed* p.d. (OA′). BD is drawn parallel and equal to OC, and OD is the resultant m.m.f. of the two coils. By suitable proportioning of the compensating coil $\angle$ AOD can be made a right angle, which is the condition for accuracy.

In practice the same effect is produced by a short-circuited ring on the pressure coil pole. This is much cheaper than using a double pressure circuit, and is equally accurate with proper adjustment.

The second method of compensation consists in making the m.m.f. due to the current lead it by the same angle that the actual lag of the pressure coil m.m.f. falls short of 90°. This is effected by a second current coil in parallel with the main one, but wound in the opposite direction: it has a greater impedance than the main one, and so carries a smaller current, and its reactance is greater in a larger ratio.

The result is shown by the vector diagram of Fig. 3.24*b*, where OA represents the p.d., OB the m.m.f. due to the p.d., and ∠ AOD is a right angle as before. The m.m.f. due to the current in the main coil is given by OC, and that of the compensating coil by OE, lagging behind the reversed current (OC′) owing to the greater relative reactance of the compensating coil. CF is drawn parallel and equal to OE, and OF then gives the resultant m.m.f. of the two. The total mains current is the vector sum of the currents (not of the m.m.f.s) in the main and compensating coils, and so is given by OG, lagging a little behind OC.

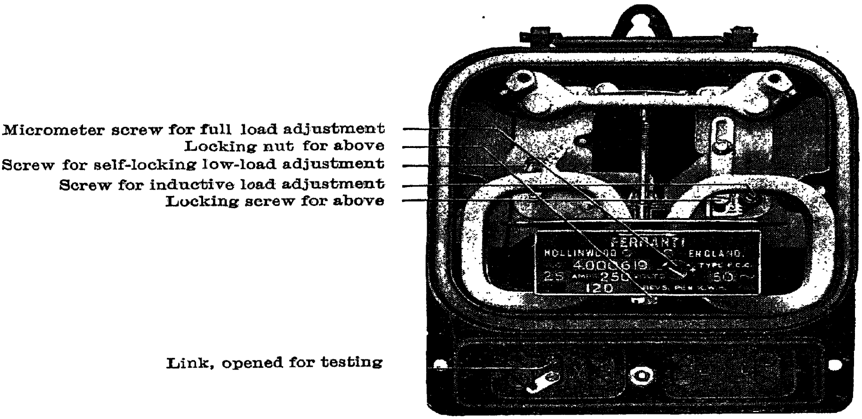


Fig. 3.25.—FERRANTI INDUCTION-TYPE METER.

By proper adjustment ∠ FOG can be made equal to ∠ BOD. Then, when the current is in phase with the p.d., the two resultant m.m.f's (OF and OB) will be at right angles, and when the current lags by ϕ (AOG in Fig.) the m.m.f.s will differ in phase by (90° — ϕ). Thus the necessary compensation is secured.

In watt-hour meters, but not in wattmeters, a compensation for friction is used, though the friction is much less than in D.C. motor meters. For instance, in the Ferranti meter (see Fig. 3.25) the adjustment for friction, so as to obtain accurate registration at low loads, is effected by a pivoted steel tongue piece, movable between the pole-tips of the current coil core by means of a micrometer screw provided for this purpose. The movement causes a slight adjustable dissymmetry in the gap flux due to the shunt coil, and this results in some torque being exerted on the disk with no current in the series coil. Two slots in the disk's periphery prevent the meter running on voltage alone.

Inductive load adjustment, for obtaining exact quadrature of the pressure and current m.m.f.s, according to the first method explained above, is effected by a short-circuited loop placed round the pole-tip of the shunt coil core. Three sides of this loop consist of a copper band, and the fourth side is the adjusting screw itself. This screw is made partly of copper, and partly of eureka. Variation of its position alters the resistance, and therefore the impedance of the loop. Thus the amount of compensation can be adjusted so as to give the desired exact quadrature of the two fluxes.

The brake magnet's field distribution can be adjusted. By moving a mild steel plate downwards the flux density is reduced. This diminishes the braking effect and so the speed is increased. This is called the full load adjustment.

The " Link " connects the pressure and current circuits and can be removed when testing (see Volume I., Chapter VII., Art. 35).

16. Clock-Type Meters

The principle utilised is the alteration in the time of swing of a pendulum when a vertical force, in addition to that due to gravity, is applied to its bob. If this force acts downwards the time of swing is reduced, and vice versa.

The connexions of this type as built by the Aron Meter Co. are shown in Fig. 3.26. Two similar pendulums have bobs made of flat coils of wire, A, A. Below each is a coil, C, carrying the main current, arranged so that one pendulum is accelerated and the other retarded by the forces between C and A. Each pendulum actuates a similar train of wheels ending in a differential gear. This turns the recording train of wheels by amounts proportional to the difference between the numbers of swings of the two pendulums. The record is thus proportional to the watt-hours supplied to the main circuit.

To eliminate slight differences between the natural times of swings of the pendulums the current in their bobs is reversed automatically every ten minutes. Simultaneously the gearing is altered so that the recording fingers are driven in the same direction as before.

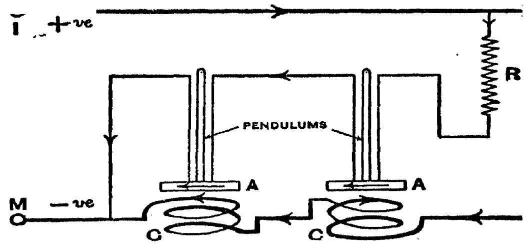


Fig. 3.26.—CONNEXIONS OF ARON METER.
A A. Flat coils of fine wire. C C. Main current coils.
M M. Mains. R. Resistance in series with A A.

The whole arrangement of pendulums, differential gear, and recording train is driven by a spring. This is rewound electrically when required by an automatic arrangement. The meter is thus self-contained, and is connected in the same way as other watt-hour meters.

It gives very good accuracy, particularly at low loads. But its cost and the difficulty of calibration have caused it to be superseded by the improved motor types of meter.

17. Polyphase and Reactive Meters

In two-phase and three-phase circuits the total power or the total energy can be obtained from a single instrument. This consists of two wattmeters (or watt-hour meters) in one case. In the dynamometer type the two moving coils are mounted on the same spindle and each is acted on by its own current coil. The spindle actuates a single pointer in a wattmeter, or a single wheel-train in a watt-hour meter. Similarly in the induction type there is only a single spindle which may carry two disks, but more often has a single one, on which the two sets of series and pressure systems act. For a 4-wire supply three sets are used.

The advantages of these instruments are that their cost is less than that of two separate similar monophase instruments, particularly in the induction type, and that they occupy less room. Their disadvantages are that their calibration is more troublesome, and that the mutual inductance of coils in different phases may introduce errors.

Meters for measuring reactive voltamperes (or voltampere-hours) are of the dynamometer type with a highly inductive pressure circuit, or of the induction type with a non-inductive pressure circuit. In the former case some compensating device is necessary just as for an induction-type power wattmeter (see Art. 15). In polyphase circuits an ordinary wattmeter may be used for this purpose, the necessary quarter-period shift in the phase of the pressure-coil current being obtained by altering its connexions (see Fig. 3.19).

18. Arnò Meter

The cost of supplying energy by A.C. depends on the power-factor as well as on the total amount of energy. This is in addition to the difference in cost for the same amounts of energy with different maximum demands (see Volume I., Chapter XVIII., Arts. 14, 15).

The effect of a low power-factor is to increase the average current required for a given rate of consumption of energy. This entails a corresponding increase in the sizes of the generators, cables, etc., and at the same time increases the losses in transmission. On the other hand, the fuel consumption and the sizes of the engines, etc., are not affected by the power-factor except to the comparatively small extent due to the increased losses in transmission. *E.g.* a power-factor of 0·5 means twice as much current as for the same power at unity power-factor. If the cross-sections of the cables are doubled the I^2R losses are doubled nevertheless, and the generators, transformers, etc., are similarly affected. If the original transmission losses were 15 per cent. of the energy delivered they will become 30 per cent., so the fuel consumption, size of boilers, etc., are increased in the ratio $130/115 = 1{\cdot}13$, *i.e.* by 13 per cent. (see further, Chapter V., Art. 14).

The running charge for A.C. supply should therefore in fairness be made to depend partly on the energy ($VI \cos \phi \times t$) and partly on the volt-ampere-hours ($VI \times t$). Prof. Arnò has suggested that the charge should be made on the "complex load" × time; defining "complex load" as $\frac{2}{3}VI \cos \phi + \frac{1}{3}VI$.*

To measure the quantity on which the charge is based a small modification is made in the meters. This consists in the case of the dynamometer type of making the pressure circuit partially inductive so that the current therein lags by a small angle behind the p.d. Similarly in the induction type the compensation is so adjusted that the pressure m.m.f. lags more than 90° by a small angle.

The result is that the record of the meter is increased in the ratio ($1 + \tan \phi \tan \alpha$) where α is the additional lag given (see Art. 6). Over a considerable range this makes the quantity recorded nearly equal to ($\frac{2}{3}VI \cos \phi + \frac{1}{3}VI$). Prof. Arnò suggests making $\alpha = 5°$

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* See *The Electrician*, Vol. 72, p. 732.

for supplies which are mainly lighting, when cos ϕ ranges from 0·8 to 1·0; and $\alpha = 14°$ for supplies which are mainly power with cos ϕ varying between 0·5 and 0·9. Even though the readings may not give the exact value of ("complex load" × time) they will always give a greater increase the lower the power factor (cf. Example 3, Art. 7), and so will be approximately fair between various consumers.

Example 6. *If in an Arnò meter $\alpha = 10°$, find for various power-factors down to 0·5 the ratios in which the readings are increased compared with the true energy, and compare this with the ratio of increase for the true "complex load." Assume the meter to be accurate at unity power-factor.*

COS ϕ	ϕ	TAN ϕ	METER RATIO	CORRECT RATIO	CORRECTION
1	0°	0	1·00	1·00	0
0·95	18·2°	0·329	1·058	1·018	− 3·8%
0·9	25·85°	0·484	1·085	1·037	− 4·4%
0·8	36·85°	0·750	1·132	1·084	− 4·2%
0·75	41·4°	0·882	1·156	1·111	− 3·9%
0·7	45·6°	1·021	1·180	1·143	− 3·1%
0·65	49·5°	1·170	1·206	1·180	− 2·2%
0·6	53·1°	1·333	1·235	1·222	− 1·1%
0·55	56·6°	1·517	1·268	1·273	+ 1·4%
0·5	60°	1·732	1·306	1·333	+ 2·1%

The "meter ratio" = (meter reading)/(true energy) = $1 + \tan 10° \tan \phi$.

The "correct ratio" = (complex load)/true load = $(\frac{2}{3} \cos \phi + \frac{1}{3})/\cos \phi$.

The "correction" = percentage to be added to or subtracted from the meter reading to give the quantity on which the charge for energy is to be made on the Arnò system. Note that if the power-factor varies between 0·7 and 0·95 only, then by adjusting the wheel train so that the meter runs 3¾ per cent. slow at unity power-factor the maximum departure from the correct ratio is reduced below 2/3rds of one per cent.

In many cases there were legal obstacles to the employment of this system. Where these do not interfere the chief drawback is that the calibration and adjustment of the meters are more troublesome than for the usual type. The system is also more difficult to explain to non-electrical people. On the other hand a given departure from the intended angle of lag causes less error than the same departure from exact quadrature in a normal induction-type meter.

19. Voltampere-Hour Meters

Instead of measuring the " complex load " the same method of charging can be applied by using an ordinary kWh. meter and a kilovoltampere-hour meter. The same instruments may be used for tariffs in which power-factors above a certain value are given a bonus, and those below a certain value have an additional penalty added. An example of such a tariff is the following:—Power-factor 0·95 or over, 8% bonus; 0·9 to 0·95, 7% bonus; 0·85 to 0·9, 5% bonus; 0·8 to 0·85, 3% bonus; 0·75 to 0·80, 1% bonus; 0·7 to 0·75, 1% penalty; 0·65 to 0·70, 4% penalty; 0·60 to 0·65, 7% penalty. These are made on both the kWh. consumption and the kW. demand. Very often the demand charge is made on kVA., and the bonus or penalty (if any) made on the kWh. only.

The method generally used in kVA.-hour meters is similar to that for Arnò meters. That is to say in an induction-type meter the m.m.f. due to the pressure is made to lag by more than 90°. If this departure from 90° is made equal to the average angle of lag the meter will give kilovoltampere-hours approximately. Thus if an accurate induction energy meter has its angle of pressure flux lag made 120° with no change of magnitude, it will record kVAh. accurately if the power-factor is cos 30°, *i.e.* 0·866. For under these conditions the current induced by the main current is in phase with the pressure flux.

Moreover the reading will be near to the true kVAh. over a considerable range of power-factors, being proportional to $\cos(\phi - \alpha)$. Thus since $\cos 14° = 0{\cdot}970$, the maximum error with $\alpha = 30°$ is 3 per cent. low for values of ϕ from 16° to 44°, *i.e.* for power-factors from 0·96 down to 0·72. And by increasing the flux by 1½ per cent., or altering the gearing to give the same result, the error over this range is made ± 1½ per cent.

Another way of applying a tariff embodying a power-factor clause is to use a reactive-kilovoltampere-hour (kVARh.) meter (see Art. 17). The power-factor is obtained as follows:—

$$\tan\phi = \text{kVARh./kWh.}, \quad \text{and} \quad \cos\phi = 1/\sqrt{1 + \tan^2\phi}.$$

In three-phase circuits the reactive meter must be a 3-element one; but the energy and kVAh. meters can be 2-element patterns for 3-wire supplies, but three elements (or three separate meters of each sort) are necessary for 4-wire supplies.

20. Iron-Cored Instruments

Many attempts have been made to increase the deflecting torque in dynamometer type wattmeters by the use of iron cores, and so to make them less affected by external fields. The most successful of these attempts are those of Dr. Drysdale and Dr. Sumpner.

The Drysdale wattmeter* consists of four current coils wound on laminated soft iron cores. In the air-gaps between these are two similar flat coils connected in the pressure circuit, each consisting of an 8-shaped winding so as to be astatic. They are connected so that the current flows in opposite directions in them and thus their inductance is cancelled out almost entirely. One is fixed, and the other is attached to a fine wire passing round the spindle of the pointer and is controlled by eight helical springs.

Owing to hysteresis the flux is not in phase with the main current, and in order to keep the error due to this source small the air gap has to be made fairly wide and the field strength to be kept low. This means that the advantages of the use of iron cores are lost to a large extent.

In the Sumpner wattmeter† the magnet winding forms the pressure circuit. The resulting flux is 90° out of phase with the p.d. (see Volume I., Chapter V., Art. 7) whether there is any hysteresis or eddy current loss present or not. The only departure from this is due to the effect of resistance, which can be kept small. The moving coil is connected to a "quadrature transformer" whose primary carries the main current. The quadrature transformer consists of a transformer with a large air-gap in its magnetic circuit so that its iron losses are negligible but its magnetising current very large (see Volume I., Chapter XVII., Art. 3).

The result is that the secondary e.m.f. is nearly 90° out of phase with the primary current, and as the moving coil circuit is nearly non-inductive its current is nearly in quadrature with the main current. If this quadrature were exact, and also that of the flux and the p.d., the instrument would record true watts. Unfortunately the two departures from quadrature cause errors in the same direction, so that the design of an accurate instrument (including its quadrature transformer) is difficult, and a considerable part of the advantage of the iron core is lost.

The same principle has been applied to voltmeters (in which the difficulties are less), and to power-factor meters.

QUESTIONS ON CHAPTER III

1. Draw a diagram of an ammeter, a voltmeter, and a wattmeter connected to a high pressure circuit through two instrument transformers, and discuss the advantages of this arrangement.

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* See *The Electrician*, Vol. 55, p. 472.

† See *J.I.E.E.*, Vol. 41, p. 230.

2. With the particulars given in Example 1, find the currents in the two coils at a frequency of 40 cycles per second, and their percentage changes from their values at 50 cycles per second.

3. A power-factor meter to be used on a three-phase circuit consists of a fixed coil, and two coils wound at right-angles on an armature that is free to rotate in the field of the fixed coil. Draw a diagram showing how the instrument should be connected, and deduce an expression to show how the position taken up by the armature depends on the power-factor. How could such an instrument be graduated by means of direct currents?

4. A wattmeter has a current coil of 0·1 ohm resistance, and a voltage coil of 6 500 ohms resistance. Calculate the percentage errors, due to resistance only, with each of the alternative methods of connexion, when reading the input to an apparatus which takes

(*a*) 12 amperes, 250 volts, $\cos \phi = 1$,

(*b*) 12 amperes, 250 volts, $\cos \phi = 0{\cdot}4$.

5. A wattmeter has a current coil of ·03 ohm resistance and a voltage coil of 6 000 ohms resistance. Give a diagram of the two possible methods of connexion, and calculate the percentage error for each when the load takes 20 amperes at 220 volts (*a*) $\cos \phi = 1$, (*b*) $\cos \phi = 0{\cdot}6$.
What loads would give equal errors with the two methods?

6. Explain why wattmeters are likely to be inaccurate on low power-factors.
If the reactance of the voltage circuit of a wattmeter is 1 per cent. of its resistance, calculate the percentage error due to this cause at power-factors of 0·8, 0·5, and 0·1 respectively.

7. If the current in the shunt coil of a wattmeter lags 2° behind the voltage, and the instrument is accurate when $\cos \phi = 1$, find the percentage error when $\cos \phi = 0{\cdot}8$, 0·6, and 0·4 respectively.

8. A wattmeter is required to measure accurately the power taken by an alternating current circuit. What corrections must be made (*a*) for the power absorbed in the pressure coil, and (*b*) for the inductance of the pressure coil? Explain fully the principles involved, and the influence, if any, of the power-factor of the load on the corrections used.

9. A certain circuit takes 10 amperes at 200 volts and the power absorbed is 1 000 watts. If the wattmeter's current coil has a resistance of 0·15 ohm, and its shunt coil a resistance of 5 000 ohms and an inductance of 0·3 henry, find (*a*) the error due to resistance for each of the two possible methods of connexion; (*b*) the error due to inductance if the frequency is 50 cycles per second; (*c*) the total error in each case.

10. What are the sources of the main errors occurring in wattmeters, and how may they be minimised? How does the frequency affect these errors?

11. The resistance of the pressure circuit of a wattmeter is 200 times its reactance, the phase error of the pressure transformer quarter of a degree, and the phase error of the current transformer 0·3° at full load, double this at $\frac{1}{5}$th load non-inductive, and 50 per cent. more on loads of power-factor 0·5. Find the percentage errors of the wattmeter at full and at $\frac{1}{5}$th load of power-factor 0·5, inductive loads.

12. Describe the "two-wattmeter method" of measuring three-phase power and give a proof that it is accurate whatever the distribution of the load between the three phases and whatever the wave form of e.m.f. and current may be. [C. & G., II.

13. Establish and explain fully a method of determining the total power, and the power-factor, of a three-phase balanced circuit, from the readings of a single wattmeter. If the current and voltage are also measured, write down an alternative expression for the power-factor. To what causes would you attribute any discrepancy between the values obtained by the two methods in a given test?

14. Calculate power and power-factor (and plot them against each other) for the following pairs of readings, with the above method:—

1st reading	..	250	400	720	1260	1700	watts.
2nd reading	..	−50	0	105	410	850	watts.

15. Draw a diagram of connexions for obtaining the power in an unbalanced three-phase circuit from two readings on a single wattmeter.

16. Describe and prove the theory of the two-wattmeter method for determining the power supplied to a three-phase motor on load. Show how from the readings of the two instruments the power-factor of the load can be found when the currents are balanced, and the line voltages are equal. If the currents, or voltages, are somewhat unequal, can the power and the power-factor be deduced accurately from the readings? Would you prefer as a matter either of convenience or accuracy to test the power-factor by means of a phase-meter of the ordinary dynamometer type? Give reasons for any preference you may have.

17. Explain the principle of action of an induction watthour meter. Show that the driving torque is proportional to the watts, and explain the action of the devices used for the power-factor adjustment and for the compensation for the running friction. [C. & G., Final, Instr.

18. Describe the construction of some good form of watt-hour meter for alternating current circuits, and explain carefully the conditions which must be met in order to secure accuracy. What must be the connexion between the torque and the speed of the brake, and how is this relationship secured in practice?

19. Describe the construction and method of action of either a two-phase or a three-phase induction type energy meter.

20. Calculate the values of the "meter ratio" for an Arnò meter for power-factors between unity and 0·5, (*a*) when $\alpha = 5°$; (*b*) when $\alpha = 14°$. Plot these, and also the ratio (complex load)/(true load), against power-factor.

If in case (*a*) the meter is to be used for power-factors from unity to 0·8, what is the maximum departure from the correct ratio? What alteration in the wheel train will make the maximum error as small as possible?

Answer the same questions for case (*b*) when the meter is used for power-factors from 0·9 to 0·5.

21. If a meter is to be used as a kVAh. meter for power-factors from 0·8 to 0·5 find suitable values for (*a*) the change in phase of pressure flux; (*b*) the change in flux magnitude (or in gearing).

CHAPTER IV

ALTERNATORS

1. Differences between A.C. and D.C. Generators

A.C. generators or **alternators** differ from D.C. generators in several respects. In the first place the former require no commutator. The alternating voltage generated is applied directly to the external circuit, thus producing an A.C. in it instead of being commutated (see Volume I., Chapter IX., Art. 14) so as to produce D.C. in the external circuit. The commutator may therefore be replaced by **slip-rings,** which enable the fixed terminals to be connected always to the same points in the rotating winding.

In the second place, the magnets usually rotate (and are often called **the rotor**) while the armature is stationary, and may be called the **stator.** The advantage of this change is that it is then easier to insulate the armature coils which may be for high pressure, while the rotating magnet coils can be supplied at a low pressure. Moreover, in polyphase alternators a rotating armature would necessitate at least three slip-rings, whereas two are sufficient for the rotating magnets.

Thirdly, alternators are nearly always separately excited. For self-excitation the current would have to be rectified by a commutator or some equivalent device. Frequently each alternator has its own source of supply for its magnetising current in the form of an **exciter,** *i.e.* a small D.C. generator, either directly coupled to the alternator or belt driven from it.

A fourth difference is, that while all modern D.C. generators have **closed-coil** armature windings (*i.e.* there is a complete circuit in the armature without coming into the external circuit at all), alternator windings are nearly always of the **open-coil** type, *i.e.* the armature windings come to terminals, and the circuit is not complete until the external circuit is closed. This arises from the same cause as the absence of commutators in alternators.

2. Slow-Speed Alternators and Turbo-Alternators

Before the introduction of the steam turbine it was necessary to use a large diameter for the magnet wheel in order to obtain a high peripheral speed and so economise material. The axial length of the armature and magnets was therefore relatively small, and so a large number of poles was required to make them of good proportion, and to give the desired frequency. The number employed varied usually between 8 and 64, the larger numbers being used on the alternators of large output.

With steam turbines, on the other hand, the r.p.m. are very much higher, and so a much smaller diameter is used than for an alternator of equal output driven by a reciprocating engine. The axial length has, therefore, to be increased in actual value, and its ratio to the diameter is very greatly increased. The result is that alternators with only four poles are now (1941) built up to outputs of 200 000 kilovoltamperes; and six is the largest number of poles used in turbo-alternators and occurs rarely. The minimum number of two poles has been used up to 60 000 kilovoltamperes.

In all alternators the number of cycles of alternating e.m.f. per revolution is equal to the number of pole-pairs (p), hence the frequency (f) in cycles per second is given by:—

$$f = \frac{p \times n}{60}$$

where n = revolutions per minute.

The maximum speed of an alternator for 25 cyc., is, therefore, 1 500 r.p.m., and as this is too low for outputs below about 4 000 kVA., mechanical gearing is employed in such cases to enable the turbine to run at a higher speed than the alternator.

3. Differential Action and Relative Output

As explained in Art. 1, the connexions of the armature windings of an alternator remain the same when the magnet is rotated, instead of being altered by a commutator as in a D.C. generator. The result of this is that if the winding of a monophase alternator extended over the whole surface of the armature (or were in slots placed all the way round) the e.m.f.s generated in different parts of the winding would be opposed during parts of each period.

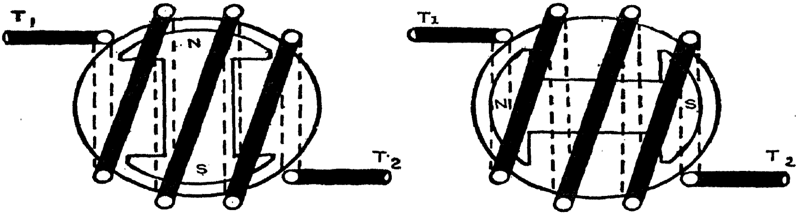
Even when only a portion of the armature is wound, some of this so-called "**differential action**" occurs. Thus in Fig. 4.01 (*a*) the e.m.f.s in all the conductors act in the same direction between the terminals (viz. from T_2 to T_1 if the magnets are rotated counter-clockwise). But when the magnets have reached the position shown in Fig. 4.01 (*b*), the two left-hand conductors are under the N. pole, and so both generate upward e.m.f.s, and these oppose each other. Similar considerations apply to the two right-hand conductors.

If the unwound portion is made equal to the pole shoe, very little differential action occurs. Consequently, in monophase alternators, the pole shoe is made generally about half the pole pitch, and the winding is spread over about half the armature. Sometimes a greater spread is used to obtain a better wave-form (see Art. 5).

If a second similar winding is placed in the unwound portion, an equal e.m.f. is produced in it. The two e.m.f.s will differ in phase by ¼-period, and thus a two-phase alternator is obtained.

The output of this will be double that of the same frame wound monophase, except in so far as the filling of all the slots reduces the current which the armature windings can carry. The armature copper is doubled, but the rest of the alternator is unchanged. Hence the cost per kVA. of a two-phase alternator is much less than that of a similar monophase one.

For a three-phase alternator the windings for each phase need not cover more than a third of the armature surface, and so the pole shoe may cover about two-thirds of the pole pitch instead of being restricted to half the pole pitch. The output of a three-phase alternator is, therefore, greater than that of the same frame wound two-phase.



(*a*) Fig. 4.01.—DIFFERENTIAL ACTION. (*b*)

The output ratio with equal maximum flux-density is not necessarily the same as that of the pole-shoe arcs in the two cases, viz. 3 to 4, because the field strength is usually not uniform under the pole shoes owing to considerations of wave-form. With sinusoidal field form and windings uniformly distributed over the whole periphery of the armature, the increase of output of the three-phase alternator is 50 per cent. compared with that of the completely wound monophase alternator (see Art. 5), but only 6 per cent. compared with a two-phase alternator.

4. Electromotive Force of an Alternator

It can be shown, as in Volume I., Chapter XVII., Art. 2, that the e.m.f. generated in a single turn of an alternator is given by:—

$$E = 4kf\Phi_m/10^8 \text{ volts (R.M.S.)},$$

where Φ_m = maximum flux linking with the turn,
f = frequency in cycles per sec.,
k = form factor of the flux wave,
= 1·11 for sinusoidal wave.

Since the e.m.f. in a single conductor varies as the flux-density (**B**) (when the relative speed is constant), the graph of E against time is of the same shape as that of **B** against distance round the periphery of the magnets. If the two sides of the turn are a pole-pitch apart (called a **full-pitch turn**), the two e.m.f.s are equal and opposite in space at every instant, and so add up round the turn, and the wave-form of the total e.m.f. is the same as that of each alone, and so is the same as the form of the **B**-space wave.

If the two sides are some other distance apart, the e.m.f. wave in general differs from that of the flux-density wave because the e.m.f.s (round the turn) due to the two sides are not exactly in phase with each other. If, however, the flux-density wave is sinusoidal, the total e.m.f. wave will be the same, since the sum of any two sinusoidal waves is itself sinusoidal. The magnitude of the e.m.f. is in all cases less than for a full-pitch coil, and so may be written:—

$$E = 4ckf\Phi_m/10^8 \text{ volts (R.M.S.)},$$

where Φ_m = armature flux per pole;
c = **coil span factor**;
= 1 for full-pitch turn (or coil)
< 1 for other pitches.

The value of c will be considered in detail later.

When a number of similar turns are connected in series, the total e.m.f. will be increased in direct proportion if the turns are all placed either in the same position on the armature, or in equivalent positions, *i.e.* at distances apart equal to twice the pole-pitch. But if they are placed otherwise the e.m.f. is increased in a smaller ratio than the number of turns, due to the phase differences between the various e.m.f.s; hence

$$E = 4bckf\phi\mho/10^8 \text{ volts},$$

where $\mho$ = number of turns *in series* and b is a factor less than unity.

The value of b depends partly on the distribution of the turns in the group corresponding to one pair of poles, and partly on the difference (if any) between positions of the various groups relative to the corresponding pole-pairs. Usually in an alternator the winding consists of a number of similar groups at distances apart equal to twice the pole-pitch, and then this latter effect is absent: when present it is taken into account by a quantity less than unity, called the group factor.

The factor which takes into account the effect of the distribution of the coils in a group may be called the **distribution factor**, or the **breadth coefficient**, because it depends on the breadth of the armature surface covered by each side of the group of coils.

The value of b is equal to (distribution factor) × (group factor).

The product (bc) of all three factors, *i.e.* (coil-span factor) × (distribution factor) × (group factor), may be called the **winding factor.**

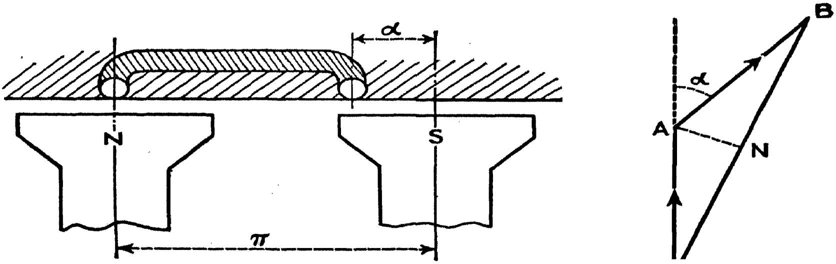
In the usual type of alternator winding (see Art. 7) the coils are of full pitch or equivalent to this, and the groups are similarly placed, so that the first and third factors are each equal to unity, and only the second factor affects the e.m.f.

The wave-form is affected by the above two causes in the same way as by coil-pitches differing from the pole-pitch; *i.e.* in general the resultant wave-form is altered (see Example 2, page 90), but if the wave-form of one coil (or of one group) is sinusoidal the resultant is of the same shape.

5. Values of the Winding Factors

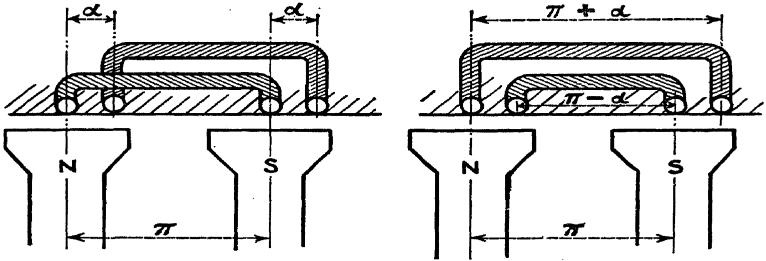
As has been stated already, when the flux distribution is sinusoidal the e.m.f. in each conductor is similar, and so vectors may be used in finding the total e.m.f.

Fig. 4.02 shows a single turn whose pitch falls short of the pole-pitch by the electrical angle α: the pole-pitch being equal to π



(*a*) Coil Diagram. (*b*) Vector Diagram.

Fig. 4.02.—Short-Pitch Coil.



(*a*) FULL-PITCH COILS. (*b*) EQUIVALENT COILS.

Fig. 4.03.—TWO COILS IN SERIES.

radians (= 180°). In the vector diagram, OA represents the e.m.f. due to the left-hand conductor. The e.m.f. in the right-hand conductor is nearly in the opposite direction across the armature, *i.e.* nearly in phase round the loop; it is, therefore, represented by the vector AB, equal in length to OA, and making an angle α with OA produced. The total e.m.f. of the turn, therefore, is given by OB.

Draw AN perpendicular to OB. Since OA = AB, $\angle$ AON = $\angle$ ABN, and each = $\alpha/2$;

$$\therefore \text{OB} = \text{ON} + \text{NB} = 2\text{OA} \cos(\alpha/2).$$

For a full-pitch coil, OB = 2OA;

$$\therefore \text{the coil-span factor (see Art. 4)} = \cos(\alpha/2).$$

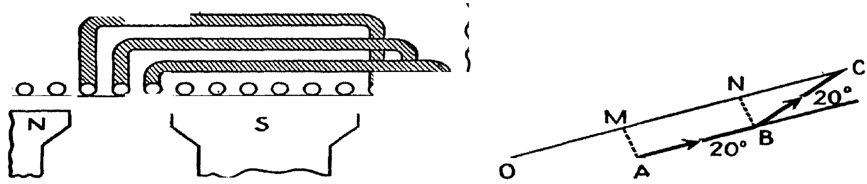
If the pitch exceeds the pole-pitch by α the same result is obtained.

In a group consisting of two turns (or coils), each of full pitch, and at a distance α apart, the same result is obtained, since the e.m.f.s of the two are equal but differ in phase by α. This can be proved also by noting that the coils are equivalent (see Fig. 4.03 *b*) to one short-pitch and one long-pitch coil, the pitch in each case differing from the pole-pitch by α, and that the e.m.f.s of these two coils are in phase since they have a common centre line.

The method of dealing with a group containing a larger number of coils is shown in the following example.

Example 1. *An armature with 9 slots per pole is wound with full-pitch coils for three-phase. Find the breadth coefficient of the winding for a sinusoidal field.*

The winding of any one phase will occupy 3 slots per pole, the other sides of the coils being in 3 other slots a pole-pitch away. The slot-pitch = $\frac{1}{9}$ of pole-pitch = $\pi/9$ radian = 20°.



(*a*) Coil Diagram. (*b*) Vector Diagram.

Fig. 4.04.—Three Coils in Series per Pole-Pair.

Thus, if the vector OA represents the e.m.f. of the left-hand coil, the e.m.f. of the centre coil is given by AB equal in length to OA, and making an angle of 20° with OA produced. Similarly the e.m.f. of the right-hand coil is given by BC, equal to OA, and making ∠ 20° with AB produced (Fig. 4.04 *b*).

The total e.m.f. is, therefore, given by OC.

Draw AM, BN, perpendicular to OC. By symmetry AB is parallel to OC, and

$$\angle \mathrm{AOM} = \angle \mathrm{BCN} = 20^\circ;$$

$$\begin{aligned} \therefore\ \mathrm{OC} &= \mathrm{OM} + \mathrm{MN} + \mathrm{NC} = \mathrm{MN} + 2\,\mathrm{OM} \\ &= \mathrm{AB} + 2\,\mathrm{OA}\cos 20^\circ \\ &= \mathrm{OA}\,(1 + 2\cos 20^\circ) \\ &= \mathrm{OA}\,(1 + 2 \times \cdot 9397) = 2{\cdot}879\,\mathrm{OA}. \end{aligned}$$

If the three e.m.f.s were in phase, the total would be 3 OA;

$$\therefore\ \text{Breadth coefficient} = 2{\cdot}879/3 = \cdot 960.$$

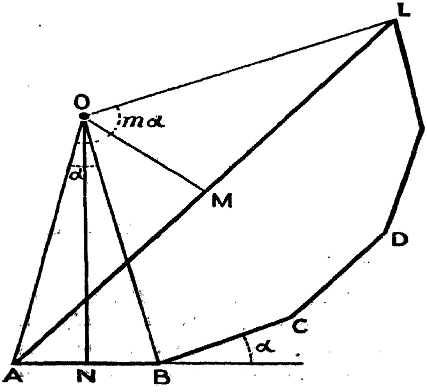


Fig. 4.05.

The general formula for cases of this sort can be obtained as follows. Let AB, BC, etc. (Fig. 4.05), be the vectors of the e.m.f.s of *m* coils, placed at equal distances apart of *α* electrical degrees. Since they are equal and make equal angles with one another they form the sides of part of a regular polygon, and a circle, centre O, can be drawn through all the points A, B, C, . . . L.

Draw ON ⊥ to AB, and OM ⊥ to the resultant voltage (AL).

Then— $\mathrm{AB} = 2\,\mathrm{AN} = 2\,\mathrm{OA} \sin \mathrm{AON} = 2\,\mathrm{OA} \sin \alpha/2;$

$$\text{and } \mathrm{AL} = 2\,\mathrm{AM} = 2\,\mathrm{OA} \sin \mathrm{AOM} = 2\,\mathrm{OA} \sin m\alpha/2;$$

$$\therefore \text{ Breadth coefficient} = \mathrm{AL}/m\,.\mathrm{AB}$$

$$= \sin (m\alpha/2)/(m \sin \alpha/2).$$

Applying this to the above example gives:—

$$\text{Breadth coefficient} = \sin 30^\circ/(3 \sin 10^\circ)$$

$$= 0\cdot500/(3 \times 0\cdot1737) = 0\cdot960.$$

As the number of slots per pole increases the angle between successive vectors diminishes, and so the vector polygon approximates more and more closely to an arc of a circle, and the resultant e.m.f. to the chord of this arc.

In Fig. 4.06, O is the start and L the end of the vector polygon in the limiting case, and C is its centre. β is the phase difference between the first coil of the group and the last. Since CO and CL are perpendicular to the vectors for these two coils:—

Breadth coefficient = (total e.m.f.)/(sum of separate e.m.f.s)

$$= (\text{Chord OL})/(\text{arc OL})$$

$$= (2\,\mathrm{OC} \sin \beta/2)/(\mathrm{OC}\,.\,\beta)$$

$$= \sin (\beta/2)/(\beta/2).$$

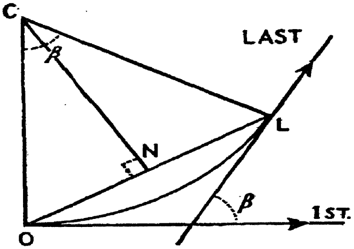


Fig. 4.06.—VECTOR DIAGRAM FOR MANY SLOTS.

The closeness of the approximation can be tested by putting $\beta = 2\pi/9$, *i.e.* 40°, as in Example 1 above. Then:—

Breadth coefficient $= \sin (\pi/9)/(\pi/9) = (\cdot3420)/(\cdot3491) = \cdot980$, *i.e.* 2 per cent. greater than the exact value.

The four most important applications of this formula are when $\beta =$ (*a*) $\pi/3$, *i.e.* 60°, (*b*) $\pi/2$, *i.e.* 90°, (*c*) π, *i.e.* 180°, and (*d*) $2\pi/3$, *i.e.* 120°. These correspond with a completely distributed single layer winding connected (*a*) three-phase; (*b*) two-phase; (*c*) monophase with all slots wound; and (*d*) monophase with two-thirds of the slots wound.

In case (*a*):—Breadth coefficient $= \sin (\pi/6)/(\pi/6) = 3/\pi$

$$= \cdot500/\cdot5236 = \cdot955,$$

which differs from the value found in Example 1 by $\frac{1}{2}$ per cent. only.

In case (*b*):— Breadth coefficient $= \sin (\pi/4)/(\pi/4)$

$$= (\cdot7071)/(\cdot7854) = \cdot900.$$

Thus the output of a two-phase alternator with sinusoidal field is ($\cdot 900/\cdot 955 =$) 94·2 per cent. of that of the same frame wound for three-phase.

In case (*c*):— Breadth coefficient $= (\sin \pi/2)/(\pi/2)$
$= 2/\pi = \cdot 637,$

i.e. the output is only two-thirds of that of the three-phase alternator.

In case (*d*):— Breadth coefficient $= (\sin \pi/3)/(\pi/3) = 3\sqrt{3}/2\pi$
$= \cdot 866/1\cdot 047 = 0\cdot 827.$

Thus the output relative to three-phase alternator is $(2/3)(\cdot 827/\cdot 955) = 1/\sqrt{3} = 0\cdot 577$.

An alternator with slots skewed by a slot-pitch gives the effect of a completely distributed winding.

When the winding consists of a number of groups of coils with the pitch of the groups different from double the pole-pitch, the value of the factor to be introduced can be found by the same methods as are used above (see also Art. 8). The effect usually is to make the result approach more closely that of a completely distributed winding.

6. Winding Factors in General Case

When the field distribution is not sinusoidal the vector method cannot be applied in its simple form. If the field is analysed into its harmonics, vector methods can be applied to each separately and the resultant e.m.f.s. combined to give the total resultant.*

Alternatively graphical methods may be used to find the resultant e.m.f. for the particular case under consideration (see Example 2). This plan has the advantage of being easy to apply, but the disadvantages of being less rapid in most cases, and of being less useful for obtaining the effect of changing the various factors.

Example 2.—*Find the breadth coefficient for the winding of Example* 1 *for a field which is uniform over two-thirds of each pole pitch, and varies uniformly between.*

In Fig. 4.07 (*a*) the full line gives the field distribution, and also the e.m.f. time graph for a single coil. The e.m.f.s of the other two coils are similar but displaced in phase by 20° and 40° respectively as shown by broken and dotted lines. By adding the ordinates at various points with due regard to sign the resultant e.m.f. is found to be as shown in Fig. 4.07 (*b*). Its maximum value is thrice the maximum value for a single coil, but as their wave forms are different the R.M.S. values must be determined. As the half-waves are symmetrical, only a quarter of a period need be taken.

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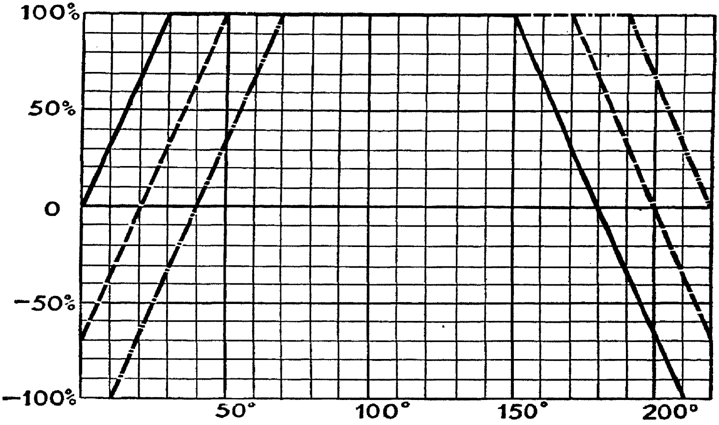
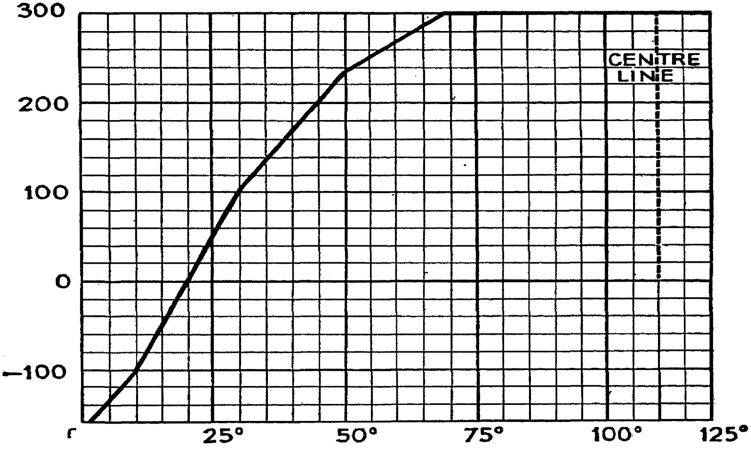
* See B. Hague, *The Electrician*, Vol. 78, p. 710.



(*a*) SEPARATE E.M.F.S.



(*b*) RESULTANT E.M.F.

Fig. 4.07.—BREADTH COEFFICIENT WITH NON-SINUSOIDAL FIELD.

Dividing into 10-degree strips, and taking mid-ordinates:—

Mean square of coil e.m.f. $= \frac{1}{9}(16{\cdot}7^2 + 50^2 + 83{\cdot}3^2 + 6 \times 100^2) = 7\,725$;

$\therefore$ R.M.S. value of coil e.m.f. $= \sqrt{(7\,725)} = 87{\cdot}9$.

(N.B.—The exact value by the calculus is 88·2.)

Mean square of total e.m.f. $= \frac{1}{9}(50^2 + 133^2 + 200^2 + 250^2 + 283^2 + 4 \times 300^2)$
$= 62\,540$;

$\therefore$ R.M.S. value of total e.m.f. $= \sqrt{(62\,540)} = 250{\cdot}1$;

$$\text{Breadth coefficient} = \frac{250{\cdot}1}{3 \times 87{\cdot}9} = {\cdot}948.$$

(By calculus, R.M.S. value of e.m.f. = 250·6, breadth coefficient = ·947.) The resultant e.m.f. is nearer sinusoidal than the e.m.f. of a single coil.

If the coil e.m.f. is analysed by the approximate method of Chapter II., Art. 9, it is found to be given approximately by:—

$$124{\cdot}4 \sin\theta + 33{\cdot}3 \sin 3\theta + 8{\cdot}9 \sin 5\theta.$$

On plotting the first quadrant of this it will be seen that it agrees very closely with the original straight line graph except in the neighbourhood of 15° and of 75°. The R.M.S. value of this e.m.f. is:—

$$\sqrt{\{(124{\cdot}4^2 + 33{\cdot}3^2 + 8{\cdot}9^2)/2\}} = \sqrt{8335} = 91{\cdot}3 \text{ V}.$$

The breadth coefficient for the fundamental is $\sin 30^\circ/(3 \sin 10^\circ)$, *i.e.* ·960 as in Example 1. Similarly that of the 3rd harmonic is $\sin 90^\circ/(3 \sin 30^\circ)$, *i.e.* ·667; and of the 5th harmonic, $\sin 150^\circ/(3 \sin 50^\circ) = {\cdot}218$. Thus the amplitudes of the resultant e.m.f. are:—

Fundamental, $3 \times 124{\cdot}4 \times {\cdot}960 = 358{\cdot}3$;
Third harmonic, $3 \times 33{\cdot}3 \times {\cdot}667 = 66{\cdot}6$;
Fifth harmonic, $3 \times 8{\cdot}9 \times {\cdot}218 = 5{\cdot}8$:

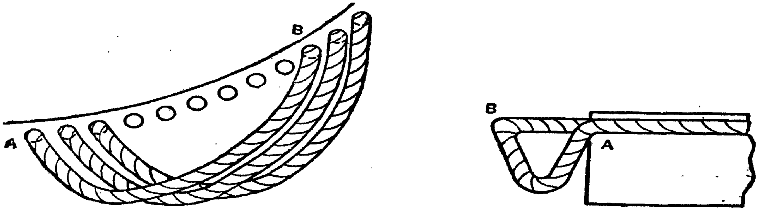
and its R.M.S. value is $\sqrt{\{(358{\cdot}3^2 + 66{\cdot}6^2 + 5{\cdot}8^2)/2\}} = \sqrt{66\,420} = 258$ V. Therefore the breadth coefficient for this winding with the stated field form is $258/(3 \times 91{\cdot}3) = {\cdot}942$.

The differences from the previous results are due to the slightly different field form.

The most important result of the complete analytical investigation is that the harmonics of frequencies $(2Q \pm 1)$ times the main frequency appear in the resultant e.m.f. wave in the same ratio as in the flux wave, where Q = No. of slots per pole; *e.g.* in the above example the 17th, 19th, 35th, etc., harmonics appear undiminished in relative amplitude in the resultant wave. All other harmonics are diminished in the resultant wave relatively to the fundamental.

7. High-Pressure Windings

In a high-pressure winding there is a large number of wires in each slot and so the arrangement of the ends of the coils requires care, whereas the connexions between successive coils, being only single wires, can be effected easily. By making all the coils of the same pitch (usually equal to the pole-pitch) their end connexions can be arranged in two layers in a way similar to a D.C. coil-wound armature (cf. Volume I., Chapter VIII., Art. 7). This **basket** (or



(*a*) SIDE VIEW. (*b*) END VIEW.

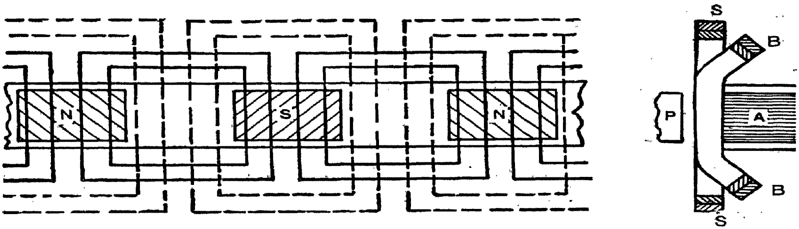
Fig. 4.08.—SKEW WINDING.

skew) **winding** differs from the latter, however, in having its coil-sides all at the same distance from the shaft instead of in two rings.

It is generally more convenient to make the coils of different pitches, and with the smaller coils fitted inside the larger ones. This enables the end connexions of a monophase alternator to be arranged without any crossings, as shown by full lines in Fig. 4.09.

For a two-phase alternator the second phase may be connected in the same way, except that the end connexions have to cross those of the first. This can be arranged by making one set " straight out " (*S*, *S* in sectional end view), and the other " bent-up " (*B*, *B*). The drawback to this is that the bent-up coils will have larger inductance than the straight out ones, and this upsets the balance of the p.d.s on load.

For a three-phase alternator the same plan can be followed, but there will have to be three " ranges " of end connexions, viz. straight out, partly bent up, and completely bent up. To avoid this, and for other reasons, it is usual to make the winding **hemitropic** (*i.e.* half-turned) in such cases. This means that the coil sides of any one phase which lie under one pole are all connected the same way round at the ends: thus in Fig. 4.09 the two left-hand coil-sides



(*a*) WINDING DIAGRAM. (*b*) SECTIONAL END VIEW.

Fig. 4.09.—TWO-PHASE COIL WINDING (CONCENTRIC).

under the S. pole would be connected to two coil-sides under the right-hand N. pole, instead of to two under the left-hand N. pole as in the diagram.

Fig. 4.10 shows a hemitropic three-phase winding. It will be seen that the end connexions can be arranged in two ranges as against three for the previous method. Moreover those of one phase are straight out and bent up in alternate groups, and so the phases are kept balanced. On the other hand the total length of the end connexions is increased by the hemitropic method.

A difficulty arises with an odd number of pole-pairs (*i.e.* 6 poles, 10 poles, etc.), as then one group of end connexions has to be straight out for one half and bent up for the other half. This increases the number of different shapes of coils and makes their clamping more difficult. The " skew " winding (see above) may be used in such cases.

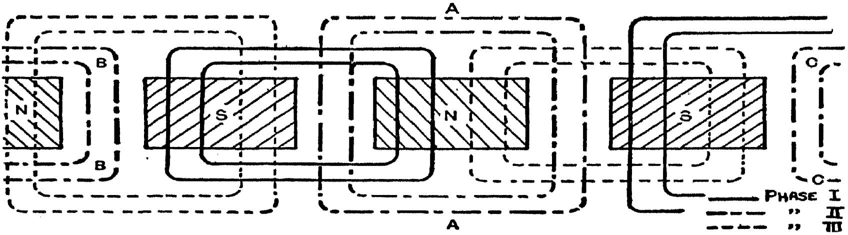


Fig. 4.10.—THREE-PHASE HEMITROPIC WINDING (CONCENTRIC).

Large armatures usually are made in two halves. When these are to be separated the unwinding of any coil can be avoided by lifting out of the slots one side of any coil whose end connexions cross the dividing plane. Only some of the single-wire connexions between coils need then be broken to divide the armature.

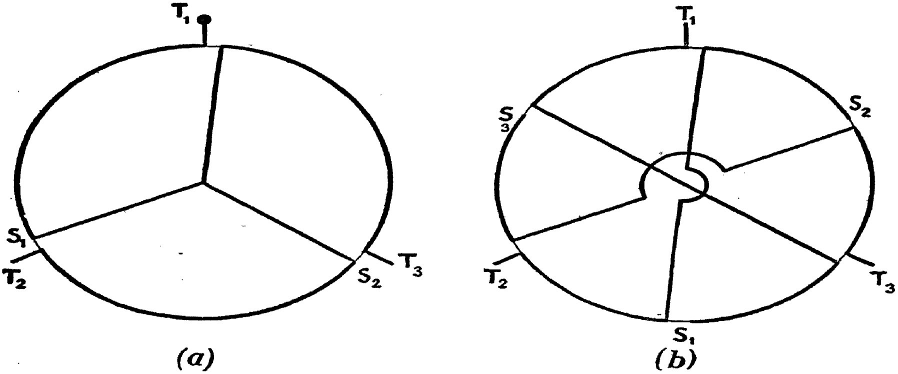
This lifting cannot be done with roofed slots nor with tunnels. By modifying the winding the required dividing points can be introduced. *E.g.* to obtain a division across AA in Fig. 4.10 the end connexions of phase II. must be reversed. The coils crossing AA then have their left-hand sides connected to the coil-sides BB, and their right-hand sides to those at CC, and so on round the armature. This will require three ranges of end connexions. As the divisions occur once for every pole-pair they will not be diametrically opposite for odd numbers of pole-pairs.

In all these cases the e.m.f. is the same as for coils of a constant pitch equal to the average value (6 slots or 1 pole-pitch in Fig. 4.10).

Moreover, if the winding is not hemitropic it may be replaced by a hemitropic winding before obtaining the average. Applying this to one phase of Fig. 4.09*a* gives average coil pitch $= \frac{1}{4}(5 + 7 + 9 + 11) = 8$ slot-pitches $= 1$ pole-pitch; *i.e.* it is equivalent to a winding with four full-pitch coils at $22\frac{1}{2}°$ phase differences.

8. Low-Pressure Windings

Low-pressure windings differ from high-pressure ones chiefly in that the coils consist of one, or at most of a few turns. *All* the end connexions, therefore, require care in their arrangement. The methods used, therefore, resemble those employed in D.C. bar windings.



(*a*) THREE OPENINGS. (*b*) SIX OPENINGS.

Fig. 4.11.—CONVERSION OF D.C. WINDING TO THREE-PHASE WINDING.

T_1 T_2 T_3, Terminals. S_1 S_2 S_3, Connections to star-point.
Fine lines represent the places at which the winding is opened.

A D.C. simple wave winding can be converted into an A.C. winding by opening it at the proper points and connecting these to the terminals. For a three-phase generator these openings will occur at points 120° (electrical) apart as shown diagrammatically in Fig. 4.11 (*a*). The commutator connexions are not required, and the winding is connected to the terminals and to the star-point as shown.

It should be noted that the openings are not at points actually 120° apart, but at points between which one-third of the total number of coils occur in following through the winding. *E.g.* in a wave-winding for 4 poles with 54 conductors and pitches of 13 and 13 (see Volume I., Chapter VIII., Art. 9), the openings could be made between (*a*) conductors Nos. 42 and 1; (*b*) Nos. 6 and 19; (*c*) Nos. 24 and 37.

An alternative method is to (i) make *six* openings in the winding [Fig. 4.11 (*b*)], (ii) connect the six sections in pairs in series (as shown by the 3 diametral connectors, which are not connected to one another), and (iii) connect one end of each of the three pairs to the star-point, and the others to the terminals. The advantage of this latter method is that it gives a higher winding factor. For winding (*a*) the factor is $(\sin \pi/3)/(\pi/3) = \cdot 827$, and for winding (*b*) it is $(\sin \pi/6)/(\pi/6) = \cdot 955$ (cf. Art. 5).

Similar windings may be made in a single layer, the end connexions being then similar to those of the " skew " winding (see Art. 7). These are called **creeping-bar** windings, because after passing once round the armature the connexions return to a bar two away from the start.

In both cases the number of bars must be of the form $2\,mp \pm 2$, where m is an integer, and so it is impossible to use this method for three-phase when the number of poles is six or any multiple thereof. For then p is divisible by three and so the total number of bars cannot be divisible by three.

For such cases, and as an alternative when the above methods are possible, a winding of the following type may be used. The number of bars per pole is a whole number. The pitch is equal to the pole-pitch for one circuit of the armature. Then a step either forwards or backwards and either one bar more or one less than the pole-pitch is taken. The pitch then becomes equal to the pole-pitch for another circuit of the armature. This is repeated until all the bars of a phase have been connected.

E.g. for a 6-pole winding with 54 bars the winding of one phase may be—

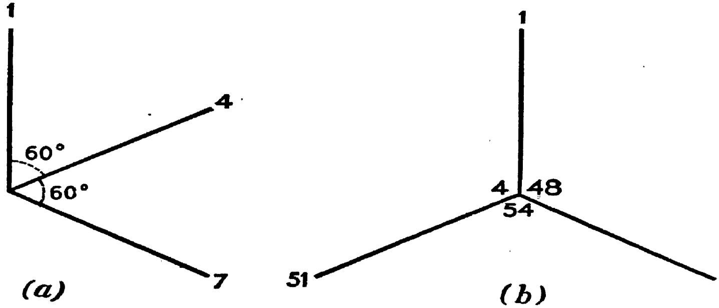
(*a*) Start 1—10—19—28—37—46—
—2—11—20—29—38—47—
—3—12—21—30—39—48— Finish.

or (*b*) Start 1—10—19—28—37—46—
—38—29—20—11— 2—47—
—3—12—21—30—39—48 Finish.

In (*a*) the numbers in each row might be one less than those in the previous row instead of one more as given. A similar modification could be made in case (*b*). The only differences between these is a small change in the end connexions.

The windings of the other two phases will be similar, starting with bars Nos. 7 and 13 respectively. The conductors may be arranged in either a single layer or in two layers.

It may be noted here (similar considerations applying to all three-phase windings) that it would be incorrect to start one of the phases at bar No. 4. This would give two phase-differences of 60° as shown in Fig. 4.12 (*a*). But if a winding is made with No. 4 as one end of it, and exactly similar to the first winding the correct result will be obtained by calling No. 4 the finish instead of the start. Thus if the finishes of phases I. and II. (viz. bars Nos. 48 and 54) are connected to the star-point, No. 4 must be connected to this too [Fig. 4.12 (*b*)], and Nos. 1, 7, and 51 will go to the terminals.



(*a*) INCORRECT. (*b*) CORRECT.

Fig. 4.12.—CONNEXIONS OF 3-PHASE WINDING TO STAR-POINT.

9. Pole-Shoe Forms

With a uniform air-gap and a rectangular pole-shoe the flux distribution will be uniform (except for the disturbance due to armature slots) under the pole-shoe, and will diminish rapidly beyond this. This gives rise to a corresponding e.m.f. wave in each coil, and though the total e.m.f. wave is of a different shape (see Example 2) it is by no means sinusoidal.

One method of obtaining approximately sinusoidal distribution of flux is to increase the air-gap from the centre to both edges of the pole-shoes. The flux-density varies inversely as the reluctance, *i.e.* approximately inversely as the length of the air-gap. Hence the desired result is obtained if the air-gap $(l) = l_0/\cos\alpha$, where:—

l_0 = length of gap at centre (AB),
α = "electrical" angle from centre.

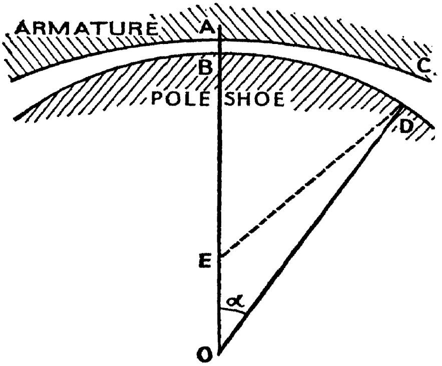


Fig. 4.13.—VARYING AIR GAP.

This makes l double l_0 when $\alpha = 60°$, and beyond this l increases rapidly, and so the pole-shoe arc may be given the value $\frac{2}{3}$ of pole-pitch. The fringing will give a good approximation to sinusoidal distribution in the interpolar space.

An approximation to this shape is obtained by making the polar surface cylindrical but of a smaller radius (ED in Fig. 4.13) than for a uniform gap.

Example 3. *A 6-pole magnet for an alternator is* 100 *cm. diam. and has an air-gap of* 0·8 *cm. at centre of pole-shoes. Find the radius of these to give approximately a sinusoidal field.*

If in Fig. 4.13: OB = 50 cm., OA = 50·8 cm.

Then when $\alpha = 60°$ electrical $= 60°/3 = 20°$ actual,

$$CD = 2 \times 0{\cdot}8 = 1{\cdot}6 \text{ cm.}$$

$$\therefore\ OD = 49{\cdot}2 \text{ cm.}$$

Let required radius $= r = EB = ED$.

$$ED^2 = OE^2 + OD^2 - 2\,OE.OD \cos EOD;$$

$$\therefore\ r^2 = (50 - r)^2 + 49{\cdot}2^2 - 2\,(50 - r) \times 49{\cdot}2 \times \cos 20°$$

$$= 2500 - 100r + r^2 + 2420 - (4920 - 98{\cdot}4r) \times 0{\cdot}94;$$

$$\therefore\ 100r - 92{\cdot}5r = 4920\,(1 - 0{\cdot}94);$$

$$\therefore\ 7{\cdot}5\,r = 295; \quad \therefore\ r = 39{\cdot}3 \text{ cm.}$$

Thus in this case the radius is made about 80 per cent. of the radius for uniform air-gap.

For other cases this percentage will be the same if the ratio of gap (AB) to radius (OB) is unchanged and α is constant. For relatively larger gaps the percentage is smaller, and for smaller values of α, *i.e.* more poles, it is likewise smaller.

A second method for approximating to a sinusoidal field is to vary the axial width of the pole-shoe. If this width varies as sin α the required effect is produced but at the expense of great magnetic leakage between adjacent pole-tips (see Fig. 4.14 *a*).

By reversing half of each pole as in Fig. 4.14 *b* the leakage is reduced and the effect is unchanged.

An approximation to this shape is obtained by making the pole-shoes parallelograms (Fig. 4.14 *c*). A pole-arc of 65 per cent. of pole-pitch, and a *skew* (i.e. *sr*) of 35 per cent. give good approximation, particularly if the blunt corners (*r*, *r*, *r*, *r*) are rounded.

When the pole-shoe is laminated it is more convenient for construction to do the skewing in a number of steps (see Fig. 4.14 *d*).

Examples of pole-construction as shown in Fig. 4.15 are suitable for alternators with many poles driven at slow speeds.

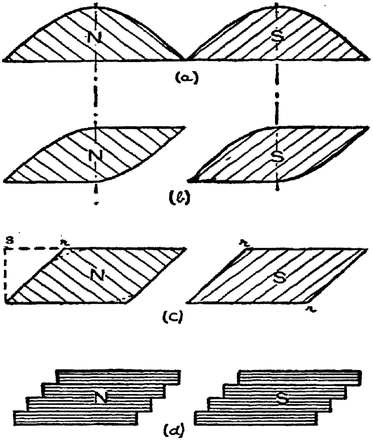
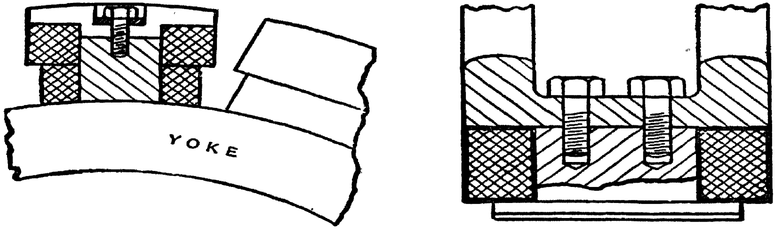


Fig. 4.14.—POLE-SHOES FOR SINUSOIDAL FIELDS.

10. Cylindrical Rotors

Owing to the difficulties of constructing magnets of the salient pole type for the high speeds required with steam turbine drive, the cylindrical type is used for such cases. This is shaped like a D.C. drum armature but with slots in part only of the circumference (Fig. 4.16). The magnetising coils are wound in the slots, and the main part of the flux passes through the unslotted portion with a very considerable fringing owing to the spread of the winding (see Example 4).

All the conductors in a group of slots such as *a*, *a*, *a*, *a* in Fig. 4.16 carry current in one direction, and those in neighbouring slots (*b*, *b* . . .) in the reverse direction. Consequently the unslotted portions are of alternate polarity. The conductors in half the slots in each group are connected to the group on its right, and those in the other half to the group on its left.



(*a*) LAMINATED POLE-SHOE. (*b*) CORE AND SHOE BOLTED TO YOKE.

Fig. 4.15.—POLE-SHOE CONSTRUCTIONS.

Example 4. *If a cylindrical rotor has slots in groups of six with the same number of conductors in each, find the flux wave-form for a slot-pitch of* 20° (*electrical*).

The m.m.f. is zero over the centre tooth of each group. On passing across each slot the m.m.f. alters by the same amount. The m.m.f. is, therefore, of the wave-form shown in Fig. 4.17, the heights of the steps being uniform.

Neglecting the variation in reluctance of the air-gap due to the slots this will be the wave-form of the flux too. A sine-wave of the same maximum height is shown dotted for comparison.

It may be noted that the effect is similar to the resultant e.m.f. of a number of coils with a rectangular flux-wave.

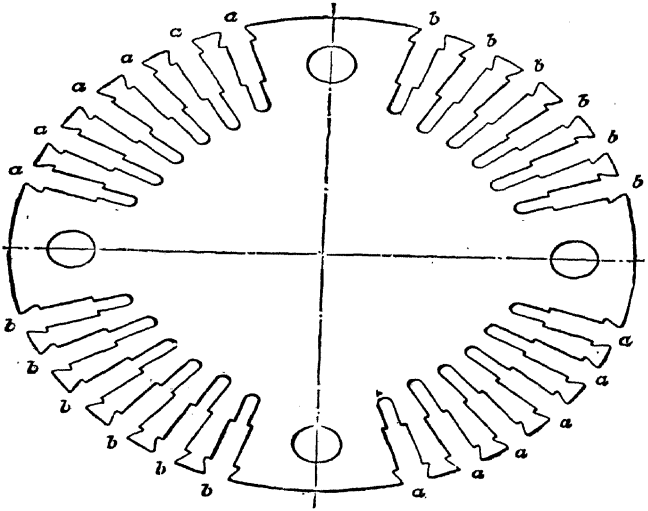


Fig. 4.16.*—CYLINDRICAL ROTOR CORE.

11. Armature Reaction

Armature reaction in an alternator affects the field very differently for different power-factors. Taking first the case of a monophase alternator with unity power-factor, the state of affairs when the e.m.f. (and therefore the current too) is at its maximum value is represented in Fig. 4.18 (*a*). The centres of the poles are opposite the centres of the coil-sides, the directions of the currents in which can be obtained by the Right-hand Rule (Volume I.,

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* Reproduced from *J.I.E.E.*, Vol. 54, p. 66.

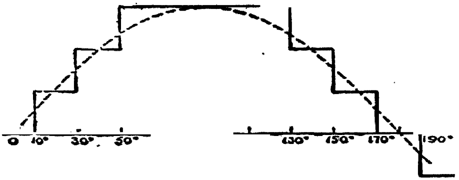


Fig. 4.17.—Flux Wave of Cylindrical Rotor.

Chapter VIII., Art. 5), remembering that the direction of motion *of the conductors* relative to the field is to be used in applying the rule.

The m.m.f. of the armature currents is as shown by the double-headed arrows. This weakens the left-hand side of the N. pole shown, and strengthens its right-hand side. It has a similar effect on the neighbouring S. poles, and so on all round the magnets. This may be summed up by saying that:—*A current in phase with the e.m.f. weakens the leading pole-tips and strengthens the trailing pole-tips*:—i.e. *it distorts the field against the direction of rotation of the magnet.*

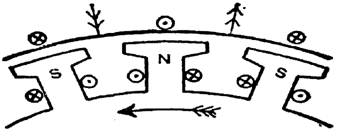
This is similar to the effect in a D.C. generator when the brushes have no lead. In this case too there is no direct weakening of the field as a whole, but there is an indirect weakening due to the changes in the permeabilities of the armature teeth (Volume I., Chapter IX., Art. 6).

It may be noted that it is when the current is in phase with the *pole position* that pure distortion occurs. The distortion delays the flux cutting and so such a current leads the voltage by a small amount.

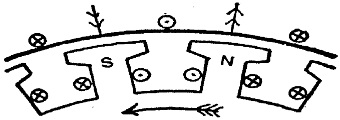
When the armature current lags by 90° (zero power-factor) it does not reach its maximum until the poles have moved half a pole-pitch beyond the point of maximum e.m.f. Consequently [see Fig. 4.18 (*b*)] the armature m.m.f. directly weakens the poles at every point.

This corresponds to the effect in a D.C. generator of giving the brushes a lead, since this lead makes the reversal of armature current occur later.

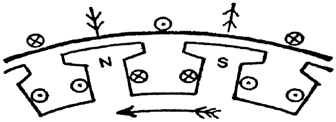
When the armature current leads the e.m.f. by 90° the opposite effect is produced, *i.e.* the field is strengthened at every point [see Fig. 4.18 (*c*)]. For the maximum current occurs when the poles have not yet reached



(*a*) Cos $\phi = 1$.



(*b*) Cos $\phi = 0$, Leading.



(*c*) Cos ϕ 0, Lagging.

Fig. 4.18.—Armature Reaction.

position (*a*) by half a pole-pitch, *i.e.* they are a whole pole-pitch away from position (*b*).

This result is evident also from the fact that a current leading by 90° is exactly the opposite to one lagging by 90°.

These two results may be summed up in the statement:—

A lagging current weakens, and a leading current strengthens, the field of an alternator.

When the power-factor lies between 0 and 1 both the weakening (or strengthening) and the distorting effects occur. The latter is approximately proportional to cos ϕ and the former to sin ϕ.

In all cases the armature m.m.f. of a monophase alternator varies between maximum and zero. Hence the resultant field fluctuates in position or magnitude or both. The former fluctuation is opposed by the eddy currents which it generates in the pole-shoes. The fluctuation in magnitude is opposed by the changes in exciting current due to the e.m.f. induced in the magnet windings by the change of field strength.

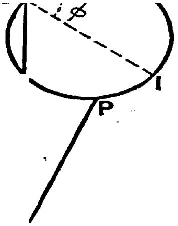


Fig. 4.19.—Variation of Armature Reaction.

These fluctuations may be represented roughly as follows. Let AB (Fig. 4.19) represent the field ampere-turns, and let $\angle$ ABC = right angle, $\angle$ CBD = angle of lag of armature current, and BD = armature ampere-turns at instant of maximum value. Draw a circle on BD as diameter. Let a point P rotate round this circle once each half-period with uniform velocity, reaching B at the instants of zero armature current. Then AP represents approximately the resultant field at any instant.

It will be seen from the above diagram that the greatest weakening of the field occurs with $\phi = 90°$, and that then the resultant field oscillates equally on both sides of AB. As ϕ diminishes the weakening becomes less and the average direction moves to the right of AB, *i.e.* there is both distortion and weakening of the field.

12. Armature Reaction in Polyphase Generators

Armature reaction in these generators produces the same general results as in monophase alternators but the fluctuation is very much reduced (cf. Volume I., Chapter VI., Art. 7).

For instance with a 3-phase armature with one slot per pole per phase the m.m.f. of the armature at the instant when the current in phase I is at its maximum is as shown in Fig. 4.20. In (*a*) the coil-sides are shown with the directions of the currents marked, those in phases II and III being negative and of half the maximum value (see vector diagram). The m.m.f.s of the separate phases are consequently as shown in (*b*). Combining these gives the resultant m.m.f. as shown in (*c*).

The maximum value of this m.m.f. is twice the maximum m.m.f. due to one phase. The average value over one pole-pitch is $1\frac{1}{3}$ times maximum for one phase, and so is just over double the time-average for one phase (which is $2/\pi$ or ·637 times the maximum). The average taken over two-thirds of pole-pitch, including all the maximum portion of wave (*c*), is $1\frac{1}{2}$ times the maximum of one phase.

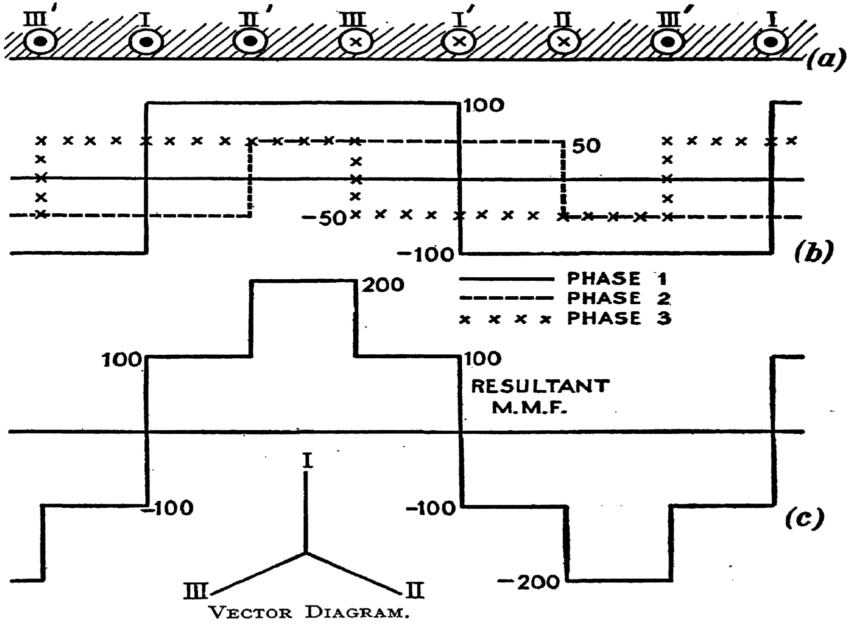


Fig. 4.20.—ARMATURE REACTION OF THREE-PHASE ALTERNATOR.

The centre line of the armature m.m.f. wave coincides at this instant with the centre line of the coils of phase I. The position of this relative to the poles depends on the phase of the armature current. When this is in phase with the e.m.f. the armature m.m.f.

centre line is midway between poles, and produces distortion without weakening. When the current lags (or leads) by 90° the m.m.f. centre coincides with the pole centre, and so directly weakens (or strengthens) the field. For intermediate power-factors both distortion and alteration of strength are produced (cf. previous Art.).

At an instant $\frac{1}{12}$th of a period later than the above, the currents are as shown in Fig. 4.21 (*a*). That in I has diminished to 86·6

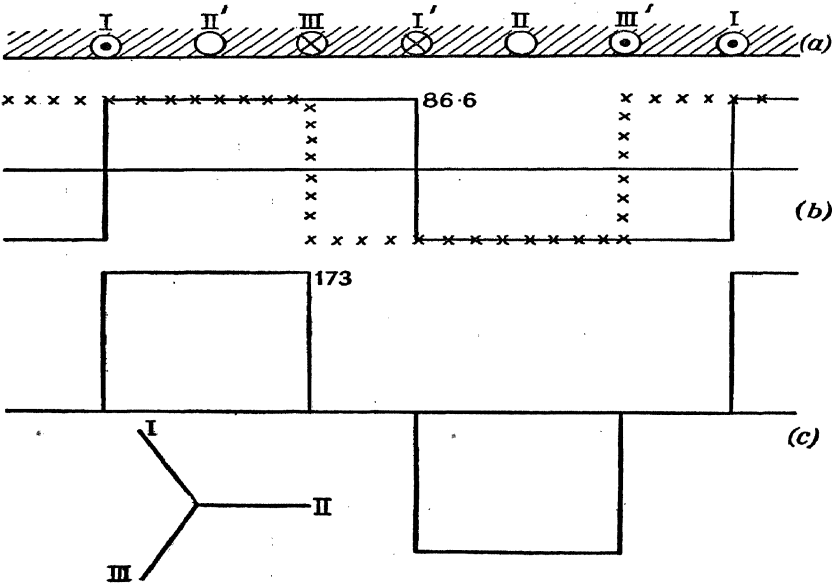


Fig. 4.21.—Armature Reaction $\frac{1}{12}$th Period Later.

per cent. of maximum value and III carries an equal current in the negative direction, while in II there is no current at this moment (see vector diagram).

The m.m.f.s of the separate phases are, therefore, as shown in (*b*), and the resultant m.m.f. as shown in (*c*).

The maximum value of this m.m.f. is 1·73 times the maximum of one phase (compared with twice in previous case), and the average over the pole-pitch is 1·15 times this (compared with $1\frac{1}{3}$ times).

But the average over two-thirds of pole-pitch is 1·73 times maximum per phase (against $1\frac{1}{2}$ times previously).

The centre line of the wave now coincides with coil-side II ′, *i.e* it has moved half a slot-pitch (= 30° electrical) to the left. And this is the distance moved by the poles in $\frac{1}{12}$th of a period. Thus the relative position of the poles and the armature m.m.f. is unaltered if the power-factor has not changed.

Another $\frac{1}{12}$th of a period later III carries maximum current in the *negative* direction, and I and II carry half maximum in positive direction. Consequently the resultant armature m.m.f. is of the same wave-form as in Fig. 4.20 (*c*), but the positive centre line is half a pole-pitch to the left of the coil-side III, instead of this distance to the left of coil-side I′. Thus the m.m.f. wave has moved to the left of its first position by $\frac{1}{3}$rd of pole-pitch (= 60°), *i.e.* the same distance as the poles have moved.

Thus in the three-phase generator with balanced loads and constant power-factor, the armature m.m.f. always acts in a position fixed relatively to the poles, rotating round the armature at the same velocity as these. Its wave form changes from that of Fig. 4.20 (*c*) to that of Fig. 4.21 (*c*) in $\frac{1}{12}$th period, and back again in the next $\frac{1}{12}$th period, and so on in succeeding twelfths. The fluctuation is thus small compared with that in a monophase generator.

The correct value to take for the average armature m.m.f. depends on the shape of the pole-shoes, or the distribution of slots in a cylindrical rotor, as well as on the disposition of armature coils.

Other three-phase armatures give similar results, except that increasing the number of slots per pole brings the waves closer to the sinusoidal form. Two-phase generators behave in much the same way, except that the fluctuations in wave-form are rather greater, and are spread over eighths of the period in place of twelfths.

When the number of slots is not too small the armature m.m.f. is approximately (see further Chapter VI., Art. 23) a sine-wave of maximum value $m\sqrt{2}I_A\mho_A$; where I_A = armature current (R.M.S.), $\mho_A$ = No. of armature turns per phase per pole, and $m = 1\frac{1}{2}$ for three-phase, 1 for two-phase.

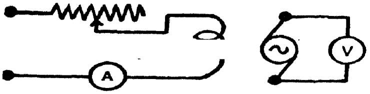
13. Experimental Determination of Regulation

The **inherent regulation** of an alternator (cf. Volume I., Chapter XVII., Art. 12) is the change in its terminal p.d. between the full load and no load when it is driven at a constant speed (and therefore constant frequency), and with constant exciting current in its magnet windings.

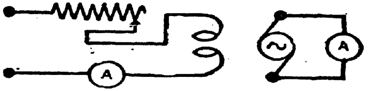
This regulation is expressed usually as a percentage of the terminal voltage at the rated output of the alternator.

The values of this will vary with the power-factor of the load in the same sort of way as in transformers. Behn-Eschenburg's method of finding them without actually putting full load on the alternator is to treat the effect of armature reaction as equivalent to a constant inductance in the armature circuit, or a constant reactance at a fixed speed. This is similar to the use of a reactance to represent the effect of magnetic leakage in a transformer (Volume I., Chapter XVII., Art. 7), the two cases being very similar. The chief differences are that the effect is much larger in an alternator owing to its poorer magnetic circuit, and that the inductance of the latter varies with the position of the magnets.

The value to give to this equivalent inductance is found in the following way. The alternator is driven at normal speed with its armature on open circuit. The excitation is varied and its values measured by an ammeter [see Fig. 4.22 (*a*)], and the corresponding values of the open-circuit p.d. are measured by a voltmeter connected to the armature terminals. If the speed varies the voltmeter readings are corrected in proportion to give the p.d. at normal speed.



(*a*) OPEN CIRCUIT TEST.



(*b*) SHORT CIRCUIT TEST.

Fig. 4.22.—REGULATION TESTS.

The values of this p.d. are then plotted as ordinates with the excitation currents as abscissae. The graph thus obtained is called the **open-circuit characteristic,** or the **magnetisation** (or **saturation**) **curve** (see Fig. 4.23). Like other magnetic curves it is nearly straight at first, and bends over at the higher values of the flux when the permeability of the iron diminishes.

The short-circuit test is made by joining the alternator terminals by leads of low resistance and an ammeter capable of measuring several times the full load current. The excitation is made very weak so as to prevent excessive current in the armature, and its values are measured by a low-reading ammeter [see Fig. 4.22 (*b*)].

The currents in the short-circuited armature are plotted as ordinates against the excitation values as abscissae, and this graph is called the **short-circuit characteristic** (see Fig. 4.23). It is as a rule very nearly but not quite straight.

The speed at which the alternator is driven should be near the normal, but considerable variations in speed make little difference in the short-circuit current. This is because the value of the latter is that required to wipe out almost entirely the part of the flux which links with the armature conductors. Another way of looking at it is to consider that the equivalent reactance of the armature varies as the frequency (the inductance being constant) and the e.m.f. varies as the speed with a given field strength. Hence, apart from the effect of resistance, the armature short-circuit current, which is equal to (e.m.f.)/(armature impedance), is independent of the speed.

The value to be given to the equivalent reactance is found from the two graphs as follows. The full load current is marked off on the scale of short-circuit current (OA in Fig. 4.23). Thence by the short-circuit characteristic the corresponding excitation (OB) is found. The open-circuit characteristic then gives the e.m.f. (BC) with the same excitation. Then the ratio (BC measured on volt scale) / (OA measured on current scale) gives the equivalent impedance ("short-circuit" or "synchronous") of the armature. From this and from the measured resistance of the armature (allowing for that of the ammeter and leads used) the equivalent reactance is found. This is then used to find the regulation in the same way as in a transformer (Volume I., Chapter XVII., Art. 12).

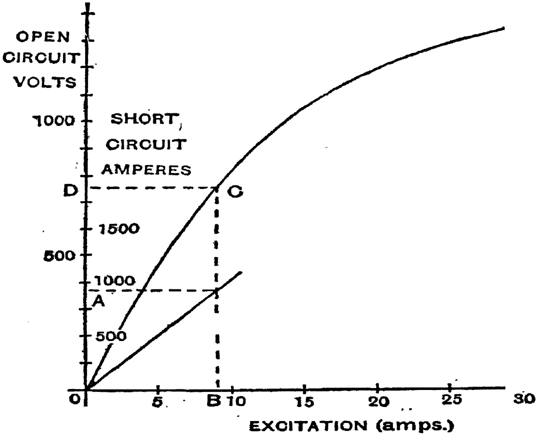


Fig. 4.23.—OPEN-CIRCUIT AND SHORT-CIRCUIT CHARACTERISTICS.

It will be seen that the whole procedure is very similar to that employed for the same purpose with transformers. The main difference is that, owing to the percentage pressure change being much greater in alternators, the approximate methods applicable to transformers cease to be sufficiently accurate. But the two-circle diagram (Volume I., Chapter XVII., Art. 12) applies, and is more useful for the alternator, as the same error in reading the results is a smaller fraction of the larger regulation.

Example 5. *An alternator requires an excitation of 1·9 amp. to produce on short-circuit its full load current of 50 amp. The same excitation produces on open-circuit an e.m.f. of 216 volts.*

If the resistance of the armature is ·785 ohm, and of the ammeter and leads used in the short-circuit test ·007 ohm, find the full load regulation (a) on non-inductive load; (b) with $\cos\phi = 0{\cdot}6$; *when the terminal p.d. on load is 1 000 volts.*

(*a*) Armature impedance $\frac{216}{50} = \; = 4{\cdot}32$ ohms;

$\therefore$ Armature reactance $= \sqrt{\{4{\cdot}32^2 - ({\cdot}785 + {\cdot}007)^2\}} = 4{\cdot}25$ ohms.

Fig. 4.24 (*a*) shows the vector diagram for non-inductive load.

OA = 1 000 volts in phase with the current.

$AB = 50 \times {\cdot}785 = 39$ volts in phase.

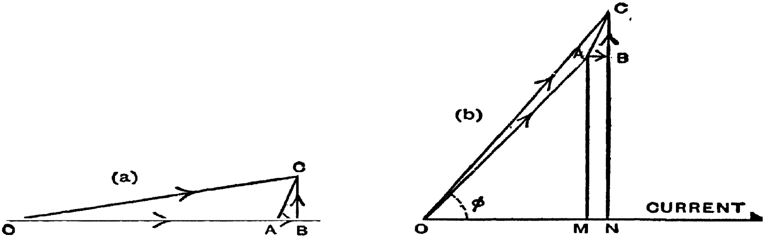
$BC = 50 \times 4{\cdot}25 = 212{\cdot}5$ volts, leading by 90°.

OC = total e.m.f. $= \sqrt{\{OB^2 + BC^2\}}$

$= \sqrt{\{(1\,039)^2 + (212{\cdot}5)^2\}} = 1\,061$ volts;

$\therefore$ Regulation $= 1\,061 - 1\,000 = 61$ volts.

N.B.—By approx. method (Vol. I., Chap. XVII., Art. 13) value = 39 volts.



NON-INDUCTIVE LOAD. INDUCTIVE LOAD.

Fig. 4.24.—VECTOR DIAGRAMS FOR REGULATION.

(*b*) Fig. 4.24 (*b*) shows the vector diagram for a lagging current.

AB, BC have the same values and phases as before.

Produce CB to meet current vector in N, and draw AM parallel to BN.

Then:— $OM = OA\cos\phi = 1\,000 \times 0{\cdot}6 = 600$ volts.

$BN = AM = \sqrt{\{OA^2 - OM^2\}} = 800$ volts;

$\therefore OC = \sqrt{\{ON^2 + NC^2\}} = \sqrt{\{(OM + MN)^2 + (NB + BC)^2\}}$

$= \sqrt{\{(600 + 39)^2 + (800 + 212{\cdot}5)^2\}} = 1\,197$ volts;

$\therefore$ Regulation $= 1\,197 - 1\,000 = 197$ volts.

N.B.—By approx. method, value $= 0{\cdot}6 \times 39 + 0{\cdot}8 \times 212{\cdot}5 = 193$ volts. The difference is less than in (*a*) because the internal power-factor differs less from the external one.

This equivalent reactance method is sometimes called the pessimistic method, because the calculated results are worse than the actual performance of the alternator. It is better to find the reactance from the values corresponding to the actual excitation, as this gives a smaller reactance (cf. Chapter VI., Art. 18). Thus the above alternator with 12·4 A. excitation produces 1 060 V. on open circuit; the short-circuit current is $50 \times 12{\cdot}4/1{\cdot}9 = 326$ A. Hence the armature impedance is $1060/326 = 3{\cdot}25$ ohms. The calculation is then as above. Therefore it is easier to apply the method when the excitation is known, instead of the voltage on load.

14. The Ampere-Turn Method

This method of calculating the regulation is due to Rothert. It employs the results of the open-circuit and short-circuit tests as the previous method does, but uses them in a different way.

Neglecting the effect of armature resistance, the total excitation required to produce a terminal p.d. of V volts, when the armature is carrying a current of I amperes at a known power-factor, cos ϕ, is found as follows. The open-circuit characteristic is used to find the excitation (OB in Fig. 4.25) which gives on open circuit the required p.d. V (= OA in fig.). The short-circuit characteristic is then used to find the excitation OD, which gives on short circuit the required current I (= OC in fig.).

The total excitation required to produce V volts with I amperes in armature is then obtained by adding the separate excitations vectorially. The angle OBE is made equal to $(90^\circ + \phi)$, and BE equal to OD. Then OE is the total excitation.

The corresponding open-circuit voltage is found by swinging OE round to OF, and measuring the ordinate FG (= OH) of the open-circuit graph. The regulation, therefore, is given by AH.

The justification for this procedure lies in the fact that a current lagging by 90° directly weakens the field (Art. 11), and that the short-circuit graph shows how much excitation is required to balance this demagnetising effect. As the angle of lag decreases the amount of weakening becomes less, and the above construction allows for this.

This method is optimistic, *i.e.* its results are generally better than the alternator will actually give on load. It should not be relied on alone, but may be used to compare alternators, or as a check on the equivalent reactance method.

The allowance for armature resistance may be made in the following way. The length OA marked off on the voltage scale is made equal to $(V + IR \cos \phi)$, where R = resistance of armature. The construction is then carried out as before. The regulation is then {OH (measured on voltage scale) — V}, *i.e.* it is equal to $(\text{AH} + IR \cos \phi)$.

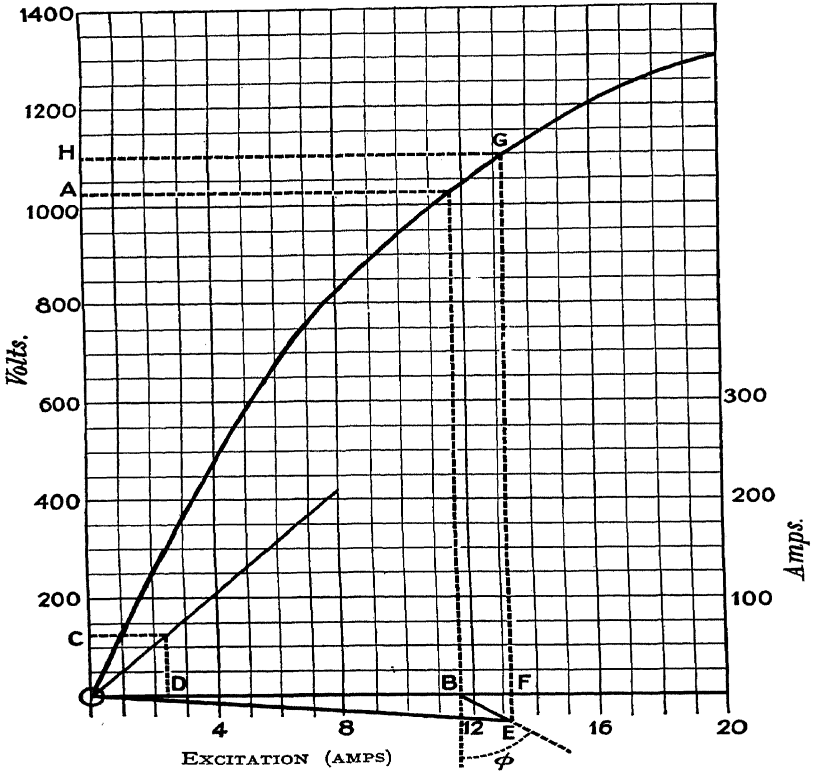


Fig. 4.25.—AMPERE-TURN METHOD FOR REGULATION.

Example 6. *The open-circuit and short-circuit tests of the alternator of Example* 5 (*Art.* 13) *gave the following results :—*

Excitation .. 4; 8; 12; 16; 20 *amp.*

E.m.f. .. 460; 820; 1 040; 1 190; 1 300 *volts on open circuit.*

Current .. 105; 220 *amp. on short-circuit.*

Find by the ampere-turn method the regulation for the same loads as in Example 5.

The open-circuit and short-circuit characteristics in Fig. 4.25 are plotted from the above figures.

(*a*) $V + IR \cos \phi = 1\,000 + 39 = 1\,039$ volts.

Whence, from graph, OB = 12·0 amp., BE = OD = 1·9 amp., and $\angle$ OBE = 90°;

$$\therefore \text{OF} = \text{OE} = \sqrt{\{(12{\cdot}0)^2 + (1{\cdot}9)^2\}} = 12{\cdot}15 \text{ amp.}$$

Whence, from graph, OH = 1 050 volts. (Not shown in Fig. 4.25.)

$\therefore$ Regulation = 50 volts.

(*b*) $V + IR \cos \phi = 1\,000 + 39 \times 0{\cdot}6 = 1\,023$ volts = OA.

Whence OB = 11·7 amp. as shown in Fig. 4.25,

BE = OD = 1·9 amp. as in (*a*),

$\angle \text{OBE} = 90° + \cos^{-1} 0{\cdot}6 = 90° + 53{\cdot}1° = 143{\cdot}1°$.

By completing the construction, OF is found to be 13·25 amp.

Thence:— OH = 1 100 volts,

Regulation = 100 volts.

15. The Zero-Power-Factor Method

The zero-power-factor method (due to Potier) of obtaining the regulation depends on the curve connecting the terminal p.d. when full-load current flows in a purely inductive load with the excitation. This zero-power-factor characteristic can be obtained without using much power, but it is not so easy to obtain as the short-circuit

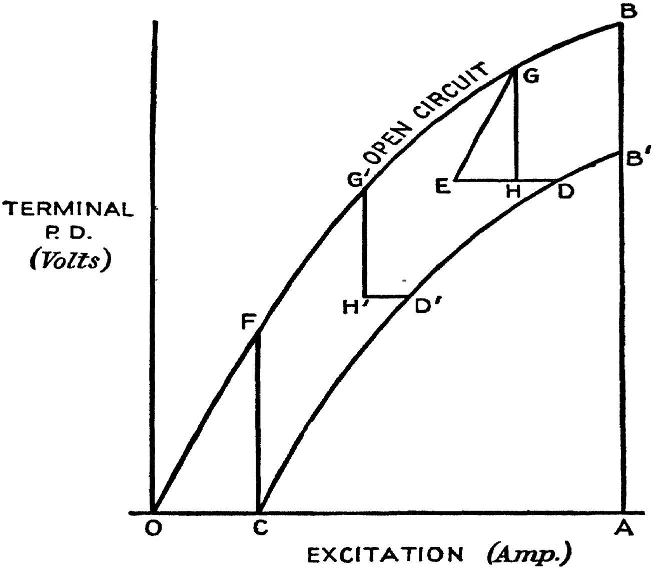
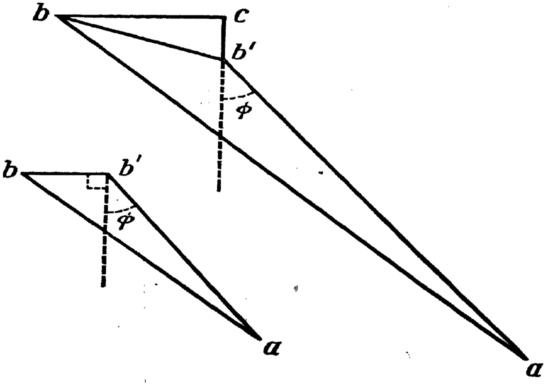


Fig. 4.26 (*a*).—ZERO-POWER-FACTOR AND OPEN CIRCUIT CHARACTERISTICS.

characteristic owing to the difficulty of providing suitable purely inductive loads. But it can be approximated to, as will be explained later.

When the zero-power-factor graph has been obtained either by direct experiment or otherwise, it is used as follows. The terminal p.d.s on open circuit [AB in Fig. 4.26 (*a*)] and at full load zero-power-factor (AB′) are obtained from the graphs for the excitation (OA) in use. A line *bb′* [Fig. 4.26 (*b*)] is drawn equal to the difference between these voltages (*i.e.* B′B). A line *b′a* is drawn, making an angle (90° + ϕ) with *bb′*. With centre *b* and radius equal to AB, an arc is drawn cutting *b′a* in *a*. Then the length of *b′a* gives the terminal p.d. with full-load current at the power-factor cos ϕ.



(*b*) NEGLECTING RES. (½ SCALE).

(*c*) ALLOWING FOR RESISTANCE.

Fig. 4.26 *b* and *c*.—ZERO-POWER-FACTOR METHOD. —These are essentially vector diagrams.

The above makes no allowance for armature resistance. The standard American method of allowing for resistance is as follows. The resistance drop (*IR*) is calculated and a line *b′c* [Fig. 4.26 (*c*)] is drawn equal to this. The angle *b′cb* is made 90° and *b′b* is made equal to B′B. A line *b′a* is drawn making an angle ϕ with *cb′* produced. The position of *a* is fixed by making *ba* equal to BA as before, and *b′a* again gives the terminal p.d. on full load of power-factor cos ϕ.

If only one point on the zero-power-factor graph is known, such as D in Fig. 4.26 (*a*), the remainder can be found in the following manner, due to Fischer-Hinnen. The initial point C can be found from the short-circuit graph, since OC is the excitation required to produce full-load current on short-circuit. From D a horizontal line DE is drawn equal in length to CO. From E a line is drawn parallel to the initial part of the open-circuit characteristic OF, and meeting this graph in G. Thence GH is drawn perpendicular to

ED. If from any other point on the open-circuit characteristic G′, a line G′H′ is drawn vertically downwards and equal to GH, and a line H′D′ horizontally to the right and equal to HD, then D′ is a point on the zero-power-factor characteristic. Any number of points may be obtained in this way.

This procedure depends on the following reasons. The constant vertical length GH represents the voltage absorbed by the inductance of the armature winding, which voltage is constant since the current is constant along the graph CD′DB′. The constant horizontal length HD represents the excitation required to balance the armature reaction. This again is constant all along CD′DB′ because the current and power-factor are both constant. And the fact that the latter is zero shows that the armature ampere-turns should be subtracted directly from the excitation (see Art. 11). It will be seen that the construction used gives the only values of GH and HD which will fit both the points C and D on the zero-power-factor graph.

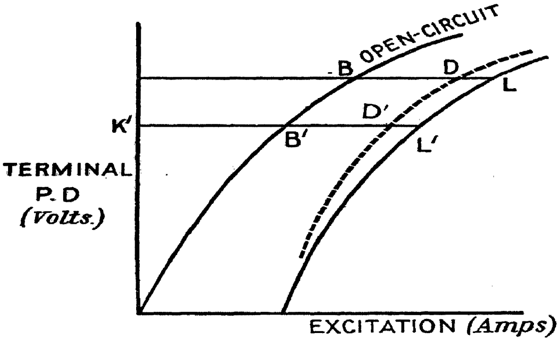


Fig. 4.27.—Approximation for Zero-Power-Factor Curve.

The equivalent reactance method (Art. 13) amounts almost to the same thing as obtaining the zero-cos ϕ graph purely by a vertical shift equal to CF. The ampere-turn method is similar to obtaining this graph by a horizontal shift alone, equal to OC.

When no point on the zero-cos ϕ graph is known, an approximation may be made as follows. The initial point C is obtained from the short-circuit characteristic as described above. The open-circuit characteristic is then shifted horizontally throughout by a distance equal to OC. The approximate zero-cos ϕ graph is then obtained by increasing the horizontal shift at normal excitation by an amount dependent on the type of alternator.

Thus, in Fig. 4.27 the open-circuit curve OB is shifted horizontally by the constant amount OC, thus obtaining the broken line curve CD. At normal p.d., OK, a further shift is made to L. In alternators of low reactance, *e.g.* turbo-alternators, DL may be 5 per cent. to 10 per cent. of BD. When the reactance is high, as in slow-speed alternators with many poles, DL may be 20 per cent. to 30 per cent. of BD. If the flux-density is high DL is large, and vice versa. For other points (B′, D′, L′) the ratio D′L′/B′D′ may be decreased in proportion to the open-circuit excitation, but the whole curve CL′L is not usually required.

16. Automatic Voltage Regulators

Since D.C. excitation is required, whereas the armature supplies A.C., an alternator cannot be compounded by the addition of series coils on the magnets as is done for D.C. generators. Moreover, since the regulation with a given current increases as the power-factor falls, the addition of a rectifying device could give the required increase of excitation at one power-factor only. When it is desired to maintain the terminal p.d. constant at all loads a method using the following principle is employed. It is applicable to D.C. generators as well.

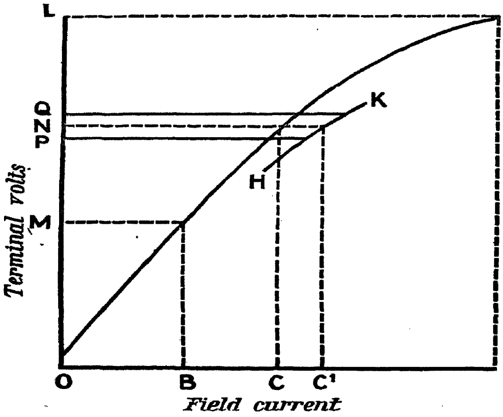


Fig. 4.28.—ACTION OF A.V. REGULATOR.

The field rheostat is divided into two parts, R_1 and R_2, the former alone being adjustable. R_1 is set at such a value that the resulting excitation would give more than the desired voltage at the heaviest load. R_2 is designed so that, with the lowest value used for R_1, the excitation with R_1 and R_2 both in circuit gives less than the desired voltage even at no load. Thus in Fig. 4.28 the field current when R_1 alone is in circuit is OA, which if continuous would produce OL volts at light load. With R_2 added the current falls to OB, which if continuous gives a voltage OM at the same load.

If the desired voltage is ON, a control magnet M (Fig. 4.29) cuts out R_2 when the voltage falls a little below this, to OP; and re-inserts R_2 when the voltage rises a little above ON, to OQ. In practice NQ and NP are about $\frac{1}{2}$ per cent. of ON. The result is that the voltage fluctuates quickly about ON by $\pm \frac{1}{2}$ per cent. The field current, owing to the inductance of the magnet windings, never rises to OA nor falls to OB, but fluctuates by small amounts above and below OC.

If the load increases the (voltage)-(field current) graph changes to some position such as HK. The voltage still fluctuates between OP and OQ; but the proportion of time with R_2 cut out is increased, so that the mean field current is increased to OC′, *i.e.* sufficiently to maintain the mean value of the terminal voltage at the increased load.

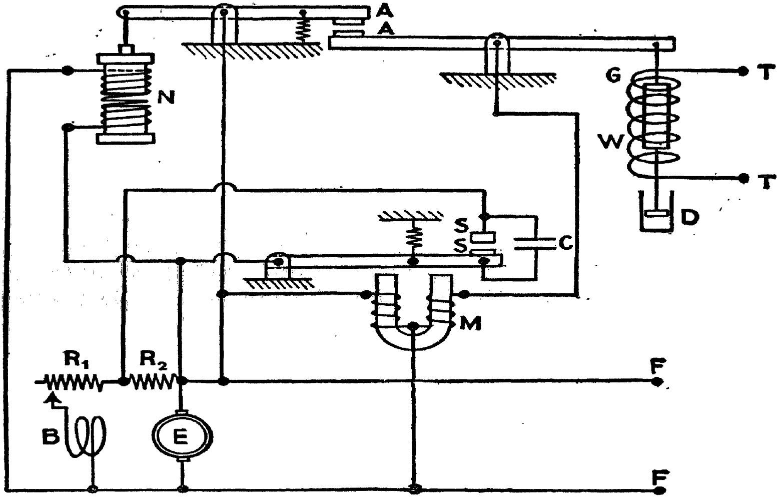


Fig. 4.29.—TIRRILL REGULATOR.

C. Condenser to suppress sparking. FF. To alternator field winding.
TT. To alternator terminals.

Usually the resistance R_2 is placed in the exciter field circuit This does not affect the principle of action and has the advantage of reducing the currents to be dealt with by the short-circuiting contacts.

One of the earliest regulators using this method was the Tirrill. The circuits employed are shown in Fig. 4.29. The field winding, B, of the exciter is connected to its armature, E, through two resistances. As long as the main contacts, A, A, are open the left half-winding of the magnet M opens the short-circuiting contacts, S, S, when the exciter voltage exceeds a certain value, and allows them to close when this voltage falls. The exciter voltage is thus kept constant in the way explained above.

If, however, the alternator voltage falls the iron weight W is no longer held up by the pull of coil G, and the main contacts, A, A, close. This causes equal currents to flow in the two half-windings of M so that it is demagnetised. This causes R_2 to remain shorted until the exciter voltage has risen sufficiently to cause the exciter control magnet, N, to separate the main contacts again. Thus the alternator voltage is kept constant, the average exciter voltage being altered to suit the load. A dashpot, D, is fitted to W so that it moves more slowly than the arm moved by N. The position of the lower A is therefore changed only when the load changes, whereas the upper one vibrates continually.

Other methods of automatic voltage regulation are:—

(*a*) The Taylor-Scotson, using a small reversible booster in the alternator field circuit. This is reversed continually, but the lengths of the times of boost and buck are altered to suit the load.

(*b*) Automatically adjusted rheostats, *e.g.* Thury, and Brown, Boveri. The latter has the advantage of giving an " overshoot " of adjustment. It thus approaches the rapidity of action of the vibrating contact (Tirrill) type, which has a large " overshoot."

17. Parallel Running of Alternators

Very often alternators are run in parallel in the same way as D.C. generators. When an additional alternator is to be connected in parallel with those which are supplying power, the adjustment of the conditions before closing the switch connecting it to the bus-bars is much more complicated than in the case of a D.C. generator. The latter only requires that its e.m.f. shall be nearly equal to the bus-bar voltage before switching it on to the bus-bars. This can be regulated either by altering the speed of the driving engine, or by altering the excitation by means of a field rheostat, whichever is more convenient.

In the case of the alternator not only must the e.m.f. be adjusted to approximate equality with the bus-bar p.d., but the frequency requires a similar adjustment. Finally, the two voltages must be brought into phase before it is safe to close the switch. This three-fold equalisation of the voltages is called the **synchronising** of the alternator.

The equality of the frequencies settles the speed of the incoming alternator. The excitation must be adjusted to give equality of

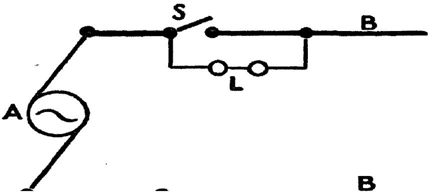


Fig. 4.30.—SYNCHRONISING CONNEXIONS.

A. Alternator. B B. Bus-bars.
L. Lamps. S. Switch.

voltages at this speed. The final adjustment is made as follows. If a low-pressure alternator is to be connected to the bus-bars by a single-pole switch, two lamps of the normal voltage are connected in series (Fig. 4.30) across the switch. The p.d. across these lamps is the difference between those of the alternator and of the bus-bars.

If these latter two voltages are equal, the lamp p.d. can have any value between zero, when the two are in phase, and double the normal voltage, when the two are exactly opposite in phase. If the frequency of the incoming alternator is not exactly the same as that of the bus-bars, the phase relationship of the two voltages will be continually changing. The R.M.S. value of the lamp p.d. will, therefore, fluctuate between zero and maximum, and the lamps will be alternately dark and of normal brilliancy.

This is illustrated in Fig. 4.31 (*a*), where OA is the vector of the *alternator's e.m.f., and* OB, *of equal length,* is the bus-bar voltage. Then AB represents the lamp p.d. If the frequencies of alternator and of bus-bars are equal, OA and OB rotate at equal speeds, and AB remains of constant length. The lamp p.d. is then of the same frequency and of constant R.M.S. value, and the lamps give a steady light. The strength of this light depends on the phase difference (∠ AOB), being greater the larger this angle up to exact phase opposition (∠ AOB = 180°).

But if the frequencies differ the vectors OA, OB rotate at different speeds, and AB varies in length between zero and twice OA. The lamp p.d., therefore, in addition to alternating at the mean frequency varies in amplitude [Fig. 4.31 (*b*)]. The slower variations

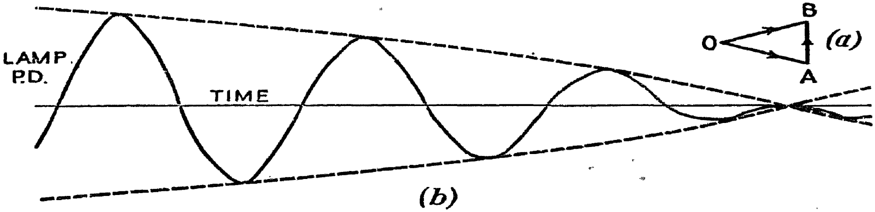
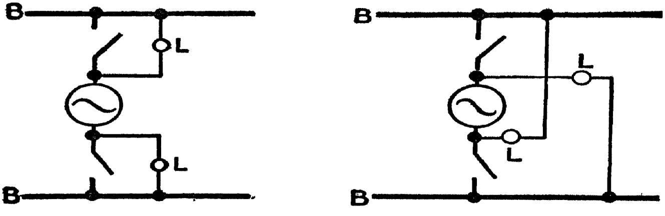


Fig. 4.31.—SYNCHRONISING LAMPS' VOLTAGE.

in amplitude (shown by the broken line in the figure) are made manifest by the brightening and extinction of the lamps.

The number of times per second that the lamps become bright is equal to the difference between the frequencies of the alternator and of the bus-bars: for this occurs whenever the vectors OA, OB are in opposite directions. Thus the cycle from bright through dark to bright occurs each time one vector gains a complete revolution on the other. The variations of voltage at about the normal frequency are too rapid to affect the lamp appreciably.

Thus the fluctuations in the light from the lamps show the difference between the actual speed of the alternator and the correct speed, and so enable the former to be adjusted closely to the proper value. When this has been done, as indicated by the slow brightening and dimming of the lamps, the switch is closed when the lamps are completely dark, as this indicates phase equality.



(*a*) DARK LAMPS. (*b*) BRIGHT LAMPS.

Fig. 4.32.—SYNCHRONISING CONNEXIONS.

18. Connexions for Synchronising Lamps

If a double-pole switch is used for connecting the alternator to the bus-bars, the connexions of Fig. 4.30 can be modified by connecting one lamp across each pole of the switch [see Fig. 4.32 (*a*)]. The effect is exactly the same as in the previous case.

A modification of the above is to cross-connect the lamps, as shown in Fig. 4.32 (*b*). The effect of this is that the lamps will be out when the alternator e.m.f. is *opposite* in phase to that of the bus-bars, and that the moment of phase equality is indicated by the greatest brightness of the lamps.

The advantage of this method is that at its normal voltage a lamp's candle-power changes by much larger percentages than the variations in the applied p.d.; whereas the p.d. can vary very considerably from zero without producing any appreciable light. Consequently the instant of phase equality can be found much more exactly with "bright lamp" synchronising than with "dark lamp" connexions.

When the pressure of the alternator is high it is no longer possible to use lamps directly, unless a number are connected in series in the places of the single lamps used for low pressure. By employing a step-down transformer with a S.P. switch, or two similar transformers with a D.P. switch, the connexions already described may still be used.

It is better, however, to use a special **synchronising transformer** for such cases. This consists (see Fig. 4.33) of a transformer whose core has three limbs. Two of these carry similar windings, one, AA, connected to the incoming alternator, the other, BB, connected to the bus-bars. The third limb carries a winding with a suitably reduced number of turns, connected to the synchronising lamp, L.

The action is as follows:—When the p.d.s applied to AA and BB are in phase, so that the m.m.f.s of these windings produce fluxes which are upwards at the same time, and then downwards at the same time, the flux in the centre limb is the sum of the two, *i.e.* double each flux. This large alternating flux produces a corresponding e.m.f. in the centre winding, and the lamp burns brightly. When the two p.d.s are exactly opposite in phase, the same applies to the fluxes, and so the whole flux up (or down) AA will return down (or up) BB, and none will pass through the centre limb, and the lamp will be completely out. For intermediate phase differences the flux in the central limb will have correspondingly reduced values, and the lamp will be dimmed to an extent depending on the phase difference.

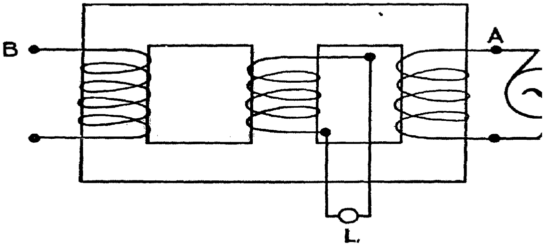


Fig. 4.33.—SYNCHRONISING TRANSFORMER.

This can be arranged for either bright or dark lamp synchronising. To change from one to the other requires only the reversal of the connexions either to AA or to BB (but not both).

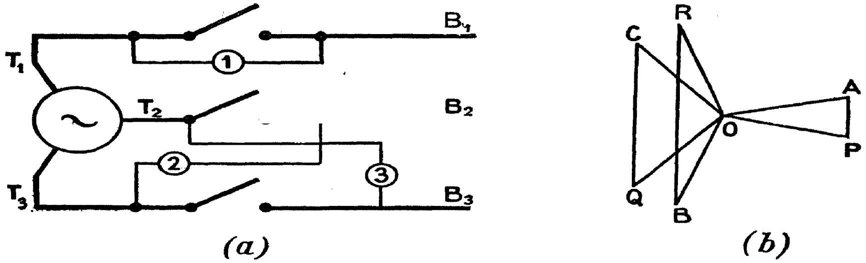
A voltmeter can be connected in parallel with the lamp so as to furnish an additional indication. In this case either the maximum reading of the voltmeter (with "bright lamp" connexions) or the zero reading (with " dark lamp " connexions) indicates the instant of phase equality.

If the two outer windings were placed on the same limb, then, when their m.m.f.s are equal and opposite, there would be no flux in this limb, and so no induced e.m.f. The windings would, therefore, carry excessive current under these conditions.

19. Method for Three-Phase Alternators

With a three-phase alternator, if one phase is correctly synchronised the other two phases will be so at the same moment, provided the connexions of the alternator to the bus-bars are made properly. This can be tested when installing a new alternator by synchronising with lamps on two (or all three) phases. With correct connexions the lamps will all light up and darken simultaneously.

When synchronising lamps are supplied on all three phases a simple change in the connexions increases the information given by the lamps. One lamp (or pair of lamps when sets of two in series are used) is connected between one terminal of the alternator and the bus-bar to which it will be connected. But the other two are cross-connected as shown in Fig. 4.34 (*a*).



(*a*) CIRCUIT DIAGRAM. (*b*) VECTOR DIAGRAM.

Fig. 4.34.—SYNCHRONISING LAMPS FOR 3-PHASE ALTERNATOR.

In the vector diagram [Fig. 4.34 (*b*)] OA, OB, OC represent the voltages of the three bus-bars B_1, B_2, B_3 respectively, and OP, OQ, OR those of the three terminals T_1, T_2, T_3 of the alternator. Consequently AP represents the p.d. on lamp No. 1, the direct-connected one, and BR that on lamp No. 2 and CQ that on lamp No. 3.

When the alternator voltages are in phase with the bus-bar voltages AP vanishes, and BR and QC are equal in length, *i.e.* lamp No. 1 is out, and in addition Nos. 2 and 3 are equally bright, giving two tests of phase equality.

More important is the effect produced by departures of the alternator speed from that corresponding to bus-bar frequency. If the machine is being driven at too high a speed OP, OQ, OR will rotate faster than OA, OB, OC. When P has caught up to A, lamp No. 1 will go out. A little later when P, Q, R have gained $\frac{1}{3}$rd of a revolution RB becomes zero and lamp No. 2 goes out. When P, Q, R have gained a further $\frac{1}{3}$rd of a revolution Q will have caught up with C and lamp No. 3 goes out.

On the other hand, if the machine is being driven too slowly OA, OB, OC will be rotating faster than OP, OQ, OR. Starting with A and P in coincidence, *i.e.* lamp No. 1 out; when P, Q, R have dropped back $\frac{1}{3}$rd of a revolution relatively to A, B, C, the points Q and C will coincide and lamp No. 3 goes out. When P, Q, R have dropped back a further $\frac{1}{3}$rd of a revolution B coincides with R and lamp No. 2 goes out. Thus the order of extinction (and equally of greatest brightness) is 1, 3, 2 in this case; and 1, 2, 3 when the machine speed is too high. The lamps, therefore, tell in which direction the alternator's speed requires adjustment, as well as the extent to which it differs from the correct value.

When transformers are used the same connexions may be employed. But it is then possible to alter the connexions so as to give "bright lamp" synchronising. In this case the instant of phase equality is shown by lamp No. 1 being at its maximum brightness, and lamps Nos. 2 and 3 equally dull. As the p.d. across the two latter is only half the maximum, against ·866 of maximum with No. 1 dark, their equal brightness cannot be judged so exactly. Nevertheless "bright lamp" synchronising is better on the whole since the equality of brightness depends on the exact equality of the lamps as well as on that of the p.d.s.

20. Synchroscopes

A number of instruments have been devised to indicate the exact moment for closing the switch when paralleling alternators. These instruments are called **synchronisers** or **synchroscopes** and two, depending on different principles, are described below.

The Weston synchroscope (Fig. 4.35) consists of a synchronising lamp combined with a modified monophase wattmeter of the dynamometer type (cf. Volume I., Chapter VII., Art. 13). The modification consists in winding the fixed coils with fine wire. The pointer lies behind the translucent scale and the lamp is mounted in the instrument behind the pointer. Consequently the latter is visible only when the lamp is bright.

The connexions are shown in Fig. 4.36. The moving coil with a condenser in series is connected across the terminals of the incoming alternator. The fixed coils are connected across the bus-bars with a resistor in series having a small amount of inductance. This is adjusted so that when the two p.d.s are in phase, or exactly opposite in phase, the currents in the dynamometer coils are exactly ¼ period out of phase. The result is that the deflecting torque is zero and so the pointer rests in its zero position which is in the centre of the scale, *i.e.* behind the black rectangle (Fig. 4.35).

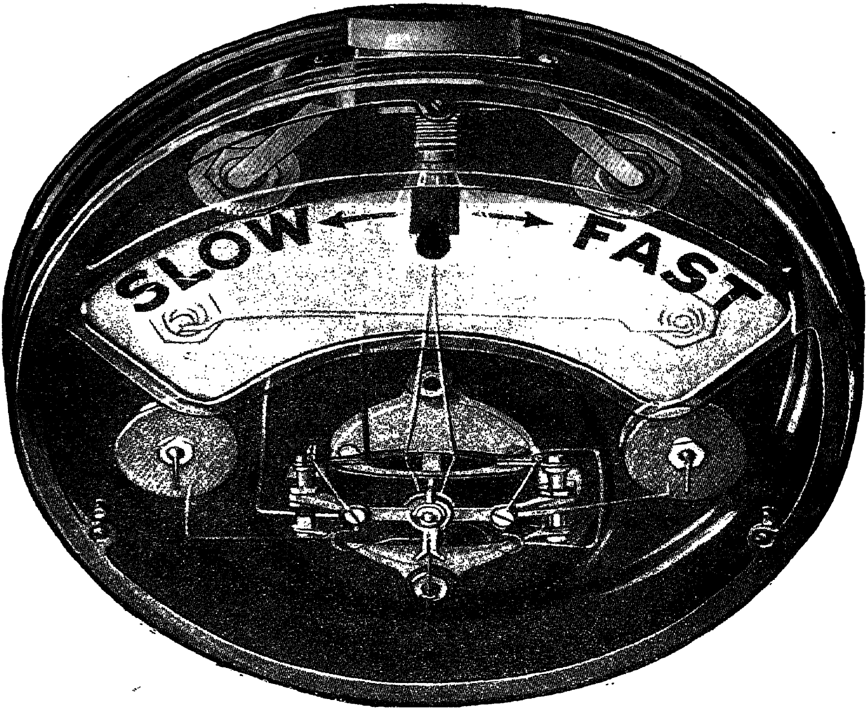


Fig. 4.35.—WESTON SYNCHROSCOPE (PHANTOM VIEW).

The lamp is connected for " bright lamp " synchronising, and so the pointer is seen at rest in the centre, only when the two p.d.s coincide in phase and frequency. If the frequencies are the same but there is a phase difference there will be a deflecting torque depending on the amount of the phase difference. The pointer will then come to rest at a place on the scale whose distance from the centre increases with the phase difference up to quarter of a period. The connexions are arranged so that this position is to the left of the centre when the alternator p.d. lags behind the bus-bar p.d., and to the right when the former leads.

When there is a difference between the frequencies the phase difference is continually changing, and so the position of the pointer changes in a corresponding way (see Art. 17). The action of the instrument is then explained by Fig. 4.37.

The first set of curves (*a*) consist of two alternating waves of the same amplitude but of different frequencies, a slower one, S, and a faster one, F. If these represent the bus-bar and alternator p.d.s the lamp p.d. is proportional to their *sum*, as shown by the broken line. For values of this lying within the shaded band the lamp gives no appreciable light, and this results in the lamp varying as shown in (*b*).

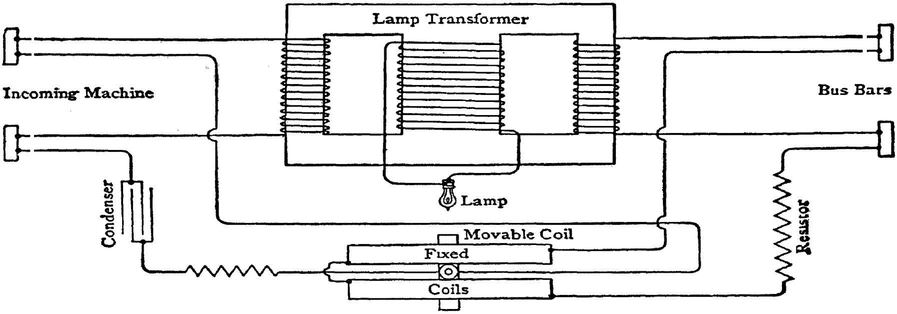


Fig. 4.36.—CONNEXIONS OF WESTON SYNCHROSCOPE.

In (*c*) is shown what occurs when the incoming alternator is running too fast, *i.e.* when F in (*a*) represents its p.d. and S in (*a*) represents the bus-bar p.d. The condenser causes the current in the moving coil to lead the alternator p.d. by 90°, as represented by F′ in (*c*); while the current in the fixed coils (S′ in *c*) is in phase with S. The deflecting torque depends on the product of these two currents at any instant, and is shown by curve T in (*c*).

The position of the pointer depends on the mean deflecting torque during a period, and is shown by the lines PPP in (*c*), positions above the line representing positions to the left of the centre of the scale. Thus the pointer is moving from left to right while the lamp is bright; and when the pointer is moving back to the left the lamp is dark and so the pointer is not seen. It appears, therefore, as if the pointer were continually rotating in the clockwise direction, *i.e.* in the direction of the arrow marked FAST in Fig. 4.35.

When the incoming alternator is running too slowly, S [in Fig. 4.37 (*a*)] represents its p.d., and S″ (in *d*) represents the current in the moving coil, leading the p.d. by 90°. The current in the fixed coils is F″ (in *d*) in phase with F (*a*), the bus-bar p.d. The torque thus becomes as shown by T′ in (*d*), and the mean torque and the pointer position become P′P′P′ in (*d*).

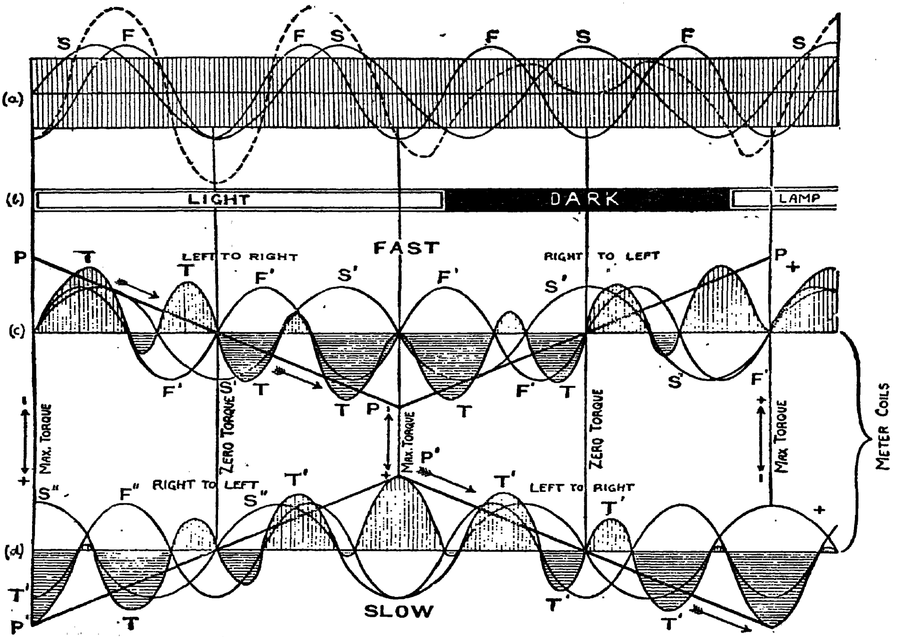
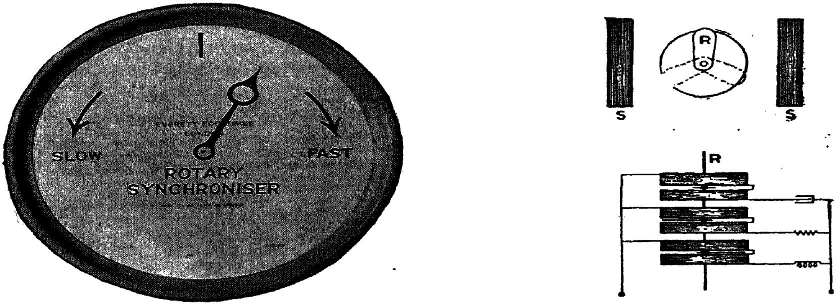


Fig. 4.37.—Action of Weston Synchroscope.

The pointer is now moving from right to left during the bright intervals, and so appears to rotate in the counterclockwise direction, *i.e.* in the direction of the arrow marked SLOW in Fig. 4.35.

In the Everett Edgecumbe synchroscope [Fig. 4.38 (*a*)] the pointer is in front of the scale and actually rotates. It is driven by a small rotor, R in Fig. 4.38 (*b*), with three iron vanes, placed 120° apart and at equal distances along a common axis. Each of these is magnetised by a pair of opposing coils. By connecting in series with the respective pairs a condenser, a resistance, and a reactance, the magnetising currents are given phase differences of 120°; consequently the resulting magnetism produced by the incoming alternator moves from vane to vane at a speed synchronous with that of the alternator. The coils S, S, are connected to the bus-bars, and so produce a field alternating in synchronism with the bus-bar voltage.

When the frequencies are equal the rotor remains stationary since the stator and rotor fields then coincide throughout the period.



(*a*) (*b*)

Fig. 4.38.—EVERETT EDGECUMBE'S ROTARY SYNCHROSCOPE.

If the alternator is running too slowly the rotor rotates in the direction of rotation of the rotor field at a rate proportional to the difference between the frequencies (cf. Chapter VI., Art. 2), thus keeping the fields coincident. On the other hand, when the alternator speed is too high the rotor revolves in the opposite direction to the field of the rotor, again keeping the fields coincident. The pointer is attached to the rotor spindle and so indicates whether the speed is too high or too low, and the extent of the difference.

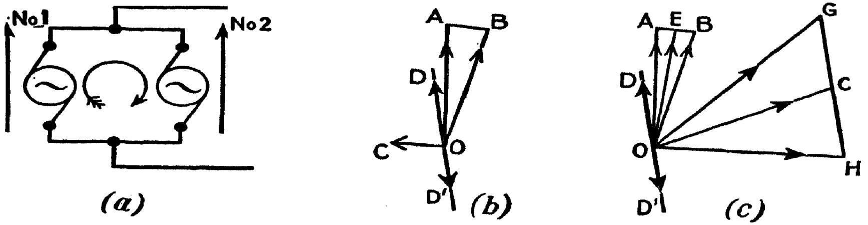
In addition a synchronising lamp may be mounted on top of the instrument. This " visual signal " can readily be seen by the engine driver.

The earlier pattern had a wound two-phase rotor supplied through slip-rings. The advantages of the present pattern are the absence of slip-rings and brushes, the robust construction, and the lowness of the power consumption which enables it to be operated by voltmeter transformers.

21. Synchronising Currents

When two alternators are running in parallel, anything tending to make them fall out of step is resisted by the changes thereby caused in their armature currents.

Consider first the case of two alternators in parallel [Fig. 4.39 (*a*)] with no external load. While their e.m.f.s are equal and in phase no current will flow. If from any cause they get slightly out of phase there will be a resultant voltage round the local circuit formed by the two armatures. *E.g.* if the e.m.f. of No. 1, represented by OA in Fig. 4.39 (*b*), leads that of No. 2, represented by OB, there will be a resultant e.m.f. in the local circuit equal to BA. This being the vector difference OA—OB acts in the positive direction in No. 1 and the negative direction in No. 2, as indicated by the arrows in Fig. 4.39 (*a*).



(*a*) (*b*) (*c*)

Fig. 4.39.—SYNCHRONISING CURRENTS.

This e.m.f. produces a current in the local circuit of magnitude BA/2Z, where Z is the impedance of each armature. As this impedance is largely reactance the current, OD, lags behind OC (equal and parallel to BA) by an angle approaching a right angle. The current in the positive direction in No. 2 is given by OD′, equal and opposite to OD.

Since OD is nearly in phase with OA, alternator No. 1 gives out power, the amount being equal to OA.OD.cos ∠ AOD. Similarly since OD′ is nearly opposite in phase to OB, alternator No. 2 absorbs power, the amount being equal to OB.OD.cos ∠ BOD. The difference between the two amounts represents power wasted in the resistances of the two armatures and of the connecting bus-bars.

The result is that No. 1 is slowed down and No. 2 is speeded up, both effects tending to bring the alternators back into phase equality.

If No. 2 gets ahead of No. 1 in phase the effects are reversed, and so again they tend to bring the machines back into phase equality.

If the alternators are on load the result of any phase difference is altered as illustrated in Fig. 4.39 (*c*). When the two are in phase, each giving an e.m.f. represented by OE, they each carry half the load current, say OF. If OA moves ahead of OE and OB lags behind it the e.m.f. in the local circuit formed by the two alternators is given by BA, and the circulating current by OD, as before.

The total current in No. 1 is, therefore, OG, the vector sum of OF and OD, and the current in No. 2 is OH, the vector sum of OF and OD′. The total load current is the vector sum of OG and OH, and this is equal to twice OF since FG and FH are equal and opposite: *i.e.* the total load current is unaltered but is no longer divided equally between the two alternators.

An inspection of the diagram shows that the current in No. 1 is increased and brought more nearly into phase with its voltage, *i.e.* the load on No. 1 is increased, tending to slow it down and so to bring OA back to OE. The current in No. 2 is decreased and its angle of lag is increased, *i.e.* the load on No. 2 is decreased, tending to speed it up and bring OB up to OE.

As the total load on either machine can be obtained by adding algebraically the amounts of power represented by the separate components of the current, it will be seen that the increase of load on No. 1 and the decrease of load on No. 2 are equal to the total amounts of power supplied by No. 1 and absorbed by No. 2 in the case of no external load with the same phase difference between OA and OB.

If the load were initially divided unequally between the two alternators the same results would follow; *i.e.* the load would be increased on the one whose e.m.f. leads and decreased on the one whose e.m.f. lags. The amount by which the loads change is very nearly proportional to the sine of half the angle of phase difference. The increase is slightly greater, and the decrease slightly less than the proportional value.

22. Phase-Swinging

As explained in the previous section, when two alternators in parallel get out of phase the leading one is slowed down and the lagging one speeded up. This action continues until they come into phase, and so by the time this occurs the originally lagging one is running at a higher speed than the other. Consequently it will proceed to run ahead in phase, and will therefore experience a retardation. This process will be repeated, each alternator in turn getting ahead in phase and then lagging. This oscillation which is known as **phase-swinging** or **hunting** will continue indefinitely except in so far as it is checked by the additional resistance losses and other actions caused by it (see Art. 23).

The effect may be made clearer by considering what happens to a single alternator running in parallel with such a number of others that their speed may be taken as practically unaffected by the behaviour of the single one. This is sometimes expressed as "running on infinite bus-bars."

Suppose that at a certain instant the single alternator is running at the correct speed but is ahead of its proper phase position. The load on it will be increased thereby, and it will slow down and so drop back in phase. This will continue with gradually diminishing load so that by the time the phase (and therefore the resisting torque) has come back to the normal value, as shown at A, Fig. 4.40, the speed will have dropped to some value such as shown at B. The phase then becomes a lag and the resisting torque drops below the normal, and so the speed begins to rise.

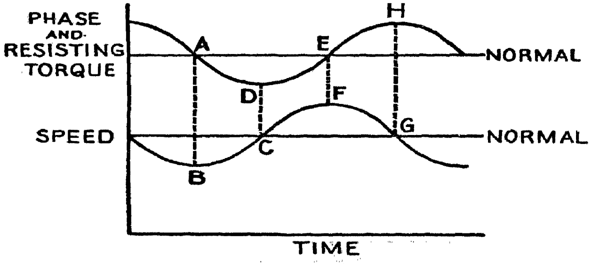


Fig. 4.40.- –Phase-Swinging.

As long as the speed is below normal the lag is increasing, so that when the speed has reached normal value again (at C) the phase and resisting torque are well below normal (at D). The opposite action then takes place, *i.e.* the phase returns to normal (at E) while the speed is rising to a maximum (at F). The phase, therefore, again becomes leading and the speed falls, and when it has returned to normal (at G) the phase lead has reached a maximum (at H).

Thereafter the same cycle of reactions follows except that the magnitude of the oscillations may be gradually reduced by various damping forces.

23. Damping Methods

When phase-swinging is occurring in an alternator the current changes in magnitude and in phase (see Art. 21). These changes, especially that of phase, alter the armature reaction and so cause fluctuations in the flux. These fluctuations set up eddy-currents in the poles, mainly in the pole-shoes.

By Lenz' Law (see Volume I., Appendix A, Art. 3), these eddy-currents, like any other induced currents, tend to oppose the change which induces them. But the change is that due to phase-swinging, so the effect of the eddy currents is to oppose this swinging in whichever direction it occurs. They act in fact like a viscous resistance to the phase-swinging, opposing to it a greater force the more rapidly it occurs. Thus the oscillations are damped down more or less rapidly according to the conditions.

When the pole-shoes are laminated eddy-currents in them are greatly reduced. Phase-swinging is, therefore, much more likely to occur in such cases. To reduce it **damping coils** may be fitted. These are merely copper rings of heavy section surrounding the poles. As long as the flux is constant no action occurs. But when the flux changes, as in phase-swinging, currents are induced in the damping coils. These currents tend to stop the phase-swinging in exactly the same way as eddy-currents in the pole-shoes.

A more complete cure for phase-swinging is afforded by an **amortisseur**. The name is the French for "damper," but is applied to a particular variety of damping coil. It is formed (see Fig. 4.41) by a number of copper bars placed in slots in the pole-shoes. All the bars in each pole-shoe are connected together at both ends by a strip of copper of large cross-section.

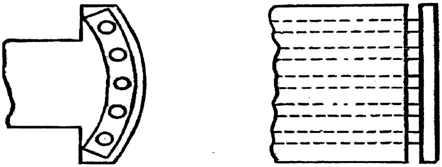

(*a*) SIDE VIEW. (*b*) END VIEW.

Fig. 4.41.—AMORTISSEUR.*

The damping grid thus formed acts in the same way as the simpler damping coils but is much more effective. The damping coil responds only to changes in the total flux. But the amortisseur, in addition to this action, has eddy-currents produced in it when the distribution of the flux is changed. Its damping action for the same amount of phase-swinging is, therefore, very much greater.

24. Adjustment of Load on Alternators

When a number of D.C. generators are running in parallel, the way in which the total load is shared between them can be altered by changing either their excitations or the settings of the governors of the engines driving them. The former is usually adopted as it is generally more convenient.

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* See further Fig. 9.28, p. 315.

The effect of increasing the excitation or one generator is to increase its e.m.f. and its current, and therefore to increase the power it supplies. It will therefore slow down slightly until the governor has altered its steam supply so as to bring the driving torque up to the increased resisting torque. It then settles down at this slightly reduced speed and supplies a greater share of the load than before. This can be continued as long as it is possible to increase the excitation, unless the drop in speed compensates for the increase in field strength.

With alternators, however, the speed is settled by the frequency so that the relative speed of all the alternators in parallel is fixed. Therefore no change of excitation alone can alter the load taken by a particular alternator, apart from a very small change due to an alteration in the copper losses.

If the excitation of an alternator is increased, the current it supplies is usually increased. But the power it supplies is not increased, owing to a reduction in the power-factor. This is because the speed is unchanged and therefore the mechanical power supplied is unaltered; consequently the electrical power given out cannot be any greater than before. In fact it will be slightly less owing to the increased current entailing a larger copper loss. The actions occurring depend upon small changes in the phase of the e.m.f. They will be considered more fully in connexion with synchronous motors in the next chapter.

Thus the adjustment of the load on an alternator can be effected only by altering the governor of the driving engine. This increases the mechanical power supplied without altering the speed, and so increases the electrical output. The necessary adjustment may be made either by hand or by means of a small motor controlled from the switchboard.

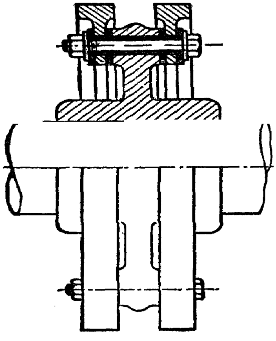


Fig. 4.42.—Slip-Rings (Upper Half in Section).

25. Some Constructional Details

The slip-rings by which the current is taken to and from the rotating magnet coils are most often of brass or gun-metal, but sometimes of cast-iron. A sleeve with three or more projecting arms is keyed to the shaft. The slip-rings are attached to these

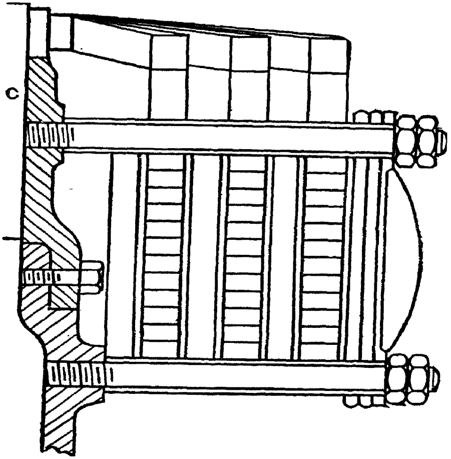


Fig. 4.43.—CLAMPING OF COIL-ENDS.

arms by means of bolts and nuts with insulating sleeves and washers. Insulated conductors are connected to the insides of the slip-rings and led along the shaft (to which they are attached) to the magnet coils. When large hollow shafts are used these conductors may be taken through the hollow. It is then possible to place the slip-rings outside the main bearings, and so reduce the distance between the latter.

One of the most important points in the construction of stators is the firm clamping of the ends of the coils. When a heavy overload or a short-circuit comes onto the alternator, very large forces are produced by magnetic action between neighbouring end-connexions. Unless these are held in place very strongly the coils will be bent. Even if an immediate breakdown does not occur, the damage to the insulation will cause breakdown ultimately.

In Fig. 4.43 is shown a method suitable for clamping the coil-ends of a large turbo-alternator. Owing to the small number of poles (2 or 4) a large number of coil-ends have to be accommodated at certain parts of the stator. In the example these are all bent away from the shaft and arranged in three layers. Sheets of insulation and spacing blocks are put between them, and clamps with strengthening ribs on top. Each clamp is held in place by nuts and lock-nuts on two set-screws. One series of set-screws is attached to the main frame. The other series is attached to the end-ring, which is bolted to the main frame, and presses against the stator-core (C, Fig. 4.43).

QUESTIONS ON CHAPTER IV

1. Draw a diagram of the winding of an alternator armature with 6 slots per pole, for three-phase supply, Y-connected,

(*a*) wound hemitropically; (*b*) wound non-hemitropically.

If one phase is connected wrong way round, what is the effect on the terminal voltages? How can the phase which requires reversing be identified?

2. If there are 6 poles, 3×10^6 lines per pole, 96 turns in series per phase, find the e.m.f. of the alternator in Question No. 1 at 1 000 r.p.m.

3. The flux per pole of a 6-pole magnet is 5 megalines. The armature has 36 slots, of which 24 are wound with 60 wires in each, all connected in series. The speed is 1 000 r.p.m.

Calculate (*a*) the frequency; (*b*) the breadth coefficient with a sinusoidal field; (*c*) the total e.m.f. generated.

4. If the above winding (Question No. 3) is used as two of the phases of a three-phase Y-connected alternator, with the third phase wound in the vacant slots, calculate:—

(*a*) the e.m.f. per phase; (*b*) the e.m.f. between terminals;

(*c*) the output with 50 amp. in the conductors;

(*d*) the output of the monophase alternator with the same current.

5. Find the root-mean-square line voltage of a six-pole star-connected three-phase alternator from the following data. Pole flux 3 megalines, 860 wires per winding (uniformly distributed), frequency 50.

6. Describe a winding for a high voltage three-phase armature with 3 slots per pole per phase.

Sketch the construction and fitting of the coils opposite to one pair of poles.

7. A three-phase winding is to be made by opening at three points a D.C. wave-winding with 33 slots, 2 bars per slot, both winding pitches being 17 conductors. Where should the openings be made, and which three conductors should be connected to form the star point?

8. If the above winding is opened at six points, where should the openings come, and how should the fresh connexions be made?

9. Show how the e.m.f. generated in an alternator depends on the angular breadth of the group of coils forming one phase. Develop an expression for the breadth coefficient assuming (*a*) that the field form is a pure sine wave; (*b*) that the field form contains third and fifth harmonics.

10. The no-load field form of a Y-connected three-phase generator has been determined experimentally. The armature slots are skewed a slot-pitch. Show how to determine the magnitudes of the harmonics, (*a*) in the phase e.m.f.; (*b*) in the e.m.f. between terminals.

11. If the flux-density in an alternator is uniform over $\frac{2}{3}$ of each pole-pitch and changes uniformly in the intervening spaces, obtain the resultant waveform (*a*) for two full-pitch coils per pair of poles $\frac{1}{6}$ of pole-pitch apart; (*b*) for three such coils per pair of poles, at the same distances as in (*a*).

If the flux were sinusoidal, what would be the "breadth coefficient" (or "distribution factor") of these two windings?

12. A three-phase star-connected generator has one slot per pole per phase and a flux that is evenly distributed under the pole faces, and is zero in the inter-polar gaps. In what ratio would the R.M.S. electromotive force of the alternator between terminals be changed, and what would be the alteration of its wave shape caused by changing the ratio of the pole-arc to the pole-pitch from $\frac{2}{3}$ to $\frac{1}{2}$, keeping the maximum flux-density unchanged?

13. If the above armature is replaced by one with the same total number of conductors, but placed in two slots per pole per phase, find for the first field-form the magnitude, relative to its original value, (*a*) of the R.M.S. e.m.f. per phase; (*b*) of the R.M.S. e.m.f. between terminals.

Comment on the results.

14. Show, by diagrams, the effect upon the magnetic circuit of an alternator of the current in the armature (*a*) when the current lags behind the terminal

voltage, (*b*) when it is in phase, and (*c*) when it leads in phase. What data would you require to find the exact effect upon the magnetic circuit ? [C. & G., II.

15. Show that the armature reaction of a single-phase alternator gives rise to a double frequency pulsation of magnetism across the pole face and in the yoke of a multi-polar machine, and examine the possible influence of this on the efficiency of the machine.

16. What two tests will enable the " regulation " of an alternator to be calculated for any load and power-factor ?

Why is the value different for different power-factors, and in what two ways can this be represented simply ?

17. An eight-pole alternator gives on open circuit 900 volts at 400 r.p.m. with an excitation of 17 amperes. The short circuit test indicates a current of 170 amperes with the same excitation. Find approximately the drop of voltage on a non-inductive load of 30 amperes and indicate how the drop will be affected if the load is inductive. [C. & G., A.

18. The following data relate to a 6 600-volt, 10 000 kVA., 50-cycle, 3-phase turbo-alternator:—

Voltage on open circuit	4 240;	5 450;	6 600;	7 300;	8 000;	9 000
Exciting ampere-turns	60 000;	80 000;	100 000;	120 000;	145 000;	220 000

Excitation needed to circulate full-load current on short-circuit: 117 000 ampere-turns. Inductive drop in stator winding at full load: 15 per cent.

Find the voltage regulation at full load, 0·8 power-factor. Explain any assumptions made in the method employed. [C. & G., A.

19. The no-load characteristic of a 6-pole three-phase generator driven at 1 000 r.p.m. is given by:—

Ampere-turns per pole	2 000	3 000	4 000	6 000	8 000	10 000
Volts at terminals	1 050	1 530	1 960	2 550	2 950	3 200

The number of armature turns per phase is 48, and the phases are Y-connected. Estimate the rise of pressure when a load of 360 amp. at 2 200 volts terminal p.d. with a power-factor of 0·7 is switched off suddenly, if this causes the speed to rise from 1 000 r.p.m. to 1 100 r.p.m. The armature resistance is 0·2 ohm, and its true reactance 1·2 ohms.

20. Describe the apparatus and method for synchronising two high voltage single-phase alternators.

What modification is made for three-phase alternators ?

21. What is meant by the synchronising current of two similar single-phase alternators running in parallel on the same load, but slightly out of phase ? Show by means of vector diagrams what happens to the armature currents, and in what way the load is divided between the two generators.

22. If each alternator in Question No. 21 has $R_a = 0{\cdot}8$ ohm, $Z_a = 4$ ohms, and $E = 1\,000$ volts, calculate and plot for phase differences from 0° to 10°:—

(*a*) the current, the load on the leading one, and the driving power developed in the lagging one, with no external load;

(*b*) the loads on each of them with an external load of 100 kVA. of power-factor 0·8.

23. Two similar alternators are running in parallel. Show that, if from any cause they get a little out of phase, their speeds tend to oscillate about the mean value. Explain the action of amortisseur coils, and state whether the frequency of the oscillations is affected by the use of such coils, or by the nature of the load on the alternators.

24. Describe a style of construction suitable for the revolving field magnet of an alternating current turbo-generator.

CHAPTER V

SYNCHRONOUS MOTORS

1. Synchronous Motors

Just as a D.C. generator may be used as a motor when supplied from D.C. mains, so an alternator may be used as a motor when supplied with A.C. power having the same number of phases as it produces when used as an alternator.

The speed of such a motor is settled by the frequency of the supply, being the same as if it were to be run as an alternator in parallel with those furnishing the supply. Its speed n in revs. per min. is given by:—$n = \frac{f \times 60}{p}$, where—

f = frequency of supply in cycles per sec.

p = number of pole-pairs of motor.

The name of this type of motor arises from the fact that it has to run at this speed, which keeps it in synchronism with the supply frequency.

It can be seen that, when running at this speed, if there is a torque in a certain direction at a particular instant, then half a period later the currents have reversed, and the S. poles have taken the places occupied previously by the N. poles, consequently the torque is in the same direction as before. This is repeated, and so there is, on the whole at least, a torque in a fixed direction.

If, however, the speed differs from synchronous, whether it is above or below this speed, the direction of the torque will reverse, more or less gradually according to the difference of the speed from synchronism. There will, therefore, be zero average torque, the torques in one direction being balanced by equal torques in the opposite direction lasting for an equal time.

The possibility of using an alternator as a synchronous motor is further shown by the synchronising currents which occur when two alternators in parallel get out of phase (Chapter IV., Art. 21). If they have no external load and the driving-power is gradually withdrawn from one and replaced by a resisting torque, it will continue to run in synchronism, unless this resisting torque becomes larger than the maximum torque which this machine can develop. The question as to what determines this maximum torque, and many other points in the behaviour of synchronous motors, can be settled by considering the vector diagram.

2. Vector Diagram of Synchronous Motor

Let OA in Fig. 5.01 represent the p.d. applied to the terminals of the motor, and let OB represent the e.m.f. generated by the motor. This e.m.f. will be nearly opposite in phase to the applied p.d., and the phase difference (*i.e.* the angle AOB) is called the angle of advance of the motor e.m.f.

The voltage available for sending current through the armature of the motor is given by the vector sum of OA and OB, viz. OC. This will be usually less than OA or OB, owing to their phases being nearly opposite. The current thereby produced in the armature will be equal to OC/(impedance of armature), and it will lag behind OC by the angle $\tan^{-1}(X_a/R_a)$, where X_a = reactance of armature, including effect of armature reaction, and R_a = resistance of armature. This current is represented by OD, and ∠ COD is its angle of lag. This will be nearly a right angle owing to the preponderance of reactance in the armature.

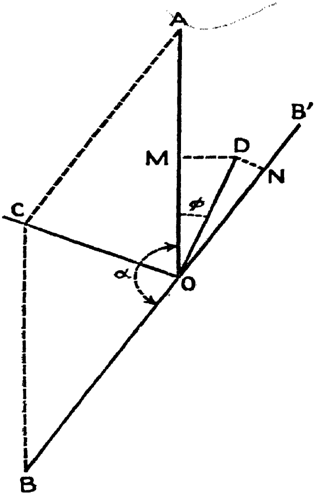


Fig. 5.01.—VECTOR DIAGRAM.

The power absorbed by the motor from the supply is given by OA . OD . cos ∠ AOD = VI cos ϕ. The angle AOD is the *external angle of lag* (or lead) of the motor, whereas ∠ COD is the *angle of lag of the armature considered as an impedance only*. The power converted into mechanical power by the motor is given by

$$\begin{aligned} & OB \,.\, OD \,.\cos(180^\circ - \angle BOD) \\ &= OB \,.\, OD \,.\cos B'OD \\ &= EI\cos(\pi - \alpha - \phi) = EI\cos\beta, \end{aligned}$$

where OB′ is BO produced;

α = angle of advance of e.m.f. = ∠ AOB

β = *internal angle of lead* of motor current = ∠ B′OD.

Note that if ϕ is a lead it must be reckoned negative, *i.e.* its value must be *added* to $\pi - \alpha$.

This mechanical power produced by the motor includes its frictional losses and the power absorbed by the iron losses. It is less than the power taken from the supply by the amount of the copper losses in the armature. In obtaining the "commercial" efficiency of the motor the power used in supplying the excitation must be taken into account in addition (see Example 1, Art. 5).

The amounts of power may be obtained semigraphically by drawing DM, DN perpendicular to OA and OB′ respectively. Then the power taken from the supply will be equal to OA.OM, and the mechanical power developed will be given by OB.ON.

3. Stability of Synchronous Motor

If a synchronous motor is running under a steady load, the whole of the vectors in the diagram shown in Fig. 5.01 will rotate at the same (synchronous) speed with no relative change. If the load is increased with fixed supply p.d. and frequency and constant excitation, changes will take place, and, under suitable conditions, the vectors will alter their phase differences and, in some cases, their lengths, and will then settle down to a new set of positions fixed relatively.

The first effect of an increase of load is to slow the motor temporarily, and so the p.d., OA, will gain on the e.m.f., OB, since the speed of the latter depends on that of the motor. If the motor is to continue to run, however, its steady speed must still be the synchronous speed, and so, after the change in the angle AOB has been effected, OA and OB must retain their new relative positions until the load changes again.

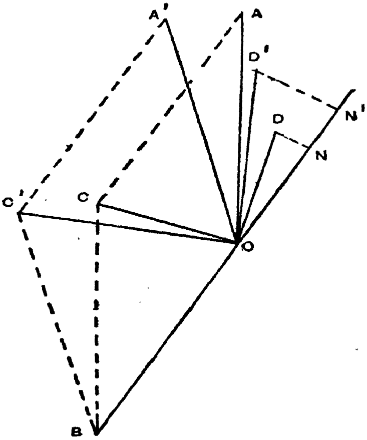


Fig. 5.02.—EFFECT OF CHANGE OF LOAD.

Since the p.d. is constant, and so is the e.m.f. at synchronous speed because the excitation is fixed, the lengths of OA and OB are fixed. If the position of OB is kept fixed in drawing the diagram, the effect of increase of load is to swing OA round in the counter-clockwise direction to some new position OA′ (see Fig. 5.02) so as to diminish α, the angle of advance of the e.m.f.

The resultant voltage will in consequence be changed from OC to OC′. Since CC′ is parallel to AA′, and OC roughly bisects $\angle$ AOB, the change will be chiefly in magnitude when α is nearly 180°, and the alteration of phase will be small. As α diminishes, however, the change of magnitude becomes relatively less, and the change of phase much greater.

The armature current is altered in the same ratio as the resultant voltage, and its angle of lag is constant ; *i.e.* it changes from OD to OD′, where $OD'/OD = OC'/OC$, and $\angle DOD' = \angle COC'$. The statements as to the magnitude and phase of the resultant voltage, therefore, apply equally to the armature current.

The mechanical power generated by the motor changes from

$$OB . OD \cos DOB' = OB . ON$$

$$\text{to } OB . OD' \cos D'OB' = OB . ON',$$

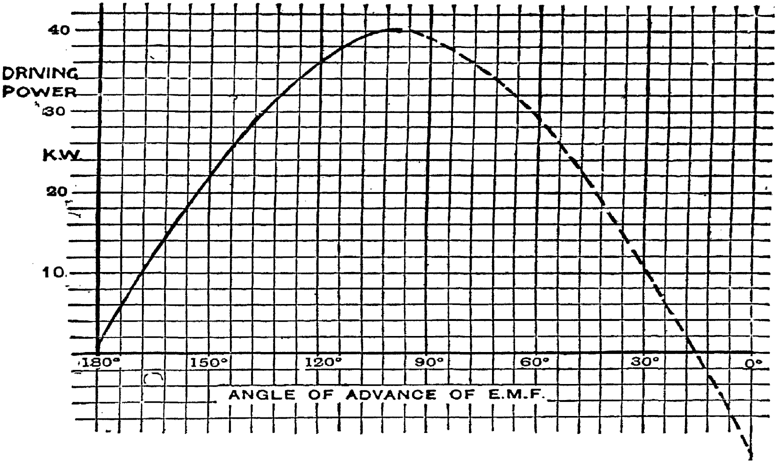


Fig. 5.03.—DRIVING POWER AND ANGLE OF ADVANCE.

where DN and D′N′ are perpendicular to OB′. Thus if ON′ is greater than ON the power developed by the motor has increased, and if this is equal to the increased load the motor will run at synchronous speed under this load. If the increase is insufficient the process will be continued until the driving power has been brought up to the value necessary for the new load.

But though the current continues to increase its internal angle of lead ($\angle B'OD$) increases at the same time. Ultimately a point is reached at which the two effects counterbalance each other and the driving power ceases to increase. Beyond this point any further diminution in ∠ AOB causes a *decrease* in the driving power. If the load is so great that this point is passed the motor will fall out of step and then stop.

This is further illustrated by Fig. 5.03 which is plotted with α diminishing, since $(180° - \alpha)$ is the lag of the e.m.f. from exact opposition to the applied p.d. The dotted portion to the right of the maximum ordinate represents unstable conditions under which the motor can be only momentarily. The left-hand portion represents stable running conditions, since an increase of load decreases the angle of advance and so increases the driving power, and vice versa.

The maximum ordinate occurs near to but not at 90°, and the value at 180° is small but not zero (see next Art.).

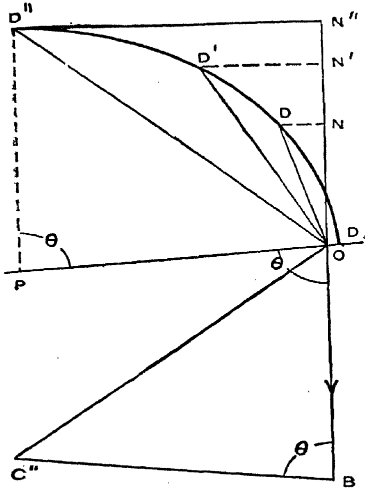


Fig. 5.04.—CURRENT ARC.

4. Locus of Current Vector

If the construction of Fig. 5.02 is repeated for a number of different positions of OA with OB fixed, it will be found that A, C, and D move on circular arcs.

It is evident that A moves on an arc with O as centre, and that C moves on a similar arc with B as centre and radius equal to OA.

Since OD is proportional to OC and the angle COD has the fixed value $\tan^{-1}(X_a/R_a) = \theta$, the locus of D is likewise a circular arc. Its centre, P, is found by drawing OP making an angle θ with OB in the lagging direction, and marking off OP so that its length measured on the scale of amperes is equal to (OB measured on scale of volts)/(impedance of armature).

In particular if the scales are so chosen that the length which represents 1 ampere is the same as that which represents Z_a volts (where Z_a = impedance of armature in ohms) the length of OP is equal to OB, and the various lengths of OD are equal to the corresponding lengths of OC.

This last arc is drawn in Fig. 5.04. As already stated (Art. 2) the driving power is proportional to ON, where N is the foot of the perpendicular from D onto BO produced. From the construction of the arc it is evident that the maximum value of ON occurs when D lies at the end of the radius from P parallel to BO, *i.e.* at D″ in the figure.

If OC″ is the corresponding position of the resultant voltage vector, $\angle D''OC'' = \theta = \angle POB$. Subtracting $\angle POC''$ from each gives $\angle D''OP = \angle C''OB$. Moreover, with the special relationship of the scales mentioned above, OP = OB, and OD″ = OC″. Therefore the triangles OBC″, OPD″ are equal in every respect and so $\angle OBC'' = \angle OPD'' = \theta$.

If other scales are used all the sides of the triangle OPD″ are altered in the same ratio, and so the two triangles are similar and $\angle OBC''$ remains equal to θ.

Now this angle $OBC'' = 180° - \alpha$ (cf. Fig. 5.01) α being the angle of advance of the motor e.m.f. Thus the maximum driving power under the given conditions occurs when $\alpha = 180° - \theta$. And since θ is generally nearly 90° but must be somewhat less, the value of α for maximum driving power is always greater than 90° but is usually close to this value.

5. Formulae for Current, etc.

From Fig. 5.01 the following relationships between various quantities in a synchronous motor can be deduced.

The angle OBC is, as noted above, equal to $180° - \alpha$, and will be denoted by γ. Then considering the triangle OBC,

$$V_R = OC = \sqrt{\{OB^2 + BC^2 - 2\,OB\,.BC \cos OBC\}};$$

$$\therefore\ V_R = \sqrt{\{E^2 + V^2 - 2EV \cos \gamma\}}$$

whence the current is found from $I = V_R/Z_\alpha$. Again—

$$(\sin OCB)/(\sin OBC) = OB/OC;$$

$$\therefore\ \angle AOC = \angle OCB = \sin^{-1}(E \sin \gamma/V_R).$$

Thence ϕ, the external angle of lag, can be obtained, since—

$$\phi = \angle AOD = \angle DOC - \angle AOC = \theta - \angle AOC.$$

If this is negative it shows that the current leads the applied p.d.

The input can be obtained since it is equal to $VI \cos \phi$ and all three quantities are known.

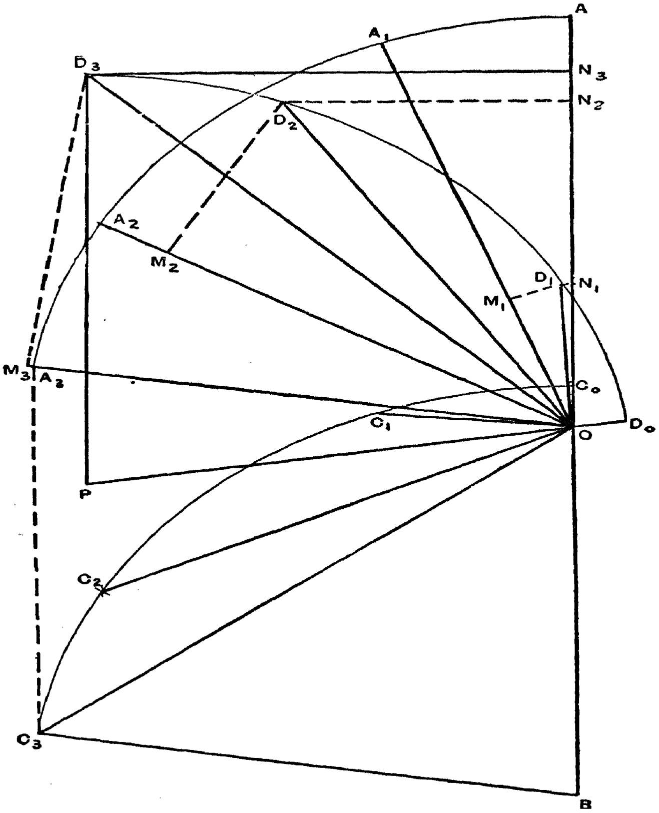
The internal angle of lag, $DOB' = \angle DOA - \angle B'OA = \phi - \gamma$.

Whence the output is obtained from $EI \cos \beta, = EI \cos (\phi - \gamma)$.

Note that if $(\phi - \gamma)$ is negative the current leads the reversed e.m.f.: and that when ϕ is a lead its value is to be taken as negative.

The input and output can be checked by calculating I^2R which is equal to (input − output).

Example 1. *A synchronous motor is running on a constant p.d. of 1 000 volts with an excitation giving an e.m.f. of 900 volts. Its armature has an equivalent impedance of 20 ohms and an equivalent resistance of 3 ohms. The iron, friction and excitation losses total 1·5 kilowatts.*



O D = Current; O M = Input; O N = Output.

Fig. 5.05.—GRAPHICAL METHOD WITH VARYING ANGLE OF ADVANCE.

Find values of the current, input, output, efficiency, and external and internal power-factors for various values of the angle of advance of the motor e.m.f., and plot the former against this last.

This example may be solved by mainly graphical methods (cf. Fig. 5.02), and Fig. 5.05 indicates the method for 4 positions. It is more accurate to use the formulae developed above, retaining the graphical method as a check and to show general tendencies.

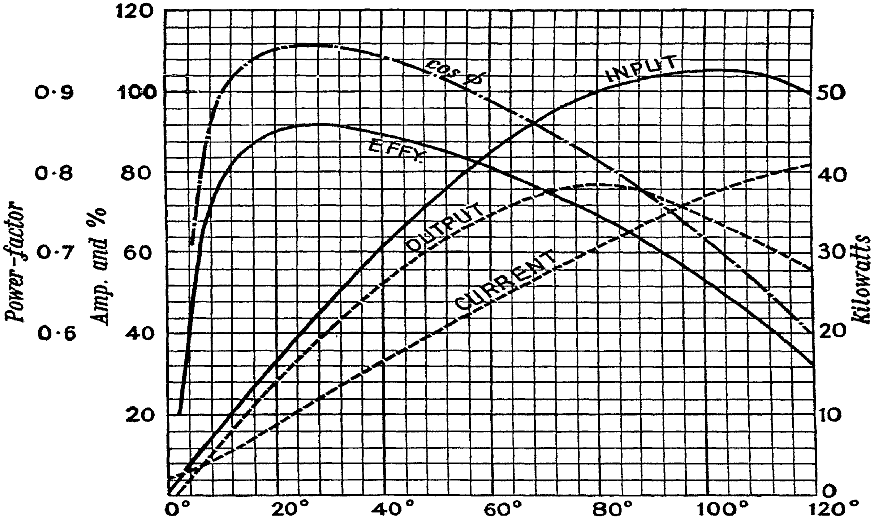
Inserting the given values in the formulae:—

$$\begin{aligned} V_R &= \sqrt{\{1\,000^2 + 900^2 - 2.1\,000.900 \cos \gamma\}} \\ &= 100 \sqrt{\{181 - 180 \cos \gamma\}} = 100 \sqrt{\{1 + 180 (1 - \cos \gamma)\}} \\ I &= V_R/20 = 5 \sqrt{\{181 - 180 \cos \gamma\}} \\ \angle\, DOC &= \theta = \cos^{-1} (3/20) = 81{\cdot}4^\circ \\ \sin AOC &= (900/V_R) \sin \gamma \\ \phi &= \theta - \angle\, AOC. \end{aligned}$$

The results of giving various values to γ are shown in the table on p. 142

6. Variation of Excitation

If the current in the magnet windings is varied the field strength will be varied correspondingly, though not in direct proportion. The relation between the two is given by the open-circuit characteristic (Chapter IV., Art. 13), whether the machine is used as an alternator or as a synchronous motor. The actual field strength is affected by armature reaction (see Art. 9), but this need not be



Supplement of angle of advance (i.e. γ).

Fig. 5.06.—EFFECTS OF VARYING ANGLE OF ADVANCE.

γ	cos γ	sin γ	V_R Volts	I amp.	sin AOC	∠AOC	Angle of Lag ϕ	cos ϕ	Input Kilo-watts	Internal Angle of Lag, B′OD (β)	cos β	Output Kilo-watts	Copper Losses, Kilo-watts	Net Output Kilo-watts	Efficiency Per Cent.
5°	·996	·087	131	6·6	·596	36·6°	44·8°	·710	4·69	39·8°	·768	4·56	0·13	3·06	65·2
10°	·985	·174	192	9·6	·816	54·7°	26·7°	·893	8·57	16·7°	·958	8·28	0·28	6·78	79·1
15°	·966	·259	266	13·3	·876	61·2°	20·2°	·938	12·5	5·2°	·996	11·9	0·53	10·4	83·2
20°	·940	·342	343	17·2	·897	63·8°	17·6°	·953	16·4	−2·4°	·999	15·5	0·89	14·0	85·4
30°	866	·500	501	25·1	·898	63·9°	17·5°	·954	23·9	−12·5°	·976	22·0	1·9	20·5	85·8
45°	707	·707	732	36·6	·869	60·3°	21·1°	·933	34·1	−23·9°	·914	30·1	4·0	28·6	83·9
60°	·500	·866	954	47·7	·815	54·6°	26·8°	·893	42·6	−33·2°	·837	35·9	6·8	34·4	80·8
90°	0	1·00	1345	67·3	·669	42·0°	39·4°	·773	52·0	−50·6°	·635	38·5	13·6	37·0	71·2
120°	− ·500	·866	1646	82·3	·474	28·3°	53·1°	·600	49·4	−66·9°	·393	29·1	20·3	27·6	55·8

Input = VI cos ϕ = 1 000 I cos ϕ watts = I cos ϕ kilowatts.

Output (driving power) = EI cos β = 900 I cos β watts = 0·9 I cos β kilowatts.

Copper losses = I^2R = 3 I^2 watts = ·003 I^2 kilowatts.

Checked by:—output + copper losses = input.

Net output = gross output − 1·5 kilowatts.

These results are plotted in Fig. 5.06.

N.B.—For cases in which E is greater than V the angle AOC is greater than 90° for small values of γ, and care must be exercised to use the correct value.

considered in drawing the vector diagram since the equivalent reactance of the armature takes this into account.

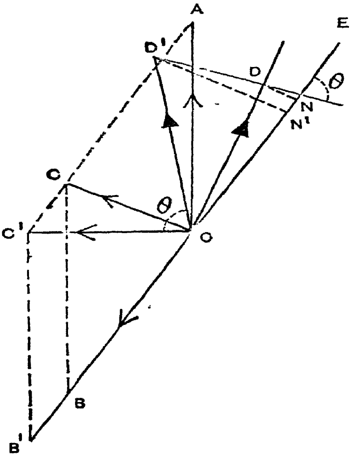


Fig. 5.07.—EFFECT OF VARYING EXCITATION.

Consider first the effect of increasing the excitation without altering the angle of advance of the e.m.f. If the increased excitation raises the e.m.f. from OB to OB′ (Fig. 5.07), the resultant voltage is thereby changed from OC to OC′, and the armature current from OD to OD′.

The changes of the current in phase and magnitude are exactly the same as the changes of the resultant voltage. When OB has a moderate value, OC is nearly perpendicular to it, and so increase of excitation affects chiefly the phase of OC (and so of OD), and increases the internal angle of lead ($\angle$ EOD). If the current originally lags behind the applied p.d. (OA), increasing the excitation will first diminish the lag and then cause the current to lead. Further increase of excitation continues to increase the lead though less and less rapidly. Hence the possibility of using synchronous motors as power-factor correctors (see Art. 12).

At the higher values of the excitation the increase in the magnitude of OC (and so of the current) becomes more important, *i.e.* the increase of current for a given increase in the e.m.f. becomes steadily larger. The minimum current under the above conditions occurs when $\angle$ BOC = 90°. If the excitation is reduced further the current again increases.

The output is changed from OB . ON to OB′ . ON′. Except for values of the angle of advance ($\angle$ AOB) close to 180°, this will be an increase. Since the triangles OCC′, ODD′ are similar, and C moves along a line parallel to OB, which is fixed, therefore D moves along a fixed line. This fixed line DD′ lags behind CC′ by θ ($= \tan^{-1} (X_a/R_a)$), *i.e.* the same as the lag of OD, OD′ behind OC, OC′. Thus DD′ is at the angle θ with BOE in Fig. 5.07, and the output changes in a smaller ratio than the e.m.f. when the excitation is altered, since ON′ is necessarily less than ON. The larger the value of the output the nearer the ratio is to that of the e.m.f., *i.e.* the percentage increase of output due to a given change of e.m.f. is greater for larger values of γ (up to $\gamma = \theta$).

Thus, if a number of curves similar to that of Fig. 5.03 (Art. 3) are obtained for different excitations and plotted, they will be as shown in Fig. 5.08. They will all have their maximum ordinates when $\alpha = 180^\circ - \theta$, *i.e.* when $\gamma = \theta$ (Art. 4). Thus OC in the figure is equal to θ, and CD is equal to $(90^\circ - \theta)$.

It may be noted here that the e.m.f. may exceed the applied p.d. (unlike the case of a D.C. motor), and the driving torque will still be positive except for values of α close to 180°, *i.e.* for small values of γ.

7. Variation of Current at Fixed Load

The curves drawn in Fig. 5.08 show that it is possible to obtain the same driving power with widely differing excitations, as illustrated by the horizontal 30 kW. line in that figure. It is of interest and importance to investigate how various quantities, particularly the armature current, vary when the excitation is varied and the load kept constant.

The question of what excitation gives the minimum current for a given load may be dealt with first, since this gives minimum copper loss and so nearly maximum efficiency, since the only other variable loss is the excitation loss.

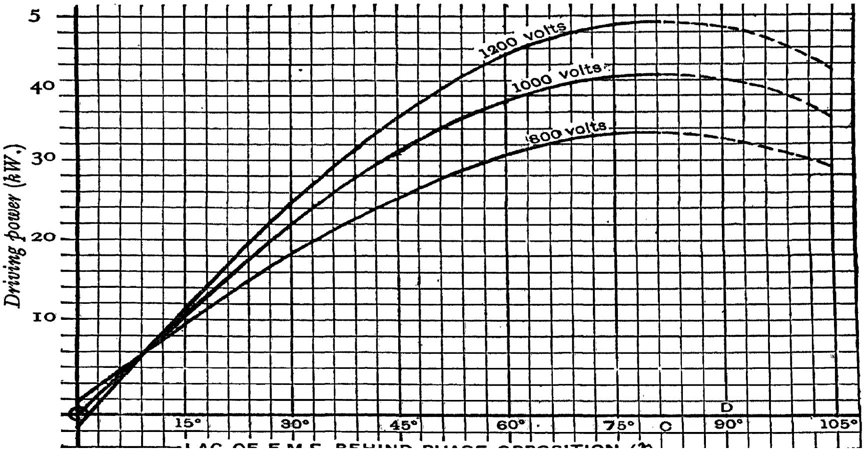


Fig. 5.08.—DRIVING POWER AT DIFFERENT EXCITATIONS.

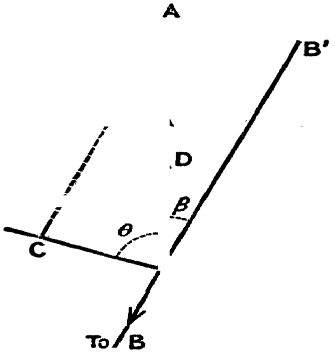


Fig. 5.09.—VECTOR DIAGRAM FOR MINIMUM CURRENT.

Since the output equals $(VI\cos\phi - I^2R)$, and the copper losses are in most cases a small percentage of the input, it is evident that the condition $\cos\phi = 1$ (*i.e.* $\phi = 0$) gives a very close approximation to the minimum current. Hence $VI - I^2R$ = output, settles the value of this current (see Example 2). Hence the resultant voltage can be found since it is equal to IZ_a, and leads the current by $\tan^{-1}(X_a/R_a)$, *i.e.* $\cos^{-1}(R_a/Z_a)$. The excitation can then be found by the vector diagram (Fig. 5.09) for this special case. This shows that

$$\text{E.m.f. of the motor} = OB = AC;$$

$$\therefore \text{ E.m.f.} = \sqrt{\{OA^2 + OC^2 - 2\,OA\,.\,OC\cos\theta\}},$$

$$\text{and } \sin\beta = \sin\gamma = OC\sin\theta/AC.$$

Example 2. *If a synchronous motor, supplied at 500 volts, has an armature of 10 ohms impedance and 1·5 ohms resistance, find the minimum currents for outputs of 6 kW. and 10 kW. respectively, and the corresponding values of the motor e.m.f. and angle of advance. (Friction, iron, and excitation losses are included in the above outputs.)*

(*a*) $6\,000 = 500I - 1{\cdot}5I^2;$

$$\therefore 500I = 6\,000 + 1{\cdot}5I^2; \quad \therefore I = 12 + 3I^2/1\,000.$$

Putting I = 12 amp. as first approximation, the right-hand side = 12·44.
Try I = 12·5 amp.; right-hand side = 12·47;

$$\therefore I = 12{\cdot}5 \text{ amp.}$$

$$\text{Hence } OC = 10 \times 12{\cdot}5 = 125 \text{ volts.}$$

$$\cos\theta = R_a/Z_a = 1{\cdot}5/10 = 0{\cdot}15;$$

$$\therefore E = \sqrt{\{500^2 + 125^2 - 2 \times 500 \times 125 \times 0{\cdot}15\}} = 497 \text{ volts;}$$

$$\therefore \sin\beta = 125\sin\theta/497 = 0{\cdot}252\sqrt{\{1 - (0{\cdot}15)^2\}} = 0{\cdot}252 \times 0{\cdot}922 = 0{\cdot}232;$$

$$\therefore \beta = 13{\cdot}4^\circ;$$

$$\therefore \text{Angle of advance} = 180^\circ - \beta = 166{\cdot}6^\circ.$$

(*b*) $10\,000 = 500I - 1{\cdot}5I^2;$

$$\therefore I = 20 + 3I^2/1\,000, \textit{ i.e. } I = 21{\cdot}4 \text{ amp.;}$$

$$\therefore OC = 214 \text{ volts;}$$

$$\therefore E = \sqrt{\{500^2 + 214^2 - 2 \times 500 \times 214 \times 0{\cdot}15\}} = 512 \text{ volts;}$$

$$\therefore \sin\beta = 214 \times 0{\cdot}922/512 = {\cdot}385;$$

$$\therefore \beta = 22{\cdot}7^\circ;$$

$$\therefore \text{Angle of advance} = 157{\cdot}3^\circ.$$

The formula and the above example show that the minimum current increases a little more rapidly than the output, and the resultant voltage (OC), therefore, does the same. An inspection of Fig. 5.09 will show that, as the output is increased, the e.m.f. for minimum current (viz. AC) increases (except when $\beta + \theta$ is less than 90°), at first slowly, and then more rapidly.

It is also known (see Art. 6) that for any given excitation there is a maximum possible load, and, therefore, for any particular load there is a minimum possible excitation. From these two considerations the general shape of the curves connecting current with excitation for a fixed load can be settled. Three such curves for different loads on the same motor with a constant applied p.d. are drawn in Fig. 5.10.

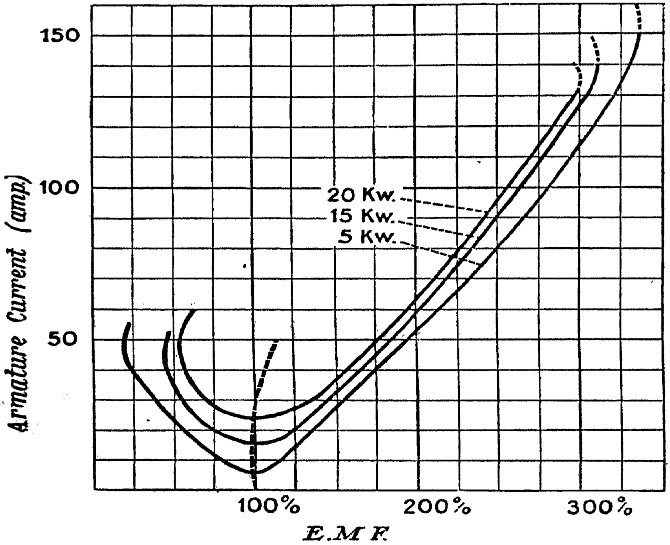


Fig. 5.10.—V-Curves.

These are known as the V-curves of the motor from their shape. The minimum points of the curves have been joined by a dotted line to show how the excitation for minimum current changes with load. It is advisable to work at an excitation somewhat above that which gives minimum current, as this procedure increases the margin of additional load which the motor can carry without falling out of step (cf. Fig. 5.08).

The method of obtaining these curves for a particular motor is shown in Art. 9.

8. The O-Curves

The mechanical power produced by a motor includes that necessary to overcome its own frictional and iron losses in addition to the power available for driving other machinery. This total mechanical power is equal to the power supplied less the I^2R losses.

Thus if the motor is producing zero mechanical power the I^2R losses are equal to the input. For a current in phase with the applied p.d. this gives $I_1{}^2R_a = VI_1$, i.e. $I_1 = V/R_a$, where R_a = resistance of the armature, or its equivalent resistance in the case of a polyphase motor. This current is equal to that which would flow if a *steady* p.d. equal to the R.M.S. value of the normal p.d. were applied to the stationary armature. The normal p.d. cannot produce so large a current, even with the armature at rest, owing to the reactance.

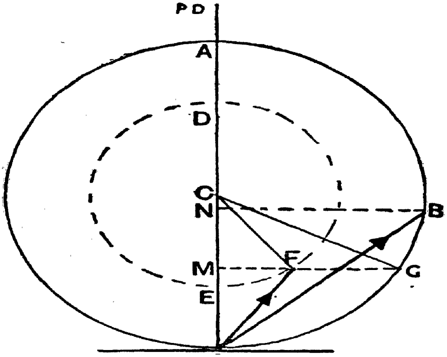


Fig. 5.11.—PROOF OF O-CURVES.

Let OA (Fig. 5.11) represent this current, I_1, in phase with the p.d. Draw a circle on OA as diameter and let B be any point on it. Then any current represented in phase and magnitude by OB gives zero mechanical power.

For, if BN is drawn perpendicular to OA, the power supplied $= VI \cos \phi = V.\text{OB} \cos \text{AOB} = V.\text{ON}$. The I^2R losses $= \text{OB}^2.\text{R}_a = \text{ON}.\text{OA}.\text{R}_a$.

And these two expressions are equal since $\text{OA} = I_1 = V/R_a$. Thus the circle on OA as diameter is the locus of the ends of all the current vectors giving zero mechanical power.

Further, if C is the centre of this circle, any smaller circle with C as centre is a similar locus for a constant value of the mechanical power.

For, if DEF (Fig. 5.11) is any circle, with centre C, cutting OA in D and E, and FM is drawn perpendicular to DE from any point on the circle, and MF is produced to meet the outer circle in G; then for a current represented by OF, the power supplied $= V.\mathrm{OM}$; the I²R losses $= \mathrm{OF}^2.R_a = \mathrm{OF}^2.V/\mathrm{OA}$; and the mechanical power being the difference between these, $= V(\mathrm{OM} - \mathrm{OF}^2/\mathrm{OA}) = V(\mathrm{OM}.\mathrm{OA} - \mathrm{OF}^2)/\mathrm{OA}$.

But $(\mathrm{OM}.\mathrm{OA} - \mathrm{OF}^2) = \mathrm{OG}^2 - \mathrm{OF}^2 = \mathrm{MG}^2 - \mathrm{MF}^2 = \mathrm{CG}^2 - \mathrm{CF}^2 = \mathrm{OC}^2 - \mathrm{CF}^2$, which is constant since CF is a radius of the circle DEF. The mechanical power developed with any current such as OF has a constant value which can be calculated for any particular circle DEF. Or the circle for a given power can be found from:—

$$\text{Radius} = \sqrt{\{\mathrm{OC}^2 - (\text{watts}/R_a)\}}.$$

If a number of such circles are drawn their mechanical powers increase as the circles become smaller. They are known as O-curves. The maximum mechanical power is developed when the circles have shrunk to the point C, *i.e.* with a current represented by OC. The expression for mechanical power then reduces to $V \times \frac{1}{2}\mathrm{OC}$: the power supplied is $V.\mathrm{OC}$, and so the "electrical" efficiency (see Volume I., Chapter XII., Art. 6) is then 50 per cent., and the true efficiency lower still. In most cases such a large current is well beyond the safe carrying capacity of the armature.

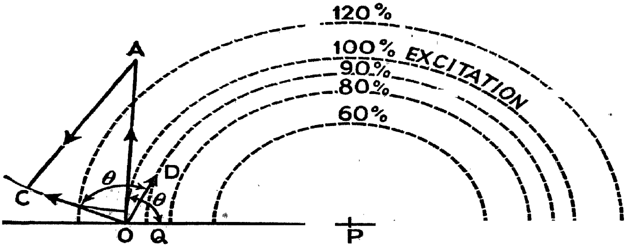


Fig. 5.12.—CURRENT ARCS.

9. Derivation of V-Curves

It can be shown exactly as in Art. 4 that if the p.d. vector OA is kept fixed then for constant excitation the ends (B, C, D) of the e.m.f. vector (OB), the resultant voltage vector (OC) and the current vector (OD) all lie on circular arcs.

Moreover, if a number of such arcs are drawn for different excitations those for the current are concentric with each other (see Fig. 5.12). In particular, if the scales of current and of pressure are so chosen that the length of the current vector is equal to that of the corresponding resultant voltage vector (OC), the common centre of the current arcs is at P; where OP = OA the applied voltage vector; and cos AOP $= R_a/Z_a$. The radius (PQ) of any particular current arc is equal to the constant excitation in volts for which it is drawn.

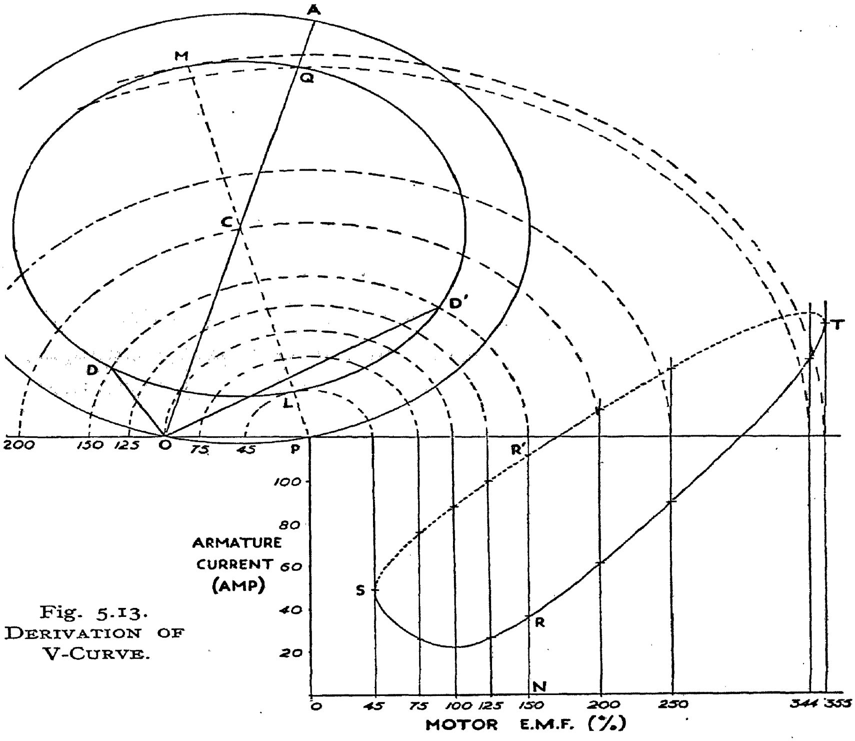


Fig. 5.13. DERIVATION OF V-CURVE.

By combining a series of such current arcs with an O-curve for constant mechanical power (Art. 8) a graph can be obtained connecting armature current with excitation (or more simply back e.m.f.). Such a curve is called a V-curve, from its shape when the power is small.

The method is illustrated in Fig. 5.13, the broken-line semicircles being the current arcs for various constant values of the back e.m.f. One O-curve (for 20 kilowatts) is shown by the thick line circle. It will be evident that there is a certain excitation below which this amount of power cannot be obtained, viz. that which makes the current arc just large enough to touch the O-curve. Similarly there is an excitation above which this power cannot be reached.

For all intermediate values of the excitation the current arc cuts the O-curve in two points (D, D′) and the lengths OD and OD′ can be plotted against the motor e.m.f. as shown at R, R′, where NR = OD, and NR′ = OD′.

By repeating this construction for a number of different possible excitations the closed curve SRTR′ is obtained. The lower portion (SRT) of this represents stable conditions and forms the V-curve.

By using other O-curves the corresponding V-curves can be obtained, and three of these are shown in Fig. 5.10.

The method of construction enables the following important values to be obtained for any particular load:—

(*a*) the minimum possible excitation, PL, 45% in the example;

(*b*) the maximum possible excitation, PM, 355% in the example;

(*c*) the excitation for minimum current, 95% in the example;

(*d*) the magnitudes of the currents for (*a*), (*b*), and (*c*), and the corresponding power-factors (unity for (*c*)).

10. Armature Reaction

Since the current in a synchronous motor is always opposed in phase to the e.m.f. the effects of the armature current on the field are the opposite of those in an alternator (see Chapter IV., Art. 11).

Thus if the current is exactly opposite in phase to the e.m.f. it will distort the field without altering its total strength, except indirectly owing to changes of permeability. But this distortion will consist of a strengthening of the leading pole tips and a weakening of the trailing ones, *i.e.* the field is distorted in the same direction as the magnet rotates. This corresponds with the distortion of the field of a D.C. motor in the direction opposite to the rotation of the armature (see Volume I., Chapter XI., Art. 3). It should be noted that the current will in this case be lagging in relation to the applied p.d. (see Fig. 5.01, Art. 2).

When the current leads on the above position by 90° the armature reaction will directly weaken the field. The lead relative to the applied p.d. will be (90° — γ), where γ as usual is the supplement of the angle of advance of the motor e.m.f.

This condition does not occur in practice, since it corresponds with no output, not even that required for its own frictional and similar losses. But it shows that when the current lies between this position and exact opposition to the e.m.f. the armature reaction will partly weaken the field, and partly distort it as described above.

Similarly a current lagging behind the position of exact opposition will partly strengthen the field in addition to distorting it. This requires the current to lag behind the applied p.d. by more than the angle γ. This will occur only at low values of the excitation. It is convenient to remember that small excitation gives lagging currents which tend to strengthen the field, while high excitation gives leading currents which tend to weaken the field.

The effect of armature reaction in synchronous motors may be dealt with by the same methods as have been described in connexion with alternators. Similarly the methods used in obtaining the V-curves, etc., may be applied with slight modifications to the case of an alternator running in parallel with others.

Phase-swinging again is liable to occur in synchronous motors when the load changes, and the same methods are used to check it as for alternators.

11. Starting Methods

A synchronous motor, since it gives no nett torque except at its correct speed, requires some means of bringing it up to this speed. The lamps and other apparatus for indicating synchronism are the same as used for alternators.

When a suitable D.C. supply is available, the motor may be brought up to speed by using its exciter as a motor during starting, supplying the excitation from the same D.C. supply. This supply may be from a battery which is recharged by the exciter, made slightly larger than is otherwise necessary. If the synchronous motor is used for driving a D.C. generator, this method is still more easy to apply.

A method which can be used in all cases, and so is frequently employed, is to bring the synchronous motor up to speed by means of a small induction motor (see Chapter VI.), called a **pony motor.** This, if directly coupled to the synchronous motor, should have one pole-pair less than the latter to enable synchronous speed to be reached.

A third method, largely adopted in the U.S. of America, is to open the field circuit and connect the armature to a supply at (say) one-half of the normal voltage. This reduced voltage can readily be obtained when the motor is supplied through a transformer, by providing additional tappings on this, and a throw-over switch. Alternatively for three-phase motors a star-delta switch may be used (see Chapter VI., Art. 3). Then, in the case of a polyphase motor, the eddy currents induced in the pole-shoes, and in the damping coils if these are used, causes the motor to run up nearly to synchronous speed in the same way as an induction motor starts.

With salient poles the hysteresis torque then raises the speed to synchronism, and the excitation can then be switched on. It is necessary to provide some means of indicating the polarity of the magnets when running, so as to avoid switching on excitation in the wrong direction. It is desirable, in addition, to be able to open the field circuit at several points to reduce the high pressure induced in it during starting by this method.

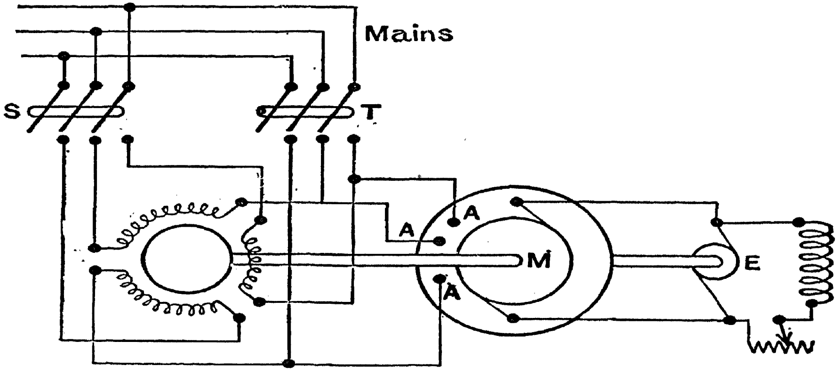


Fig. 5.14.—B.T.H. Co.'s STARTING METHOD.

A A A. Terminals of armature of main motor.
M. Magnets of main motor.
S. Starting switch.
E. Exciter.
R. Rotor of starting motor.
T. Running switch.

In the B.T.H. Co.'s method the stator of the pony motor is connected in series with the armature of the synchronous motor. This machine has its own exciter, and so, as the speed rises, the excitation supplied to the magnets (M, Fig. 5.14) of the synchronous motor is increased gradually. When the speed has risen nearly to synchronous value the motor pulls itself into synchronism. The main switch (T) is then closed, short-circuiting the pony motor's stator.

12. Self-Synchronising Motors

In order to reduce the complication of the apparatus required for starting, and the difficulties of synchronising, a number of motors have been devised which automatically bring themselves into synchronism. They all start up as induction motors, and so are more suitable for polyphase circuits than for monophase. Their differences in operation lie in the manner in which the change from induction motor action to synchronous motor action is effected.

They are built like induction motors (see Chapter VI.), with the addition of an exciter driven by the main motor. They start as induction motors with a starting resistance in the rotor circuit, which is gradually reduced as the speed rises (see Fig. 5.15). When the speed has risen nearly to synchronous value (say $1\frac{1}{2}$ per cent. below this), the rotor circuit is momentarily broken, and is then connected to the exciter by opening S_1 and closing S_2. The motor then pulls into synchronism unless the speed is too low. The excitation can then be adjusted to give the power-factor desired. This may be unity, or a leading power-factor down to 0·7 if power-factor correction is wanted on the system (see Art. 13).

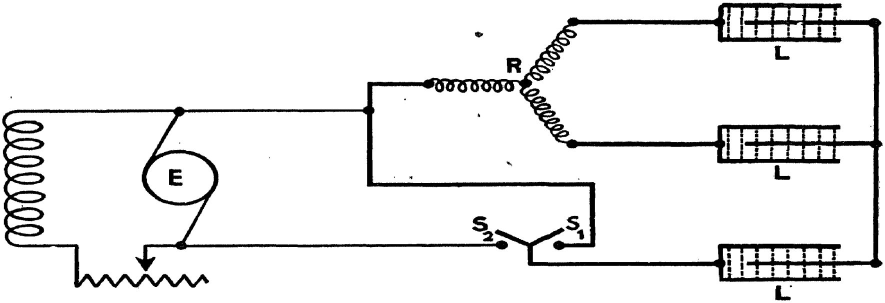


Fig. 5.15.—SELF-SYNCHRONISING MOTOR.

When the voltage induced in the rotor on starting is high (say over 1 000 volts), triple pole switches are used instead of the S.P. ones shown in the figure. The exciter is then entirely isolated during the start, and the starting resistance is disconnected during running.

Sometimes a double-wound rotor is used, one set of three-phase windings being used for starting, and the two sets in series when the D.C. excitation is switched on.

The advantages of this type of synchronous motor are that it can start against more than full-load torque, whereas a plain synchronous motor with dampers cannot overcome more than half this torque when coming into synchronism.*

13. Power-Factor Improvement

Since a power load usually consists chiefly of induction motors which take a lagging current, anything which takes a leading current will improve the power-factor of the whole system. The advantage of this is that the current for a given amount of power varies inversely as the power-factor. A reduction in the current reduces the sizes of the mains necessary for transmission or the losses therein or both. It likewise reduces the size of the alternators though not of their driving engines.

When synchronous motors are in use as well they may be made to take leading currents by raising their excitation above the value which gives minimum current. This increases the losses in the motors but will often reduce to a much greater extent the losses in the mains (see Example 3). If the synchronous motors are under the control of the supply authority this is easily arranged. If they are used by consumers the latter may be encouraged to run at leading power-factors by suitable tariffs and meters (see Chapter III., Arts. 17, 18).

If no synchronous motors are in use it may be worth while to instal such a motor and run it at a high excitation under no load. It then takes from the mains a current leading by a large angle and so has a great correcting effect on the power-factor. The power absorbed is merely that required to supply the motor's losses and these may be only a small fraction of the saving effected. Before deciding that its use is justified allowance must be made for interest and depreciation on its capital cost, and for the extra supervision necessitated.

It is important to note that the power-factor is improved only on the portion of the transmission system between the alternators and the synchronous motor: *e.g.* if the motor is installed in the generating station the gain is only in the reduction of size of the alternators and the mains are unaffected.

If the motor is installed at a transforming point it should be connected on the secondary side, both because its cost will be lower if wound for low pressure, and because in this way it will reduce the output required from the transformers. It will be still better if the motor can be installed at a point closer to the lagging low power-factor load.

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* See further "Induction type synchronous motors" (L. H. A. Carr), *Journ. I.E.E.*, Vol. 60, p. 165, and Vol. 61, p. 692.

Condensers may be used for the same purpose and a suitable type is made in units of about 1 microfarad capacity. They can be worked at a pressure of 600 volts. If the line pressure differs much from this it is advisable to use an autotransformer to bring the condenser pressure to 600 volts (or an exact multiple). The combined cost is less than that of condensers alone to take the same kVA., since the amount per condenser varies as the *square* of the p.d.

Condensers to give the desired total capacitance and working pressure are connected together and placed under oil in a tank. No attention is required, and the power wasted is only about 3 watts per kVA., apart from losses in the autotransformer when one is used.

The control is by oil switches with "no-volt" and overload releases, and with auxiliary contacts to short-circuit the condensers when the main circuit is opened.

Example 3. *If the load on a 500-volt system consists of a synchronous motor taking 50 kilowatts at unity power-factor, and induction motors taking a total of 150 kilowatts at an average power-factor of 0·6, find the power-factor of the whole load, and the values to which this rises when the synchronous motor is made to take leading currents with power-factors of* (*a*) 0·8, (*b*) 0·5.

If the armature resistance of the synchronous motor is 0·15 ohm, and the resistance of the transmission mains, etc., 0·07 ohm, find the changes in the losses.

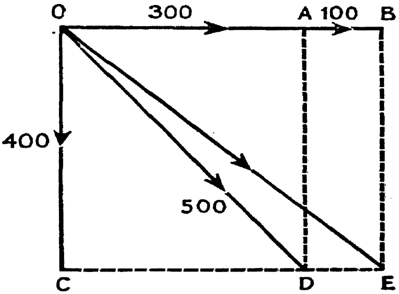


Fig. 5.16.—COMBINED LOADS.

Current taken by synchronous motor

$$= (50 \times 1\,000)/500 = 100 \text{ amp. (AB, Fig. 5.16).}$$

Current taken by induction motors

$$= (150 \times 1\,000)/(500 \times 0{\cdot}6) = 500 \text{ amp. (OD)}$$

$$\text{Power component} = 0{\cdot}6 \times 500 = 300 \text{ amp. (OA)}$$

$$\text{Reactive component} = \sqrt{\{500^2 - 300^2\}} = 400 \text{ amp. (OC);}$$

$$\therefore \text{ Total current} = \sqrt{\{(100 + 300)^2 + (400)^2\}} = 566 \text{ amp. (OE).}$$

$$\text{Hence power-factor of whole load} = 400/566 = {\cdot}71$$

$$\text{or alternatively:— power-factor} = \frac{(50 + 150) \times 1\,000}{500 \times 566} = {\cdot}71.$$

(*a*) Total current in synchronous motor

$$= 100/0{\cdot}8 = 125 \text{ amp. [OE in Fig. 5.17 (}a\text{)].}$$

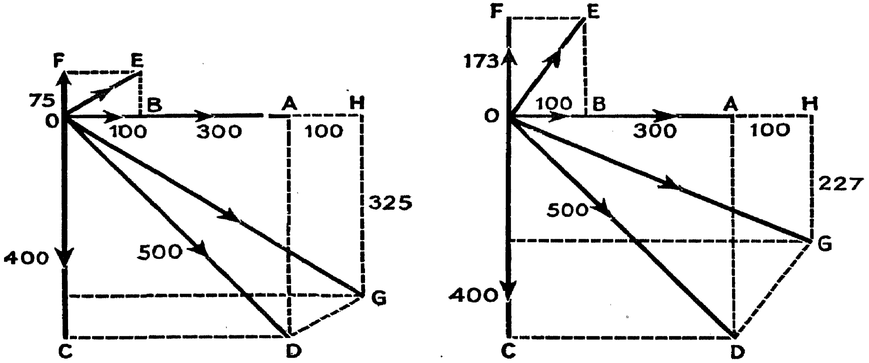
$$\text{Capacitance component} = \sqrt{\{125^2 - 100^2\}} = 75 \text{ amp. (OF)}$$

Hence total current in mains (OG)

$$= \text{vector sum of OD and OE} = \sqrt{\{(100 + 300)^2 + (400 - 75)^2\}} = 515 \text{ amp.}$$

Hence power-factor of whole load $= 400/515 = \cdot 78$
or power-factor $= (200 \times 1\,000)/(500 \times 515) = \cdot 78$.
Increase of loss in synchronous motor $= (125^2 - 100^2) \times 0{\cdot}15 = 844$ watts.
Decrease of loss in transmission line $= (566^2 - 515^2) \times 0{\cdot}07 = 3\,100$ watts.
Nett saving $= 3\,100 - 844 = 2\,260$ watts.

(*b*) Total current in synchronous motor
$= 100/0{\cdot}5 = 200$ amp. (OE in Fig. 5.17*b*).
Capacitance component $= \sqrt{\{200^2 - (100)^2\}} = 173$ amp. (OF);
$\therefore$ Total current in mains (OG) $= \sqrt{\{(100 + 300)^2 + (400 - 173)^2\}} = 460$ amp.
Hence power-factor $= 400/460 = 0{\cdot}87$.
Increase of loss in synchronous motor $= (200^2 - 100^2) \times 0{\cdot}15 = 4\,500$ watts.
Decrease of loss in mains $= (566^2 - 460^2) \times 0{\cdot}07 = 7\,610$ watts.
Nett saving $= 3\,110$ watts.



(*a*) cos $\phi = 0{\cdot}8$.

(*b*) cos $\phi = 0{\cdot}5$.

Fig. 5.17.—SYNCHRONOUS MOTOR TAKING LEADING CURRENT.

It will be noted that the nett saving in the above case with the lower leading power-factor is not much greater than with the higher one, owing to the greatly increased loss in the synchronous motor. This suggests finding what power-factor will give the greatest nett saving.

Let R = resistance of synchronous motor armature.
R' = resistance of transmission lines, etc.
I = power component of synchronous motor current.
I_L = leading component of synchronous motor current.
I_x = reactive component of current in rest of load.

Then, compared with the case of unity power-factor for the synchronous motor, the *additional* copper loss therein is

$$(I^2 + I_L^2)\,R - I^2R = I_L^2R \text{ watts.}$$

And the reduction of loss in the mains—

$$= \{I_x^2 - (I_x - I_L)^2\}R' \text{ watts} = (2I_x - I_L)\,I_LR' \text{ watts;}$$

$$\therefore \text{ nett reduction} = 2I_xI_LR' - I_L^2\,(R + R').$$

Differentiating this with respect to I_L and equating to zero gives that for maximum nett reduction of losses $I_L = I_x . R'/(R + R')$.

This is not quite accurate as it does not allow for the increased excitation and iron losses required to produce an increase in the leading current. Applying this formula to the case of Example 3 gives:—

$$I_L = 400 \times \cdot07/(\cdot15 + \cdot07) = 127 \text{ amperes,}$$

i.e. total current of synchronous motor $= \sqrt{\{100^2 + 127^2\}} = 162$ A.

and total line current $= \sqrt{\{400^2 + (400 - 127)^2\}} = 484$ A.;

$\therefore$ increase of loss in motor = 2 420 watts.

And reduction of loss in line = 5 980 watts;

$\therefore$ nett saving = 3 560 watts.

The synchronous motor's power-factor for this is equal to 100/162 = ·62 leading; and the total power-factor is 400/484 = ·83 lagging.

The above assumes that the synchronous motor can carry the increased current without overheating, under which conditions it enables the cheapest total running cost to be found.

14. Standing Costs and Power-Factor

As pointed out in Chapter III., Art. 17, a low power-factor increases the running costs to some extent and the standing costs to a much larger extent. In the case of a power-factor of 0·5 the cost of the conductors (but not the insulation) will be doubled, and that of the alternators and transformers will be nearly doubled. Though the steam side will be affected only slightly the total result will be an increase of 60 per cent. to 70 per cent.

If the power-factor improvement is effected by condensers the best value to which to raise it can be found in the following way:—

let P = combined cost in £ per kVA. of the line conductors, alternators and transformers;

Q = cost per kVAR. of condensers

and $a = Q/P$.

Let OA in Fig. 5.18 represent the kVA. of the load, with a power-factor $\cos \phi_0 = \cos \mathrm{VOA}$. Draw AN ⊥ to OV, and let AB represent the leading kVAR. taken by the condensers: and $\angle \mathrm{VOB} = \phi$. Then:—

Saving in cost of line

$$= P\,(\mathrm{OA} - \mathrm{OB})$$

$$= P\left(\frac{\mathrm{ON}}{\cos \phi_0} - \frac{\mathrm{ON}}{\cos \phi}\right) = P.\mathrm{ON}\left(\frac{1}{\cos \phi_0} - \frac{1}{\cos \phi}\right)$$

Cost of the condensers is

$$Q.\mathrm{AB} = Q\,(\mathrm{AN} - \mathrm{BN}) = Q.\mathrm{ON}\,(\tan \phi_0 - \tan \phi)$$

The nett saving in capital cost is the difference between these two, and equals

$$P.\mathrm{ON}\,\{(1/\cos \phi_0 - 1/\cos \phi) - a\,(\tan \phi_0 - \tan \phi)\}.$$

To find the maximum nett saving differentiate the expression inside { }, which gives:—

$$-\sin \phi/\cos^2 \phi + a/\cos^2 \phi,$$

which is zero for $\sin \phi = a$.

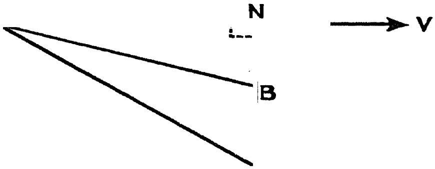


Fig. 5.18.

For instance, if condensers cost £2 per kVA., and the line conductors and associated apparatus £8 per kVA. the greatest nett saving is when sufficient condensers are used to make $\sin \phi = 0{\cdot}25$; this gives $\phi = 14\frac{1}{2}^\circ$, and so $\cos \phi$ is 0·968.

Note that the original power-factor makes no difference to this result, unless the original value is higher. The correctness of this can be seen by considering that if condensers are added in steps the question of whether it will pay to add more depends only on the power-factor reached, and not on the power-factor at the start.

If a synchronous motor is used for the same purpose similar considerations apply. But if a synchronous motor is used for driving purposes and power-factor improvement combined, two further points come in. Firstly, the total kVA. of the motor is the vector sum of the driving kVA. and the leading kVA., and so less than their arithmetical sum. Secondly, the total kVA. of the motor increases faster than its current. For the excitation has to be increased when it takes a leading current, and its size (kVA.) depends on e.m.f. × current, not on applied volts × current.

The above form of the rule is most suitable for use when the supply authority purchases the condenser or synchronous motor. It can be adapted to the case where the purchase is by a consumer charged on his kVA. demand plus a flat rate per kWh. All that is necessary is to make *a* the ratio between the annual total cost per kVA. for interest, depreciation, and value of space occupied by the condensers and the annual charge per kVA. of maximum demand.

QUESTIONS ON CHAPTER V

1. State the advantages and disadvantages of synchronous motors. What is the effect in such a motor of gradually increasing the load with a constant field current?

2. How is it that a synchronous motor working at constant potential difference can maintain a given torque when the current in its excitation coil is varied? [C. & G., II.

3. A synchronous motor on constant load and taking a leading current has its excitation gradually reduced. Show with the aid of vector diagrams the effects on the current and on the power-factor.

What are the advantages of running at an excitation higher than that which gives minimum current?

4. Draw the O-curves for the motor of Example 1, p. 140, (*a*) for zero driving power, (*b*) for 25 kW. driving power.

In case (*b*) what is the largest angle of lag of the current, and what is the current value then?

What is the minimum current and what excitation will give this current?

5. Draw the vector diagram of a synchronous motor. Explain how to find the locus of the end of the current vector with a fixed p.d. vector: (*a*) for any constant excitation, (*b*) for any constant load.

Sketch the graphs obtained in the above cases, and show how they can be combined to obtain the V-curves (*i.e.* current plotted against the e.m.f. of the motor on constant load).

6. Show how to derive graphically the relation between excitation, power-factor, and current, in a synchronous motor driving a direct current generator on constant load.

7. What is meant by the V-curve of a synchronous motor? A three-phase 60 h.p. motor, intended for a 500-volt 50 frequency circuit, gives when running as a generator with full excitation 150 amperes on the short circuit test. The armature is star coupled, and each phase has a resistance of 0·15 ohm. To what voltage must the motor be excited to give full output at unity power-factor? What will be the armature current in this case? Take the armature efficiency as 92 per cent. [C. & G., II.

8. The no-load characteristic of a monophase 4-pole 220 kVA. alternator when driven at 1 500 r.p.m. is given by:—

Field amperes	20	40	60	80	100	120	140	160
Volts	270	520	760	980	1 160	1 280	1 380	1 470

To produce its full-load current of 200 amp. on short-circuit requires 31 amp. field current.

Plot the current taken by this machine when run as a synchronous motor with a constant input of 150 kW. at 1 000 volts against the field current, when the latter is varied from 50 amp. to 110 amp.

Armature resistance = 0·5 ohm.

9. Describe the methods used for starting synchronous motors and the apparatus required.

10. Show exactly how you would calculate the capacitance of a condenser capable of adjusting the power-factor of an inductive circuit to unity value when the condenser is placed in parallel circuit.

Explain carefully why it is that an alternating current synchronous motor can be made to take a leading current. [C. & G., II.

11. A factory driven by three-phase induction motors has a total load of 600 kW. at a lagging power-factor of 0·85. A synchronous motor is to be installed taking an additional load of 150 kW. Calculate the value of the power-factor of the synchronous motor and the kVA. supplied to it, if it raises the power-factor of the whole installation to 0·95.

12. The average load on an installation of motors is 1 000 kW. with a lagging power-factor of 0·8. A synchronous motor is to be added taking 200 kW. Calculate the capacity of the motor in kVA. and its power-factor if the power-factor of the whole load is to be raised (*a*) to unity; (*b*) to 0·9. Which of these two is more economical ?

13. A consumer receives three-phase power under a line voltage of 2 000 at 50 frequency. The power-factor is 0·70. It is desired to bring this up to 0·85 by the use of condensers. If the condensers are arranged in mesh connexion, what capacitance will be required ? [C. & G., II.

14. If the load is 1 000 kW. at a lagging power-factor of 0·7; the cost of line and associated apparatus £9 per kVA., and of condensers £3 per kVAR.; calculate the separate and total costs for p.f.s of 0·7, 0·8, 0·9, unity, and the value which gives maximum nett saving. Plot all three costs against power-factor.

15. A load of 500 kW. at a lagging power-factor of 0·75 is taken by an industrial consumer. The tariff is £2 a year per kVA demand plus a flat rate per kWh. Phase advancing condensers cost £3 per kVA. and all charges associated with the condensers total 12 per cent. per annum of the capital cost. Calculate (*a*) the most economical power-factor at which the installation should be operated, (*b*) the annual saving in cost effected by improving the power-factor to this value. [C. & G., C.

16. An industrial load of 4 000 kW. has a bulk supply at 11 000 volts, the power-factor being 0·8. A synchronous motor is required to meet an additional load of 1 500 h.p. and at the same time to raise the resultant power-factor of the installation to 0·95 lagging. Find the kVA. rating of the motor and the power-factor at which it must be operated. Motor efficiency: 0·92.

Give a diagram to illustrate these conditions and an explanation of the control of power-factor by the synchronous machine. [C. & G., C.

CHAPTER VI

POLYPHASE INDUCTION MOTORS

1. Polyphase Induction Motors

These motors form the bulk of the A.C. motors in use. Their main advantages are as follows:—

(*a*) They start up from rest without needing any starting motor, and do not require to be synchronised. Their starting arrangements are very simple, especially in the short-circuited rotor type (see below).

(*b*) They are very robust, and seldom break down.

(*c*) When running, no brushes are needed, and so their frictional losses are low.

(*d*) Their efficiency is good.

(*e*) Their cost is low.

Their main disadvantages are:—

(*a*) The power-factor is rather low, especially at small loads.

(*b*) Their speed cannot be varied except by a corresponding sacrifice of efficiency.

(*c*) Their starting torque is inferior to that of D.C. shunt-wound motors.

(*d*) Their speed drops with increase of load, in a manner similar to that of a shunt-wound motor.

They consist of a **stator** constructed just like the stationary armature of an alternator: and a **rotor**, with a laminated iron core and insulated copper conductors. In the **short-circuited rotor** type, the rotor conductors form permanently closed circuits with no external connexions. In one pattern of this there is one bar in each slot, and these bars are all connected together at each end by a massive copper ring. This is known as a **squirrel-cage rotor.** Other patterns of short-circuited rotor are electrically equivalent to the squirrel-cage.

In the other type, called the **wound rotor,** the rotor conductors form a polyphase winding, usually three-phase, even when the supply is two-phase. The ends of the winding are brought to three or more **slip-rings** on which the **brushes** rest. By means of these the resistances of the rotor circuits can be increased by introducing external resistances. This is useful during starting, and for regulating the speed.

The **bearings** may be of the self-oiling type usual in D.C. dynamos (see Volume I., Chapter X., Art. 20), but often ball-bearings are preferred because they keep the air-gap more exactly to its original value. This is of great importance because narrow air-gaps (about ½ mm.) are employed generally in induction motors.

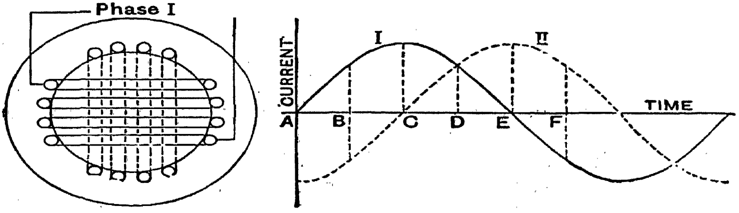
2. Production of Rotating Field in Two-Phase Motor

Consider a stator with a set of coils tending to produce a vertical field, and a second similar set tending to produce a horizontal field [see Fig. 6.01 (*a*)]. If the first set is connected to one phase of the supply, and the other to the second phase, a rotating field will be produced.

Let the currents flowing in the two sets of coils be as shown in Fig. 6.01 (*b*), viz. two sinusoidal curves with ¼-period phase difference. A positive current in I will produce a vertical m.m.f. in (say) an upward direction. Similarly, a positive current in II will produce an m.m.f. (say) from right to left. Denote the field due to the maximum current in either by 100.

Then at the instant denoted by A there is no current in I, and the current in II is at its *negative* maximum, and, therefore, produces a field of 100 from left to right [see Fig. 6.02 (*a*)]. One-eighth of a period later (B) the current in I is positive and at 71 per cent. of its maximum, while that in B has fallen to an equal negative value. The two together produce a resultant field of value 100 inclined at 45° to the horizontal [Fig. 6.02 (*b*)].

Another one-eighth period later (at C) the current in I has reached its maximum, and that in II fallen to zero. The field is, therefore, 100 vertically upwards [Fig. 6.02 (*c*)]. A further one-eighth period later (at D) both currents are at 71 per cent. of maximum value, as at B, but *both* are now positive. The resultant field is, therefore, as shown in Fig. 6.02 (*d*).



(*a*) Stator Coils. (*b*) Current-Time Graphs.

Fig. 6.01.—Two-Phase Stator.

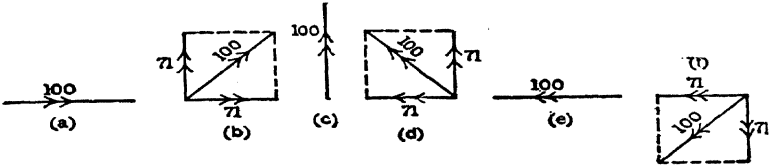


Fig. 6.02.—RESULTANT FIELDS OF TWO-PHASE STATOR.

When the instant E is reached the current in I is zero, and that in II at its maximum as at A, but the latter is now positive, so that the field is from right to left, exactly opposite to its original position. The field has thus made half a turn, retaining its magnitude throughout.

During the next half period a similar series of changes occurs, but with the currents, and therefore the resultant fields, reversed: *e.g.* at F the field is opposite to its position at B [see Fig. 6.02 (*f*), etc. Moreover, if intermediate instants are taken, the resultant field will be found to be still 100, and occupying intermediate positions corresponding to the instants chosen.

Therefore, a two-phase stator, when supplied with two-phase currents varying sinusoidally, produces a field of *constant magnitude rotating once per period.*

If the stator winding had more than one pair of poles per phase, the rate of rotation would be diminished inversely, so that $n = (f \times 60)/p$, where n = r.p.m. of field, f = frequency, and p = number of pole-pairs per phase. For this general case, therefore the last part of the above statement may be changed to " rotating at synchronous speed."

3. Production of Rotating Field in Three-Phase Motor

In a similar way a rotating field is produced if three-phase currents [Fig. 6.03 (*a*)] are supplied to a stator provided with 3 similar windings [Fig. 6.03 (*b*)] I, II, and III, 120° apart which exert m.m.f.s in the three directions which are respectively at right angles to the coils.

Starting with the instant A when there is no current in I, negative current in II and positive current in III,

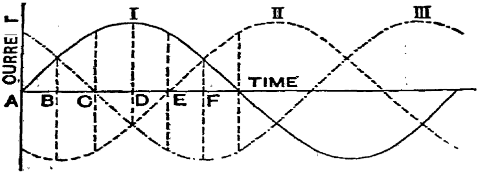


Fig. 6.03.—THREE-PHASE STATOR.
(*a*) CURRENT-TIME GRAPHS.

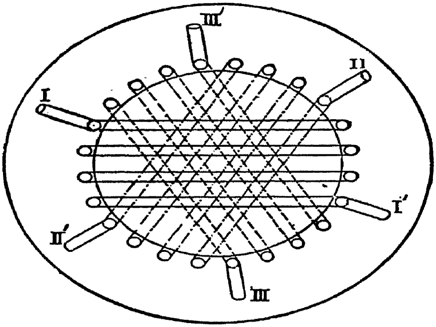


Fig. 6.03.—THREE-PHASE STATOR. (*b*) STATOR COILS.

both of 87 per cent. of maximum value, the resultant field bisects the angle between the magnetic axes of phase III and of phase II reversed, *i.e.* it is horizontal and from right to left. Its magnitude is $2 \times 87 \times \cos 30^\circ = 150$ [see Fig. 6.04 (*a*)].

One-twelfth of a period later (B) the current in II has reached its negative maximum, and that in III has fallen to 50 per cent., while that in I has risen to an equal value. The resultant of these last two m.m.f.s is 50 in line with that of II [Fig. 6.04 (*b*)], and so the total resultant is again 150, but at 30° with the horizontal.

Another one-twelfth of period later (C) the state of affairs is similar to that at A, but it is now III which carries no current and I which has 87 per cent. positive. Consequently, the resultant of 150 now bisects the angle between the magnetic axes of coil

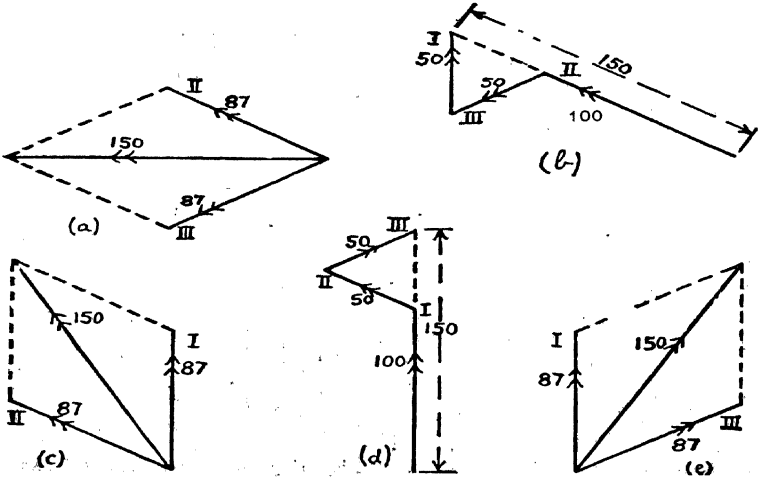


Fig. 6.04.—RESULTANT FIELDS OF THREE-PHASE STATOR.

I and of coil II reversed [Fig. 6.04 (*c*)], and so makes 60° with horizontal.

Similarly, at D, one-twelfth period after C, the diagram [Fig. 6.04 (*d*)] is like that for B, but I is at its maximum, and the resultant of 150 is vertical. And proceeding by one-twelfth periods the m.m.f. diagrams will be alternately of the two types, giving in every case a resultant of 150 in a direction changing by 30° for each one-twelfth period.

The final result is, therefore, the same as for the two-phase stator; viz. a field of constant magnitude is produced which rotates once in every period in a two-pole stator, and at synchronous speed for any number of poles.

4. Effect of Rotating Field on Rotor

When a rotating field has been produced by a two-phase or by a three-phase stator, it will produce a torque on a short-circuited rotor in the following way.

Suppose the field is rotating anticlockwise (Fig. 6.05), with a velocity v, measured at the circle on which the centres of the rotor conductors lie, and that the velocity of the latter is v' in the same direction. Then e.m.f.s are induced in the conductors proportional to $(v - v')$, and to the strength of the field at various points, and to the axial length of the conductors, *i.e.* $E = Bl\,(v - v')$.

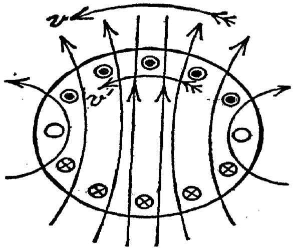


Fig. 6.05.—ROTOR IN ROTATING FIELD.

By applying the Right-hand Rule, as in the case of alternators (Chapter IV., Art. 4), it will be found that when the field is upwards, as in Fig. 6.05, the e.m.f.s are upwards in the conductors in the upper half, and downwards in those in lower half, as marked in the figure.

Since the rotor circuits are closed, currents will flow in them more or less corresponding with the e.m.f.s. The reaction between these currents and the field will produce forces on these conductors. By applying the Left-hand Rule it will be found that the forces on the upper conductors are to the left, and those on the lower ones to the right, *i.e.* the forces all tend to make the rotor rotate in the anticlockwise direction. Thus it is proved that the *rotor is pulled round in the same direction as the field rotates.*

This driving force will act as long as the rotor speed (v') is less than the field speed (v). But when the two speeds are equal the e.m.f. becomes zero, and, therefore, the rotor currents and the torque vanish. Thus the speed of an induction motor is always below synchronism by a variable amount, hence they are sometimes called " asynchronous."

This result can be proved also from Lenz' Law, since the cause of the induced e.m.f. is the relative rotation of the field and rotor, and, therefore, the induced currents must tend to prevent this relative rotation, *i.e.* to make the rotor rotate at the same speed as the field.

5. Modified Vector Diagram

The field at a fixed point of the stator varies sinusoidally if the magnetising stator currents do so, and if variations of permeability are negligible. Hence it may be represented by a rotating vector in the usual way.

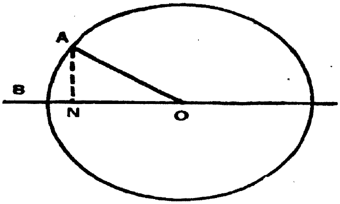


Fig. 6.06.—VECTOR REPRESENTATION OF FIELD.

But the field, as a whole, rotates at synchronous speed without change of magnitude, and so may be represented by a vector rotating once per period. In a two-pole stator this vector will be directed at every instant from the centre to the point of maximum field. In a multipolar likewise the vector will represent the position of maximum field, the circumference swept out by the end of the vector corresponding to a movement of twice the pole-pitch on the stator.

These two methods can be combined as follows. The length of the vector, OA, in Fig. 6.06, represents the maximum flux-density, and it rotates once per period. The direction in which it points indicates the position of this maximum flux-density at the instant for which the diagram is drawn.

The flux-density at any other point, *e.g.* B, at the same instant, is obtained by projecting OA on OB. If B is kept fixed while OA rotates, the length of the projection ON varies sinusoidally with time.

Thus the same vector is used to represent (*a*) the field, which is of constant magnitude with respect to time, but varies in direction; and (*b*) the field at a fixed point of the stator which varies with time.

This dual representation depends on the assumption that the flux-density at any instant at different points round the stator varies sinusoidally in space.

It will be found that the same method may be used for other quantities in an induction motor.

6. Reaction of Rotor Currents on Stator

The currents in the rotor exert a m.m.f. which would greatly affect the field if the stator currents retained their original values. But, just as in the case of the transformer (see Volume I., Chapter XVII., Art. 3), any change in the rotor current is nearly balanced by an opposite change in the stator current.

The reason is the same as for the transformer, viz. that, neglecting stator resistance, the field has a strength proportional to the applied p.d., and so is constant if the latter is kept constant. This is in order that the e.m.f. induced in the stator may balance the applied p.d.

Hence the stator currents must balance the m.m.f. of the rotor currents, and provide a further m.m.f. to produce the necessary field, and also supply the power lost in the iron: *i.e.* the relation between the two currents is the same as between the primary and secondary currents of a transformer. The stator current is built up of the three components (*a*) true magnetising current, (*b*) iron-loss current, (*c*) current to balance the rotor m.m.f.

The first two components, (*a*) and (*b*), together form the no-load current, or full (synchronous) speed current. This is very nearly the same as the current actually taken by a motor when running light, but differs slightly from it, as the motor has then to overcome its own frictional losses, and so the rotor has to carry some current to produce a small torque for this purpose.

7. Vector Diagram

As will be seen later (Art. 8), the rotor current under running conditions lags by only a small angle behind the e.m.f., hence the vector diagram (drawn as described in Art. 5) is as shown in Fig. 6.07, neglecting stator impedance. In this OA represents the flux and also, to a suitable scale, the e.m.f. in the rotor. The rotor current is, therefore, given by OB, lagging behind OA by a small angle.

A reference to Fig. 6.05 shows that for anticlockwise rotation of the flux the positive direction of current is up through the plane of diagram. Moreover, to produce an upward flux, the magnetising current must flow upwards on the left, and downwards on the right. Hence the vector (OC) representing the magnetising current is drawn to the left at right angles to OA.

The e.m.f. induced in the stator is equal to that in the rotor multiplied by the ratio (No. of turns in stator)/(No. of turns in rotor). It, too, can be represented by OA if the scale for stator volts, compared with that for rotor volts, is altered inversely as the numbers of turns (as in transformer diagrams, see Volume I., Chapter XVII., Art. 3). The applied p.d., neglecting stator resistance, is equal and opposite to this, *i.e.* it is given by OD.

Consequently the iron-loss current is given by OF, in phase with OD. The no-load current is the vector sum of OC and OF, *i.e.* OG, and this lags behind the applied p.d. by an angle approaching 90°.

The component (OH) of the stator current to balance the m.m.f. of the rotor is opposite in phase to OB. If the method used in transformer diagrams is followed, and the current scales made proportional to the respective numbers of turns in the stator and rotor, OH will be equal in length to OB. It is the ampere-turns which actually balance.

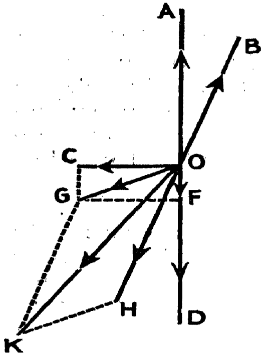


Fig. 6.07.—VECTOR DIAGRAM.

The total stator current is the vector sum of OH, OF, and OC, *i.e.* OH and OG, and so is given by OK. It will be seen that this lags behind the applied p.d. (OD) by a larger angle than the rotor current (OB) lags behind the rotor e.m.f. (OH). Moreover that the extra amount of lag diminishes as the load increases. The whole diagram, in fact, is similar to the corresponding transformer diagram.

If the stator resistance is taken into account, the applied p.d. will have an additional component in phase with the current (OK). This will reduce somewhat the lag of the current behind the applied p.d. It also increases the magnitude of the p.d. The stator leakage reactance likewise must be taken into account for accuracy, and so the current lag may be increased instead of being diminished. But the magnitude of the applied p.d. relative to the rotor e.m.f. is further increased by this leakage (see further Art. 24).

8. Rotor Current

The e.m.f. produced in each rotor conductor is given by:—

$$\text{e.m.f.} = \frac{\mathbf{B}l\,(v - v')}{10^8} \text{ volts,}$$

where $\mathbf{B}$ = flux density (radial) in lines per cm.^{2}.
l = axial length of rotor conductors in cm.
v = velocity of field in cm. per sec.
v' = velocity of rotor in cm. per sec.

The frequency of the currents in the rotor is $f(v - v')/v$, where f is the supply frequency. This is because for a complete cycle in a rotor in a 2-pole field the field must make one more revolution than the rotor; f = rev. per *sec.* of field; $f.v'/v$ = rev. per sec. of rotor; hence the above expression gives the number of times per sec. that the field gains a revolution on the rotor. The same proof applies to a multipolar stator if "twice the pole-pitch" is substituted for "revolution" throughout.

Thus the rotor frequency falls as its speed rises, and the reactance of the conductors falls in proportion to the frequency. If X_0 is the value of the reactance when the rotor is at rest, its value in general is $X_0(v - v')/v$.

The fraction $(v - v')/v$ may be denoted by s and is called the **fractional slip.** The amount by which the rotor speed falls short of the synchronous speed is called the **slip,** and this may be expressed in cm. per sec., i.e. $(v - v')$, or in any other unit of linear velocity, or in revolutions per minute (or per sec.). Dividing the slip by the synchronous speed, both being expressed in the same units, gives the fractional slip. If this is multiplied by 100 the *percentage slip* is obtained.

Thus the rotor reactance is sX_0 and therefore the rotor impedance is $\sqrt{\{R_2^2 + (sX_0)^2\}}$ where R_2 is the rotor resistance.

The maximum e.m.f., if $\mathbf{B}_m$ is maximum radial flux density, is $\mathbf{B}_m l(v - v')/10^8$, which may be written $\mathbf{B}_m lvs/10^8$. Hence the rotor current $(I_2) = \cdot 707\mathbf{B}_m lv.s/[10^8.\sqrt{\{R_2^2 + s^2X_0^2\}}]$ amperes (R.M.S.).

If the supply frequency is constant, v is fixed in a particular motor; and if in addition the supply p.d. is constant $\mathbf{B}_m$ is constant apart from the small effect of stator resistance. Hence I_2 is proportional to $s/\sqrt{\{R_2^2 + s^2X_0^2\}}$. As the slip increases both numerator and denominator of this expression increase, but the former increases in a greater ratio. The increase of the denominator, however, becomes nearer to that of the numerator as s becomes larger, since the effect of R_2 is then relatively less.

Thus with increasing slip the rotor current always increases. Starting with zero slip, *i.e.* at synchronous speed the current increases nearly in proportion to the slip. For larger values of the slip the rate of increase of the current becomes steadily less, but it never ceases to increase (see Example 1 and Fig. 6.09).

The angle of lag of the rotor current is given by $\tan \phi_2 = sX_0/R_2$, *i.e.* $\tan \phi_2$ is directly proportional to the slip. The angle (ϕ_2) itself therefore increases at first in direct proportion to the slip, but less rapidly than this for large values of the slip: it must be of course always less than 90°.

9. Torque

The force on each conductor of the rotor is $\mathbf{B}li_2/10$ dynes, and its contribution to the torque is $\mathbf{B}li_2r_2/10$ dyne-cm.; where r_2 is the radius of the rotor in cm. and i_2 is the current in the particular conductor under consideration. The total torque is obtained by adding up the various terms of the above form.

As shown by Fig. 6.07 the maximum flux-density (OA) and the maximum rotor current (OB) do not occur at the same point, and the same applies to the zero points of the two. Thus the reversals of direction of the flux and of the current flow occur at different points (Fig. 6.08) and so over certain sections of the rotor the torques will oppose the rotation. These must be subtracted from the positive torques in obtaining the nett torque.

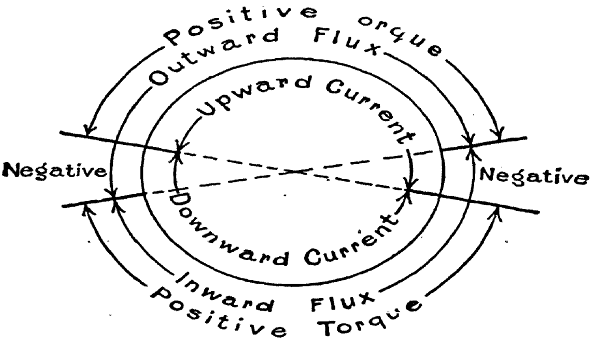


Fig. 6.08.—FLUX, CURRENT AND TORQUE RELATIONS.

If the flux-density and current-flow are both distributed sinusoidally the problem of finding the nett torque is exactly analogous to that of finding the power supplied by a current out of phase with the voltage (Volume I., Chapter V., Art. 9). Thus

$$\text{Mean torque} = (\text{R.M.S. value of } \mathbf{B}) . l . I_2 . N_2 r_2 \cos \phi_2/10 \text{ dyne-cm.,}$$
$$= \cdot 707 \mathbf{B}_m l . I_2 . N_2 r_2 \cos \phi_2/10 \text{ dyne-cm.,}$$

where N_2 = number of rotor conductors.

$$\text{From Art. 8; } I_2 = \cdot707\,\mathbf{B}_m lv\,.\,s/[10^8\,.\,Z_2],$$

where Z_2 = rotor impedance at slip s.

$$\text{Again } \cos\phi_2 = R_2/Z_2;$$

$$\therefore\ T = \tfrac{1}{2}\mathbf{B}_m{}^2 l^2 vsN_2r_2R_2/[10^9\,.\,Z_2{}^2] \text{ dyne-cm.},$$

since $\cdot707 = 1/\sqrt{2}$ and so $(\cdot707)^2 = \frac{1}{2}$.

$$\text{Now } Z_2{}^2 = R_2{}^2 + (sX_0)^2;$$

$$\therefore\ T = \tfrac{1}{2}\mathbf{B}_m{}^2 l^2 vN_2r_2\,.\,sR_2/[10^9\,(R_2{}^2 + s^2X_0{}^2)] \text{ dyne-cm.}$$

Hence for constant supply frequency and p.d. in a given motor:—
T is proportional to $sR_2/(R_2{}^2 + s^2X_0{}^2)$.

Example 1. *Calculate and plot the variations of rotor current and of torque with speed in an induction motor; for the rotor of which:*

resistance = 8×10^{-4}; *stationary reactance* = 3×10^{-3}.

By the above formula the torque is proportional to

$$s \times 8 \times 10^{-4}/(64 \times 10^{-8} + s^2 \times 9 \times 10^{-6}) = s \times 8 \times 10^4/(64 + 900s^2).$$

By the formula of Art. 8 the current is proportional to

$$s/\sqrt{(64 \times 10^{-8} + s^2 \times 9 \times 10^{-6})} = s \times 10^4/\sqrt{(64 + 900s^2)}.$$

For convenience of calculation these may be written:—

$$T = 1\,000s/(64 + 900s^2),$$

and $$I = 100s/\sqrt{(64 + 900s^2)},$$

the units in which T and I are measured being undetermined.

s	$100s^2$	$900s^2$	$64 + 900s^2$	$\sqrt{(64+900s^2)}$	I	T
·05	0·25	2·25	66·25	8·14	0·614	0·755
·1	1	9	73	8·52	1·17	1·37
·2	4·0	36	100	10·00	2·00	2·00
·25	6·25	56·25	120·25	10·96	2·28	2·07
·3	9	81	145	12·04	2·49	2·07
·4	16	144	208	14·42	2·77	1·92
·5	25	225	289	17·00	2·94	1·73
·6	36	324	388	19·70	3·05	1·546
·7	49	441	505	22·47	3·12	1·387
·8	64	576	640	25·30	3·16	1·250
·9	81	729	793	28·16	3·20	1·135
1·0	100	900	964	31·05	3·22	1·037

These results are plotted in Fig. 6.09. Note that as the slip increases the torque increases at first nearly in proportion, and then more slowly: it reaches a maximum, and then diminishes.

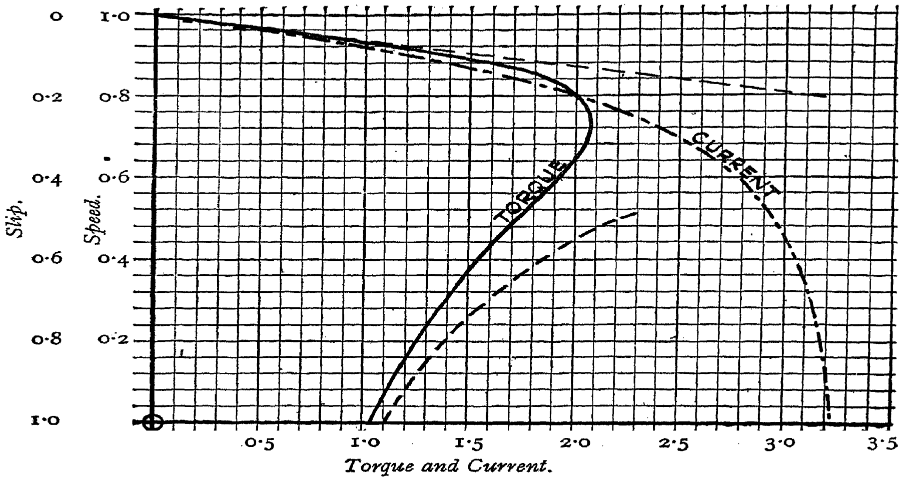


Fig. 6.06.—RELATION OF SPEED TO CURRENT AND TORQUE.

10. Maximum Torque

A number of deductions may be made from the relation $T = c.sR_2/(R_2^2 + s^2X_0^2)$, where c is a constant.

When s is small the second term in the denominator is very small, therefore $T\ (= csR_2/R_2^2) = cs/R_2$ approximately. Thus the torque is directly proportional to the slip, as is shown by the upper broken line (straight) in Fig. 6.09. Similarly when s is large the second term becomes relatively very large, and so $T\ (= csR_2/s^2X_0^2) = cR_2/sX_0^2$ approximately. Thus the torque varies inversely as the slip, and this relation is shown by the rectangular hyperbola drawn as a broken line in the lower part of Fig. 6.09.

These two approximate graphs give a fair idea of the general shape of the torque-slip curve; and if the maximum torque and the slip at which it occurs are determined in addition the curve is fixed fairly closely.

By differentiating the expression for T with respect to s it will be found that the value of the slip to produce maximum torque is equal to R_2/X_0. This can be proved in another way by writing $T = cR_2/(R_2^2/s + sX_0^2)$. The product of the two terms in the denominator is independent of s, and therefore their sum is a minimum, and so the torque a maximum, when the two are equal (cf. Volume I., Chapter XVII., Art. 11), which gives the above value of slip for maximum torque. *E.g.* in Example 1, the maximum torque occurs when $s = 8 \times 10^{-4}/(3 \times 10^{-3}) = \cdot 267$.

Substituting the above value of slip in the expression for the torque gives:— $T_m = (c \, . R_2{}^2/X_0)/(R_2{}^2 + R_2{}^2) = c/2X_0$, where T_m is the maximum value of the torque.

In Example 1 the value of c was taken equal to 1/80. Hence $T_m = (1/80)/(2 \times 3 \times 10^{-3}) = 10_3/(2 \times 3 \times 80) = 2 \cdot 083$ in the unit chosen.

It may be noted further that at the slip which gives the maximum torque the straight line which gives the torque-slip relation approximately for small slips gives twice the correct maximum torque. This results from the fact that the neglected term in the denominator is equal to the term used. Hence this straight line gives the maximum torque at half the correct slip. This supplies a convenient way of drawing this line.

11. Effects of Variation of Rotor Resistance

The expression for the maximum torque shows that this is independent of the rotor resistance. But the slip at which T_m occurs ($= R_2/X_0$) is directly proportional to the rotor resistance. Thus the effect of increasing the resistance of the rotor is to lower the speed at which the maximum torque occurs without altering the magnitude of this torque (see S, T, V in Fig. 6.10).

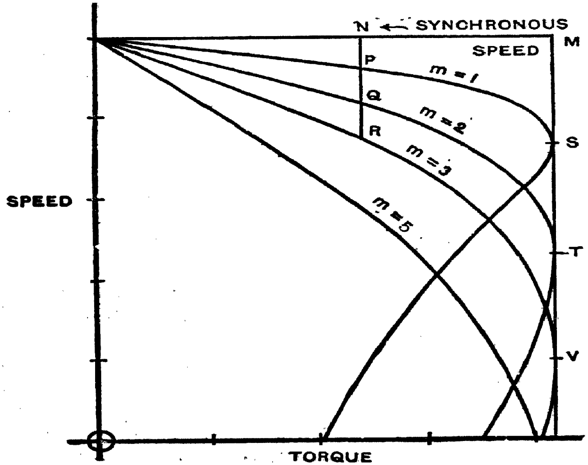


Fig. 6.10.—EFFECT OF INCREASING ROTOR RESISTANCE.

An examination of the torque-speed curve (Fig. 6.09) shows that lowering the speed for maximum torque increases the torque at the start and at low speeds, but decreases the torque when the speed is close to synchronism. Hence the advantage of the slip-ring type in which the resistance of the rotor circuits can be varied (see further Art. 29).

The effect of changing the rotor resistance can be seen more exactly by noting that in the expression $T = csR_2/(R_2^2 + s^2X_0^2)$, if R_2 and s are both increased in the same ratio (say m times) then the numerator and both terms of the denominator are increased m^2 times, and so the value of the torque is unaltered. In other words if R_2 is altered to mR_2 then any value of the torque (T_1) originally obtained with a slip s_1 is obtained at a slip ms_1 after the change of resistance.

Thus from any point P on the original torque-speed curve (see Fig. 6.10) a point Q on the torque-speed curve for doubled rotor resistance can be obtained by drawing a vertical line through P and making NQ = 2NP, N being on the line of synchronous speed, *i.e.* zero slip. Similarly a point R can be obtained on the torque-speed curve for trebled resistance by making NR = 3NP. And for any other change of resistance a point on the new torque-speed curve can be found in the same way.

By repeating the construction for a number of points the new curve can be obtained. The new points of maximum torque (T, V) are found by making TM = 2SM, and VM = 3SM. If the curve has been calculated the points found can be transferred readily to the new graph (see Example 2).

It is further evident that to obtain the maximum starting torque, R_2 must be increased till it becomes equal to X_0, for then the maximum torque occurs when $s = R_2/X_0 = 1$, *i.e.* when the speed is zero.

By increasing the resistance still further the starting torque is again reduced, but as the starting current is reduced in a greater ratio this is often done. This is shown by the curve marked $m = 5$ in Fig. 6.10, and a numerical case is given in Example 3.

Example 2. (*a*) *Draw the speed-torque curve for the rotor of Example* 1 (*Art.* 9) *when a resistance equal to that of the rotor is placed in series with it.*

(*b*) *Find the additional resistance required to give maximum torque at start, and draw the torque-speed curve when this resistance is in use.*

(*a*) R_2 is doubled, hence the torques calculated in Example 1 occur for double the values of slip in that case. This gives:—

$s =$	0·1	0·2	0·4	0·5	0·6	0·8	1·0
$T =$	·755	1·37	2·00	2·07	2·07	1·92	1·73

Moreover the maximum torque of 2·083 (see Art. 10) occurs when

$$s = 2 \times 8 \times 10^{-4}/(3 \times 10^{-3}) = \cdot 533.$$

These results are plotted in Fig. 6.11, the original curve being given for comparison.

(*b*) For maximum starting torque $R_2 = X_0 = 3 \times 10^{-3}$, *i.e.* R_2 is increased in the ratio $3 \times 10^{-3}/(8 \times 10^{-4}) = 3{\cdot}75$ times.

The *additional* resistance is 2·75 times the original, *i.e.* $2{\cdot}2 \times 10^{-3}$.

Using the results of Example 1 gives the following:—

$s =$	·1875	·375	·75	·9375	1·0
$T =$	·755	1·37	2·00	2·07	2·083

To enable the curve to be plotted accurately the formula may be modified to suit the resistance in use, giving:—

$$T = 3750s/(900 + 900s^2) = 37{\cdot}5s/9\,(1 + s^2),$$

Whence for	$s =$	0·1	0·3	0·5	·6
	$T =$	·413	1·15	1·67	1·84

These and the preceding results are plotted in Fig. 6.11.

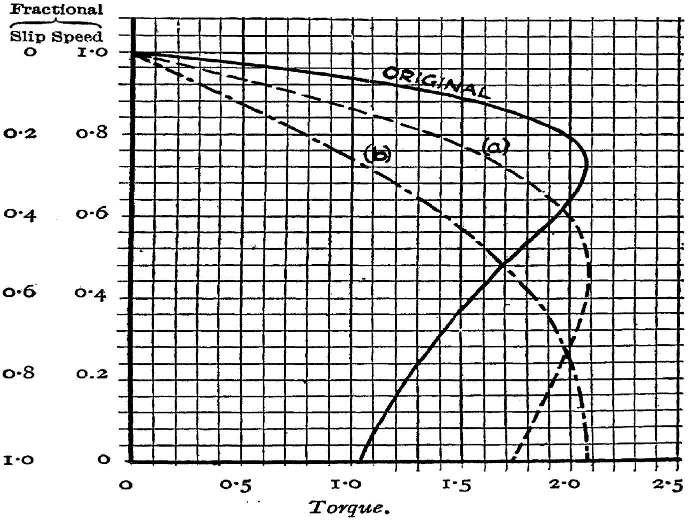


Fig. 6.11.—EFFECT OF INCREASING ROTOR RESISTANCE.

(*a*) Rotor resistance doubled. (*b*) Maximum starting torque.

12. Starting Conditions

As shown by Example 2 the addition of resistance to the rotor increases the starting torque. Since it reduces the starting current at the same time there is a two-fold improvement.

It is advisable in many cases to increase the resistance beyond the amount which gives maximum starting torque so as to reduce further the starting current (see Example 3).

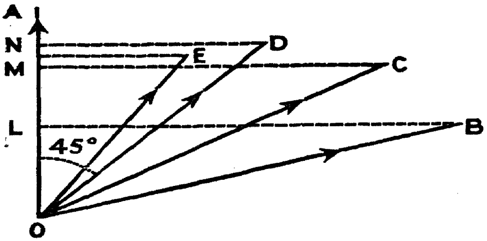


Fig. 6.12.—STARTING CURRENTS IN ROTOR.

The effect of additional rotor resistance in improving the starting torque is further illustrated by the vector diagrams of Fig. 6.12. In these OA is the rotor e.m.f., which is constant if stator impedance is neglected (cf. Art. 7). The current with no added resistance is given by OB, and the torque by OA.OL, where L is the foot of the perpendicular from B onto OA.

When an additional resistance equal to that of the rotor is used the current is reduced somewhat to the new value OC (see Example 3). But as the angle of lag is reduced at the same time the resulting torque, OA.OM is greater in spite of the reduction of current.

Further increases of resistance produce the same effect until the angle of lag is reduced to 45°, *i.e.* $R_2 = X_0$, as shown by OD. Beyond this point the reduction of current is not compensated for by the reduction in the phase difference, and so the torque diminishes again, as shown by OE.

Example 3. *Calculate the starting currents and angles of lag for the two cases of Example 2; and find the resistance which will give the same starting torque as the smaller resistance, and the corresponding current and angle of lag.*

Using the same units as in Example 1 (Art. 9) the starting current with rotor resistance doubled

$$= 100/\sqrt{(16^2 + 900)} = 100/34\cdot0 = 2\cdot94.$$

Similarly, with resistance increased 3·75 times,

$$\text{starting current} = 100/\sqrt{(900 + 900)} = 100/42\cdot4 = 2\cdot36.$$

In the first case $\tan \phi_2 = \frac{30}{16} = 1\cdot875; \quad \therefore \ \phi_2 = 61\cdot9°.$

In the second case $\tan \phi_2 = 30/30 = 1; \quad \therefore \ \phi_2 = 45°.$

Note that the relative torques are

$$2\cdot94 \cos 61\cdot9° = 1\cdot385, \text{ and } 2\cdot36 \cos 45° = 1\cdot68,$$

which are in the same ratio as 1·73 to 2·08, the values found in Example 2.

To obtain the same starting torque as with a total rotor resistance of 16×10^{-4} the value of R_2 is given by:—

$$R_2/(R_2^2 + 9 \times 10^{-6}) = 16 \times 10^{-6}/(2\cdot56 \times 10^{-6} + 9 \times 10^{-6})$$

which is satisfied by $R_2 = (9 \times 10^{-6})/(16 \times 10^{-4})$, *i.e.* R_2 greater than X_0 in inverse ratio to the previous value of R_2/X_0;

$$\therefore \ R_2 \doteqdot 5\cdot625 \times 10^{-3}.$$

With this value of R_2 and the same units as before,

$$\text{starting current} = 100/\sqrt{\{(5\cdot625 \times 10)^2 + 900\}} = 100/63\cdot7 = 1\cdot57$$

$$\text{and } \tan \phi_2 = 30/56\cdot25 = \cdot533; \quad \therefore \ \phi_2 = 28\cdot1°.$$

As a check, relative torque = 1·57 cos 28·1° = 1·39 which agrees well with the previous value of 1·385.

If the above three currents and that found in Example 1 are plotted at the corresponding angles of lag as in Fig. 6.12, it will be found that the ends of the vectors lie on a semicircle with its diameter perpendicular to OA, and touching OA at O. It can be proved in the following way that this will be always true in such cases.

Let OB (Fig. 6.13) be perpendicular to OA and equal to E/X_0, and let a semicircle OPB be drawn on OB as diameter. Join O to any point P on the semicircle and let $\angle$ POA = ϕ_2.

Then since $\angle \text{OPB} = 90°$, $\angle \text{OBP} = \angle \text{POA} = \phi_2$;

$$\therefore \text{OP} = \text{OB} \sin \text{OBP} = E \sin \phi_2 / X_0.$$

But when a voltage E is applied to a circuit of impedance Z, the current = E/Z, and $\sin \phi = X/Z$, therefore $E \sin \phi_2/X_0$ gives the magnitude of the current sent through the rotor at standstill when the resistance is such that ϕ_2 is the angle of lag of the current; *i.e.* the ends of the current vectors for all values of the rotor resistance lie on the semicircle OPB.

13. Resistance Equivalent to Load

It is shown in Art. 9 that the total torque can be expressed as $N_2 \mathbf{B} l I_2 r_2 \cos \phi_2/10$ where $\mathbf{B}$ = R.M.S. value of the flux-density. The power developed is $2\pi n T/(60 \times 10^7)$ watts, and $v_2 = 2\pi n r_2/60$. So the power developed is $N_2 \mathbf{B} l I_2 v_2 \cos \phi_2/10^8$.

But $\quad \mathbf{B} l v_2/10^8 = \mathbf{B} l\,(1 - s)\, v_1/10^8 = E_0\,(1 - s)$,

where E_0 = R.M.S. value of e.m.f. induced in each rotor conductor when the rotor is at rest;

$\therefore$ Power developed $= N_2 E_0 I_2 \times \cos \phi_2\,(1 - s)$.

The copper losses in the rotor $= N_2 . I_2^2 R_2 = N_2 I_2 . E_2 \cos \phi_2 = N_2 E_0 I_2 \cos \phi_2 . s$ (see Art. 8). Therefore the total power supplied to the rotor, which is the sum of these two, is given by:—

Total power supplied $= N_2 E_{02} I \cos \phi_2$.

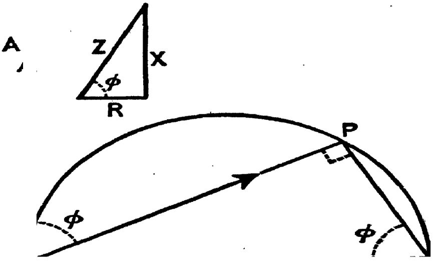


Fig. 6.13.—STARTING CURRENT SEMICIRCLE.

Thus the power developed is equivalent to adding to the rotor a resistance $(1 - s)/s$ times the actual resistance, *i.e.* making the total resistance $1/s$ times the actual resistance.

The same result can be reached from the fact that the expression for I_2 in Art. 8, viz. constant $\times\ s/\sqrt{\{R_2^2 + s^2X_0^2\}}$, may be re-written, constant $\Big/\sqrt{\left\{\left(\frac{R^2}{s}\right)^2 + X_0^2\right\}}$. Thus the rotor current varies in the same way as that produced by a constant voltage applied to a circuit made up of a constant reactance and a variable resistance, as shown in Fig. 6.14. In this the voltage applied across AD is E_0/N_2, *i.e.* the e.m.f. generated in each rotor conductor with rotor at rest. CD is X_0, BC is R_2, AB is the resistance equivalent to power developed per conductor (*i.e.* load plus friction and iron losses) and equals $R_2\,(1 - s)/s$; consequently ABC equals R_2/s.

14. Simple Circle Diagram

It has been shown in the previous Art. that the variation of load is equivalent to the variation of rotor resistance with the rotor at rest. Consequently under running conditions the end of the rotor current vector traces out a semicircle, as has been proved for the equivalent case of starting current with variable rotor resistance (Art. 12).

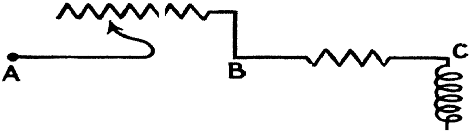


Fig. 6.14.—EQUIVALENT ROTOR CIRCUIT.

Neglecting stator losses, the stator current is the vector sum of a current balancing the rotor m.m.f., and the true magnetising current (see Art. 7). Therefore if OA (Fig. 6.15) represents the constant applied p.d., and OB the magnetising current at right angles to this, the stator currents at various loads can be obtained as follows.

Produce OB to C and make BC represent the reversed rotor current for the ideal case of zero rotor resistance; *i.e.* BC in Fig. 6.15 is equal to OB in Fig. 6.13, multiplied by the ratio of rotor turns to stator turns $(= N_2/N_1)$. Then if a semicircle BPC is described on BC as diameter the end of the stator current vector lies on this semicircle.

Thus if P is any point on the semicircle OP represents the stator current and PB the rotor current to a scale (N_2/N_1) times as great, and their combined m.m.f.s are equal to a constant current OB in the stator.

The point B corresponds with infinitely great total equivalent rotor resistance, and therefore with zero rotor current. This occurs with no load on the motor, so that (neglecting losses) the rotor runs at synchronous speed. The stator current is then simply the magnetising current OB.

The point C corresponds with zero rotor resistance, and therefore with zero output. This would occur when the rotor is clamped if it had no resistance. The line CB represents on the rotor current scale the current flowing in the rotor under these conditions, *i.e.* E_0/X_0 (Art. 12). Thus the whole of the e.m.f. induced in the rotor by the constant air-gap flux is used in balancing the e.m.f. self-induced in the rotor. This latter e.m.f. is due to the changes in the flux produced by the rotor current, and as this is not to affect the air-gap flux it must link only with the rotor conductors, *i.e.* it is a leakage flux.

Alternatively it may be noted that with the rotor clamped the induction motor acts like a short-circuited transformer (see Volume I., Chapter XVII., Art. 10). If the equivalent circuit is taken with the leakage reactance concentrated in the secondary circuit (see Volume I., Chapter XVII., Arts. 7, 8), the primary (*i.e.* stator) current is made up of two components, the larger of which flows in the leakage reactance, and the smaller in the open-circuit (synchronous speed) reactance. As resistances are being neglected these components both lag 90° and so are in phase, and the total current is their arithmetical sum.

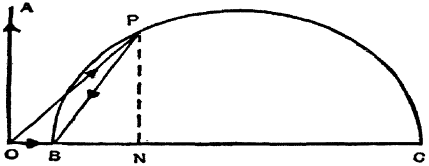


Fig. 6.15.—SIMPLE CIRCLE DIAGRAM.

The two components are represented in the present diagram (Fig. 6.15) by BC (leakage component) and OB (synchronous speed component), and the total stator current at standstill (*i.e.* with rotor clamped) is $(OB + BC) = OC$.

This alternative way of treating the motor concentrates the leakage in the stator. The previous method concentrated it in the rotor, the stator being supposed to produce a flux of constant magnitude all of which linked with the rotor conductors. Actually the leakage belongs partly to the stator and partly to the rotor, but as an approximation it may be considered as concentrated in either, just as in the case of the transformer.

The ratio OB/BC is called the **leakage** (or "**dispersion**") coefficient of the motor. A small value for this means a large value for the leakage component of the standstill current. This in turn means a large value for the reluctance of the leakage paths, since the leakage flux at standstill depends on the applied p.d. and the number of turns.

The input to the motor under any load is given by $\sqrt{3}$ VI cos ϕ (for 3-phase), and so is proportional to OP cos AOP, *i.e.* to PN in Fig. 6.15. The output is the same, since losses are neglected. And, since there is no slip with zero rotor resistance, PN further gives the torque to a scale which can be determined.

15. Deductions from Simple Circle Diagram

The maximum power-factor of the motor occurs when $\angle$ AOP is a minimum, *i.e.* when OP has the position OT (Fig. 6.16), where OT is a tangent to the semicircle. If D is the centre of the semicircle, DT is perpendicular to OT. Therefore $\angle$ ODT = $\angle$ AOT

$$\therefore \text{maximum power-factor} = \cos ODT = DT/OD = BD/(OB+BD)$$
$$= BC/(2\,OB + BC).$$

But OB = σ.BC, where σ = leakage coefficient of motor (Art. 14);

$$\therefore \text{ maximum power-factor} = 1/(1 + 2\sigma).$$

$$\text{Again since } OT^2 = OB\,.\,OC,$$

$$OB/OT = OT/OC = \sqrt{OB/OC} = \sqrt{\sigma/(1 + \sigma)},$$

$$\textit{i.e. } \text{magnetising current} = \sqrt{\sigma/(1 + \sigma)} \times \text{full-load current},$$

$$\text{and ideal standstill current} = \sqrt{(1 + \sigma)/\sigma} \times \text{full-load current};$$

provided the full-load current coincides with maximum power-factor.

If the motor is designed to have its maximum power-factor at full load, the corresponding torque is given by TN, where TN is perpendicular to BC. The maximum torque which the motor can exert occurs when P occupies the position Q, vertically above D, the torque being then given by QD. Thus the overload capacity as measured by the ratio (maximum torque)/(full-load torque) is equal to

$$QD/TN = TD/TN$$

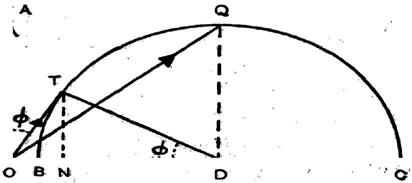


Fig. 6.16.—CIRCLE DIAGRAM RELATIONS.

$$= 1/\sin TDN = OD/OT = (OB + BD)/\sqrt{(OB \times OC)}$$

$$= (2\,OB + BC)/2\sqrt{(OB \times \overline{OB + BC})} = (1 + 2\sigma)/2\sqrt{\sigma\,(1 + \sigma)}$$

Since σ is always small this expression is approximately equal to

$$(1 + 2\sigma)/2\sqrt{\sigma} = 1/(2\sqrt{\sigma}) + \sqrt{\sigma}.$$

The current taken when the maximum torque is developed is given by $OQ = \sqrt{(QD^2 + OD^2)} = \frac{1}{2}\sqrt{(BC^2 + \overline{2OB + BC}^2)}$

$$= \tfrac{1}{2}BC\sqrt{(2 + 4\sigma + 4\sigma^2)}.$$

The full-load current is $OT = \sqrt{(OB \times OC)} = \sqrt{(OB \times \overline{OB + BC})}$

$$= BC\sqrt{\sigma\,(1 + \sigma)} = BC\sqrt{(\sigma + \sigma^2)};$$

∴ (current for maximum torque)/(full-load current)

$$= \tfrac{1}{2}\sqrt{(2 + 4\sigma + 4\sigma^2)}/\sqrt{(\sigma + \sigma^2)}$$

$$= \sqrt{\{1 + 1/2\,(\sigma + \sigma^2)\}} = 1/\sqrt{2\sigma} \text{ approx.}$$

The following table gives the values given by the above formulae for a number of values of σ:—

Leakage Coefficient σ	Maximum Power-Factor $\cos\phi_m$	Minimum Angle of Lag ϕ_m	(Magnetising Current / Full Load Current)	(Maximum Torque / Full Load Torque)	(Current at Max. Torque / Full Load Current)
·02	·96	16°	·140	3·63	5·05
·025	·95	17·8°	·156	3·27	4·53
·03	·94	19·5°	·171	3·00	4·14
·035	·935	20·8°	·184	2·82	3·85
·04	·926	22·2°	·196	2·65	3·61
·045	·917	23·5°	·208	2·50	3·41
·05	·909	24·6°	·217	2·40	3·24
·06	·893	26·7°	·238	2·22	2·98
·07	·877	28·7°	·256	2·08	2·77
·08	·862	30·5°	·272	1·97	2·61
·09	·847	32·1°	·287	1·88	2·47
·10	·833	33·6°	·302	1·81	2·35
·125	·800	36·9°	·333	1·67	2·14
·15	·769	39·7°	·361	1·56	1·97
·175	·741	42·2°	·386	1·49	1·85
·20	·714	44·4°	·408	1·43	1·76

The values in the table are from the exact formulae. For the overload capacity the approximate formula gives larger values, and for the ratio (current at max. torque)/(full load current) the approximate formula gives smaller values. In both cases the percentage difference increases with σ and reaches 10 per cent. in the last line of the table.

16. Allowances for Resistance Losses

In the actual motor the output is diminished by the iron and friction losses and by the stator and rotor copper losses. This last is proportional to BP^2 (Fig. 6.17). If PN is drawn perpendicular to BC, then since BPC is a right angle, $BP^2 = BN.BC$. Therefore the rotor resistance loss is proportional to BN. Therefore if two fixed straight lines (BV, BT) are drawn through B the length (RS) intercepted by them on PN in its various positions is proportional to the rotor copper loss.

The stator copper loss is proportional to OP^2. But this varies nearly as BP^2 except when BP is small. Hence the stator resistance loss is approximately proportional to the intercepts on PN of the same fixed lines as used for the rotor or of any other pair. By a suitable choice of the second pair the scale for the stator I^2R loss can be made the same as for the rotor I^2R loss (see Art. 19).

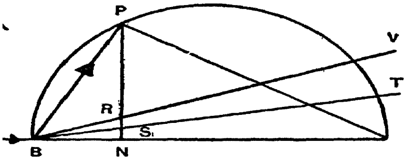


Fig. 6.17.—ALLOWANCES FOR RESISTANCE LOSSES.

The percentage error in the stator loss caused by this approximate method (which is due to Behrend) is greatest, as stated above, for small currents. As the copper loss is then small the approximation is close enough for most purposes.

Since $OP^2 = BP^2 + 2\,OB.BN + OB^2$, and the first two terms on the right hand are each proportional to BN, the error depends on the neglect of the constancy of OB^2. Thus if the line is drawn to give the true stator loss for the ideal standstill current, OC, the approximate loss for a stator current OP is too small by $OB^2\,(1 - BN/BC).R_1$, *i.e.* by $OB^2\,(NC/BC).R_1$; where R_1 = stator resistance.

If, however, the stator resistance loss at synchronous speed (*i.e.* $OB^2.R_1$ nearly) is added throughout, the approximate loss is too high by $OB^2\,(BN/BC).R_1$. This will be small over the working range of the motor.

If BC is taken as one of the stator resistance lines and BST (Fig. 6.17) is the other, and the latter (BST) is taken as the lower rotor resistance line and BRV is the upper one, the total resistance losses are given by $(RS + SN) = RN$. Therefore the output is represented by PR to the same scale as PN represents the input (cf. Art. 14).

17. Slip and Torque from Circle Diagram

The line PS in Fig. 6.17 represents the *input* to the rotor, which is equal to $N_2E_0I_2 \cos \phi_2$ (Art. 13), *i.e.* to $BlvI_2 . R_2/Z_2$. The line RS represents the rotor copper loss, *i.e.* $N_2I_2^2R_2$;

$$\therefore\ RS/PS = E_2/E_0 = Bl\,(v - v')/Blv$$
$$= (v - v')/v = \text{fractional slip} = s.$$

Again from the above $PR/PS = v'/v$.

Now the output PR is proportional to (torque) × (rotor speed) and so torque is proportional to PR/v', *i.e.* to PS/v. Since v is constant;

PS represents the torque to some scale (cf. Art. 14).

The scale may be made such that the length which represents 746 watts on the input and output scales, *i.e.* 1 h.p. on the output scale, is taken as 1 *synchronous* h.p. on the torque scale. A torque of 1 synchronous h.p. means the torque which would give an output of 1 h.p. if the motor were running at its synchronous speed.

Alternatively consider the rotor equivalent circuit, Fig. 6.14. This shows that the slip is equal to the ratio of $BC(R_2)$ to ABC (R_2/s); and therefore to the ratio of rotor copper loss (I^2R_2) to total rotor input, *i.e.* to RS/PS in Fig. 6.17.

18. Allowances for Iron and Friction Losses

The iron losses in the stator are nearly constant, since the flux-density is nearly so and the frequency is constant. There is a small diminution at large values of the current owing to the voltage absorbed by resistance, but this is negligible over the working range.

The iron losses in the rotor are affected in two opposite ways. As the currents increase, the flux-density diminishes owing to the effects of both stator resistance and rotor leakage. But at the same time the increasing slip increases the frequency in the rotor, so that on balance the rotor iron losses increase.

The frictional torque is nearly constant, and therefore the power lost in friction varies nearly as the speed.

The sum of the iron and frictional losses is therefore nearly constant and may be taken as exactly constant with sufficient accuracy. When full particulars of a motor are known it is possible

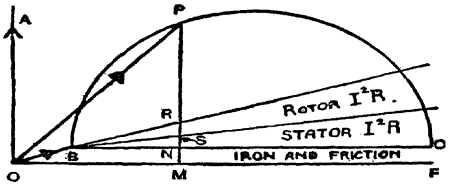


Fig. 6.18.—Allowances for all Losses.

to estimate the variation in the losses, but the trouble entailed is not as a rule worth the slight increase in accuracy.

The allowances for the effect of these losses can therefore be made by drawing the diameter of the semicircle parallel to the line (OF, Fig. 6.18) drawn through the origin perpendicular to the applied p.d. vector (OA), and at a distance above OF equal to the combined iron and friction losses measured on the input scale.

Then for any stator current OP, the input is given by PM, the various losses by RS, SN and NM, the output by PR, and the efficiency therefore by PR/RM.

19. Circle Diagram from Experimental Results

The experiments necessary for drawing the circle diagram, and thence determining the performance of an induction motor, are similar to those for finding the equivalent circuits and the performance of a transformer.

The motor is first run at normal p.d. with no load on it, and the current and power-factor measured. The latter is obtained by measuring the power taken by the two-wattmeter method for balanced loads (Chapter III., Arts. 10, 11). This determines the position and length of OB. This test corresponds with the open-circuit test of a transformer (Volume I., Chapter XVII., Art. 9).

The rotor is then clamped, or allowed to rotate *slowly*, and a reduced voltage applied to the stator sufficient to send about full-load current through it. The current, power-factor, and voltage (V_s) are measured as before. The normal standstill current is then calculated by multiplying the measured current by (V/V_s), where V is the normal applied p.d.; and the power-factor is taken at the measured value. This test corresponds with the short-circuit test on a transformer (Volume I., Chapter XVII., Art. 10).

The reason for obtaining the standstill current in this way is that the normal p.d. would send a very large current through the stator with the rotor clamped (see Example 1, Art. 9). Even if the windings can carry this without serious overheating it would cause saturation of the iron in the leakage paths, particularly the tips of the teeth. This would give a very different value of the equivalent reactance to the value under normal running conditions.

As the accuracy of the circle diagram depends on the assumed constancy of this reactance the results for normal running would be incorrect if the actual standstill current were used in constructing the diagram.

The standstill current is then plotted to scale in its proper phase relation (OE, Fig. 6.19), and this gives a second point on the semicircle. The centre of this can now be found by bisecting BE at right angles. The centre lies at the point (D) where this bisector cuts the line through B which is perpendicular to OA. With this centre and DB as radius the semicircle is drawn, and EL is drawn perpendicular to BD produced, *i.e.* perpendicular to the diameter of the semicircle.

EL is then divided at T so that TL is equal to (stator I^2R loss) with standstill current (OE) flowing. The point T is then joined to B by a straight line.

Then for any point P on the semicircle; stator current = OP, angle of lag = ∠ POA, input = PM, iron and friction losses = NM, stator copper loss = SN, rotor copper loss = RS, output = PR, efficiency = PR/PM, torque = PS, slip = RS/PS.

The method of settling the scales is shown in Example 4 below.

It may be noted that the stator copper loss at no load, *i.e.* $OB^2 . R_T$, is included with the iron and friction losses and the total

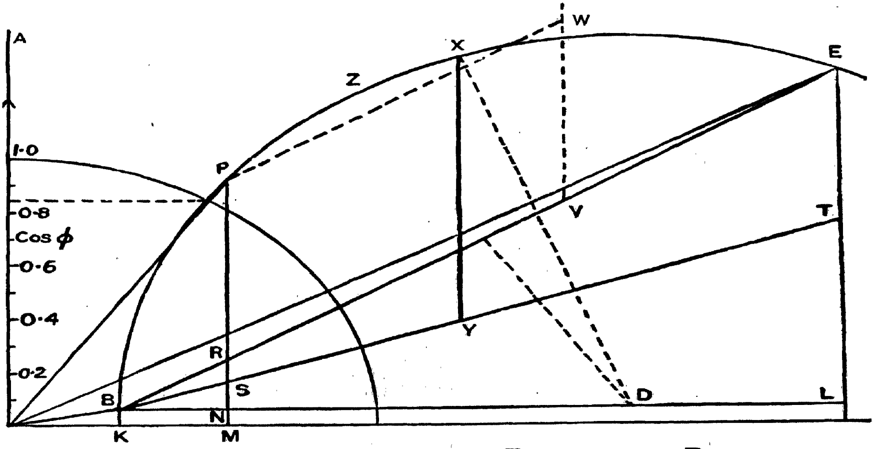


Fig. 6.19.—CIRCLE DIAGRAM FROM EXPERIMENTAL RESULTS.

XY = Maximum Torque.

assumed constant. Thus, as shown in Art. 16, the error in the stator I^2R allowance is small over the working range. Moreover, if TL is made equal to $(OE^2 - OB^2) R_1$, *i.e.* the *additional* stator copper loss, the allowance becomes accurate throughout.

Example 4. *The following test results were obtained for a 3-phase Y-connected 5 h.p., 190-volt, 50 cyc., 4-pole motor.*

Volts	190	50	68	104
Current (A.)	5·0	12	16	23
Watts	330	510	940	2100
Conditions	*No load*	*Standstill*	*Standstill*	*Standstill*

Draw its circle diagram and find the full-load current, power-factor, torque, slip, and efficiency.

Draw graphs showing the variation of these quantities with load.

Stator resistance between terminals = 1·30 ohms.

$$\text{At no load:—}\cos\phi = 330/(\sqrt{3}\cdot 190 \times 5{\cdot}0) = 0{\cdot}20.$$

When squared paper is used for the diagram a power-factor scale is readily obtained as follows. Any convenient length along OA is taken to represent unity. A quadrant is drawn with this as radius (see Fig. 6.19). Then the direction of the vector representing a current of any given p.f. is the line joining O to the point in which the horizontal line giving the power-factor cuts the quadrant.

Assuming 80 per cent. efficiency and 0·85 p.f. the full-load current equals

$$(5 \times 746 \times 100)/(80 \times \sqrt{3} \times 190 \times 0{\cdot}85) = 16{\cdot}7 \text{ amp.}$$

To produce this current at standstill will require 71½ volts and absorb 1030 watts (by interpolating between the results given);

$$\therefore \text{ Standstill current} = 16{\cdot}7 \times 190/71{\cdot}5 = 44{\cdot}4 \text{ amp.}$$

And $$\text{power-factor} = 1\ 070/(\sqrt{3} \times 71{\cdot}5 \times 16{\cdot}7) = 0{\cdot}50.$$

This is plotted (OE in Fig. 6.19) to a scale of 1″ = 10 A., and the magnetising current is plotted to the same scale.

B is joined to E, and the semicircle BEC is then drawn as described above, EL is drawn perpendicular to BC.

The scale for input, output, and losses is

$$1'' = \sqrt{3} \times 190 \times 10 \text{ watts} = 3{\cdot}29 \text{ kilowatts,}$$

or $$1'' = 3{\cdot}29 \times 1\ 000/746 \text{ h.p.} = 4{\cdot}41 \text{ h.p.}$$

Stator copper loss at standstill $= (44{\cdot}4)^2 \times 1{\cdot}30 \times (3/2)$ watts $= 3{\cdot}84$ kw.

On the above scale this is represented by 3·84/3·29, *i.e.* 1·17″. LT is made of this length, and the remainder, TE, represents the rotor copper loss at standstill. B is joined to T to give the torque line.

Note that the stator copper loss at no load is only $5^2 \times 1{\cdot}30 \times 3/2$ watts. *i.e.* 49 watts. Thus the error due to neglecting this is only a little over 1% of the stator copper loss at standstill. And, as shown in § 16, the error is less at smaller loads.

Again, the full load of 5 h.p. is developed where PR = 5/4·41 = 1·13″ approx.

To find the corresponding position of P draw a line, VW, of this length vertically upwards from any convenient point V on BE. Draw WP, parallel to EB, meeting the semicircle in P, the required point.

Join OP, the length of which measured on the current scale gives the full-load current, while its intersection with the power-factor quadrant gives the corresponding power-factor. From the diagram:—the full-load current = 18·2 amp., at a power-factor of 0·84.

The scale for torque in synchronous watts or synchronous h.p. is the same as that for output. If it is required in lb.-ft. proceed as follows:—

$$\text{Synchronous speed} = 50 \times 60/2 = 1\,500 \text{ r.p.m.}$$

1 h.p. at 1 500 r.p.m. requires a torque equal to

$$33\,000/(2\pi \times 1\,500) \text{ ft.-lb.} = 3{\cdot}50 \text{ lb.-ft.};$$

$$\therefore\ 1'' = 4{\cdot}41 \times 3{\cdot}50 \text{ ft.-lb.} = 15{\cdot}4 \text{ lb.-ft.}$$

Hence full-load torque = PS = 19·5 lb.-ft.

The slip is RS/PS = 11 per cent.

The efficiency is PR/PM = 74 per cent.

By taking other heights along VW, and finding the corresponding positions of P, the values at various other loads can be found in the same way as above. This has been done, and the results are plotted in Fig. 6.20.

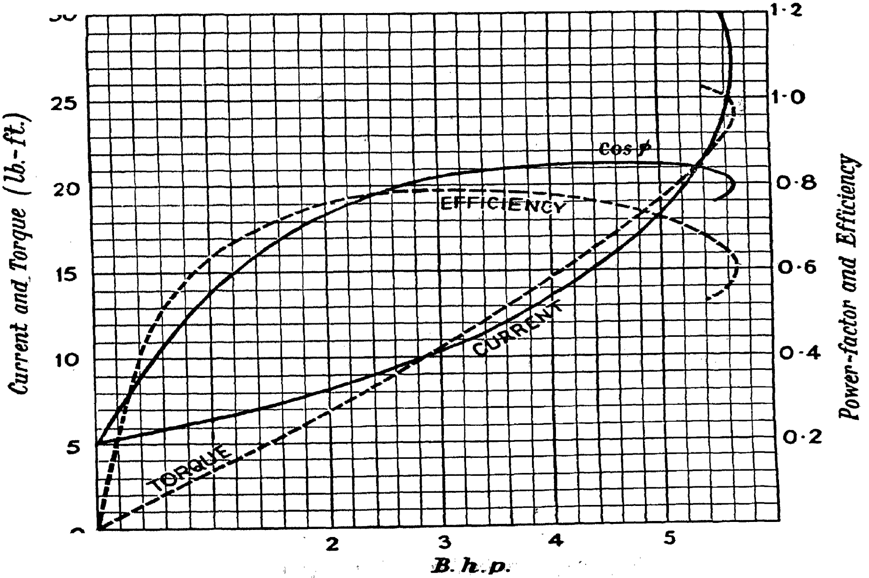


Fig. 6.20.—PERFORMANCE CURVES.

20. Ossanna Circle

The method described above for dealing with the effects of the resistance of the stator departs from strict accuracy in several ways. It is, however, quite sufficiently accurate for all ordinary induction motors.

Ossanna* has proved that if the allowance is made in a strictly accurate way the locus of the ends of the stator and rotor current vectors is still a circle. The radius of this accurate circle differs only very slightly (by about $\frac{1}{10}$ of 1 per cent.), from that of the approximate circle. At the same time its centre is raised slightly. The amount of this shift is of the order of 1 per cent. of the ideal standstill current OC.

If, as suggested above, the length of LT is made equal to the additional stator copper loss at standstill the ordinary circle diagram gives reasonably accurate results.

21. Equivalent Resistances

The consideration of the actions of the induction motor is simplified considerably by dealing with equivalent resistances, reactances, currents, etc., instead of with the actual values, and this method has been used above. Either of two methods may be used. In the first the actual values are taken for the stator, and the values for the rotor are replaced by the equivalent values for an equal number of turns. The equivalent current for the same number of turns as the stator is equal to (actual current) $\times N_2/N_1$, and the equivalent resistance is equal to (actual resistance) $\div (N_2/N_1)^2$. Compare the case of the transformer (Volume I., Chapter XVII., Art. 5).

If (N_2/N_1) is not known it can be measured by clamping the rotor and comparing the p.d. between the slip-rings on open-circuit with the p.d. applied to the stator.

In the second the actual values for both stator and rotor are replaced by the equivalent values per centimetre of periphery. The second method is perhaps more convenient for design purposes, but the first is simpler when dealing with tests on actual motors. The way in which the equivalent resistances per centimetre can be found when the dimensions of the motor are known is indicated below.

The current per centimetre is equal to (current per conductor) $\times$ (No. of conductors per cm.). The stator resistance per cm. is equal to (average resistance of stator per half-turn) *divided by* (No. of conductors per cm.); *i.e.* it is the resistance *in parallel* of the conductors in a centimetre.

---

* *Zeit. für E. Vienna*, 1899. See further L. H. A. Carr, *J.I.E.E.*, vol, 66, p. 1174, and *The Electrician*, vol. ciii., p. 24.

Or in symbols,
$$\left.\begin{aligned} I_e &= I_1 \times N_1/c \\ \text{and} \quad R_e &= R_c \div (N_1/c) \end{aligned}\right\} \text{where } c = \text{circumference of stator face.}$$

$$\therefore\ I_e^2R_e = I_1^2R_c \times N_1/c = \text{resistance loss per cm.}$$

since $I_1^2R_c =$ average resistance loss per conductor including the loss in the end connexions.

In a wound rotor the current and resistance per cm. are obtained in exactly the same way. Thus $I_{e2} = I_2 \times N_2/c$, and $R_{e2} = R_{c2} \div (N_2/c)$; where I_{e2}, R_{e2} and R_{c2} refer to the rotor. Hence the rotor copper loss per cm. $= I_2^2R_{c2} \times N_2/c = I_{e2}^2\, R_{e2}$.

In this case, as in the first method, the scales for stator and rotor currents in the circle diagram become identical; since $I_{e2}/I_e = (I_2/I_1) \times (N_2/N_1)$.

Note that in a three-phase stator the phase resistance is $R_c \times N_1/3$; and so R_e equals $R_{ph} \div (N_1^2/3c)$. Similarly in a three-phase wound rotor R_{e2} equals $R_{ph2} \div (N_2^2/3c)$.

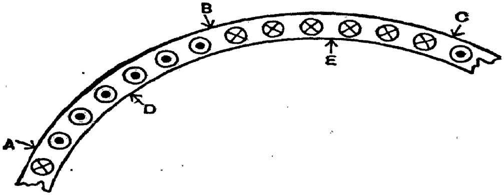


FIG. 6.21.—EQUIVALENT RESISTANCE OF END RINGS; DIRECTIONS OF CURRENTS IN BARS.

22. Equivalent Resistance of Squirrel-Cage Rotor

The squirrel-cage rotor presents some difficulty because the end rings form the common end connexions for all the conductors, and so the average resistance per turn cannot be obtained straightforwardly. If the current is assumed to be distributed sinusoidally the allowance to be made for the end rings can be obtained as follows:—

Let $R =$ resistance of each rotor bar,

$R' =$ resistance of an end ring between two neighbouring bars,

$I =$ R.M.S. rotor current per bar,

$N_2 =$ No. of rotor bars,

$p =$ No. of pole-pairs in motor,

$\tau =$ pole-pitch in cm.

Consider a double pole-pitch, ABC, of the end ring, *i.e.* $1/p$ of whole ring (Fig. 6.21). Over half of this the rotor bar currents will at a particular instant flow in one direction, and over the other

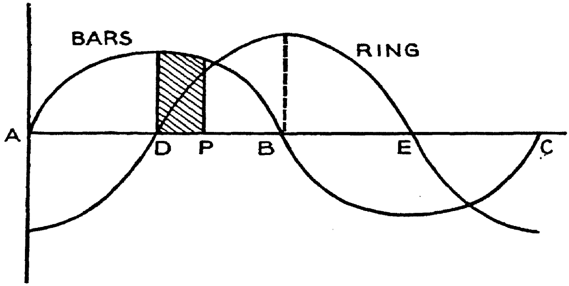


Fig. 6.22.—Equivalent Resistance of End Rings; Magnitudes of Currents in Bars and Ring.

pole-pitch in the opposite direction, *i.e.* upwards in AB and downwards in BC in figure. Hence *half* the sum of the currents over one pole-pitch will flow from left to right through the part, B, of the end ring which separates the two sets of bars. The other half of the total current of the bars in AB will flow from right to left through A.

At D and E, the centre points of AB and BC respectively, there will be no current in the end ring. With sinusoidal distribution of the bar currents (Fig. 6.22), the end-ring current will likewise vary sinusoidally but will be displaced relatively by half a pole-pitch. This follows from the fact that at any point P between D and B the end-ring current is the sum of the currents between D and P; *i.e.* the ordinate of the ring current at P is proportional to the shaded area above DP.

The end-ring current at B (*i.e.* maximum end-ring current) = (average current per bar) × (No. of bars between D and B)

$$= (2\sqrt{2}/\pi)\,\mathrm{I} \times \mathrm{N}_2/4p;$$

$\therefore$ R.M.S. value of the end-ring current = $(1/\sqrt{2})$ × max. value

$$= (2/\pi)\,\mathrm{I} \times \mathrm{N}_2/4p.$$

The resistance of ABC $= \mathrm{R}' \times \mathrm{N}_2/p$;

$\therefore$ Copper loss in the *two* end rings over twice the pole-pitch

$$= 2\,.\,(2/\pi)^2\mathrm{I}^2 \times (\mathrm{N}_2/4p)^2 \times \mathrm{R}' \times \mathrm{N}_2/p$$

$$= (\mathrm{N}_2{}^3/2p^3\pi^2)\,\mathrm{I}^2\mathrm{R}'.$$

The copper loss in the bars over twice the pole-pitch $= (\mathrm{N}_2/p)\mathrm{I}^2\mathrm{R}$;

$\therefore$ Total rotor copper loss over twice the pole-pitch

$$= (\mathrm{N}_2/p)\,.\,\mathrm{I}^2\,\{\mathrm{R} + 2\,(\mathrm{N}_2/2p\pi)^2\mathrm{R}'\}$$

and total rotor copper loss $= \mathrm{N}_2\mathrm{I}^2\,\{\mathrm{R} + 2\,(\mathrm{N}_2/2p\pi)^2\mathrm{R}'\}$.

Thus $\mathrm{N}_2\,\{\mathrm{R} + 2\,(\mathrm{N}_2/2p\pi)^2\mathrm{R}'\}$ is the total rotor resistance for all phases.

The rotor current per cm. $= I \times N_2/2p.\tau = I_e$,
the bar resistance per cm. $= R \div (N_2/2p\tau) = R_e$,
the end-ring resistance per cm. $= R' \times (N_2/2p\tau) = R_e'$.
Whence copper loss per cm. $=$ loss per double pole-pitch $\div 2\tau$

$$= I_e^2 \{R_e + 2\,(\tau/\pi)^2 R_e'\}$$

$$= I_e^2 \{R_e + 2\,(d/2p)^2 R_e'\}$$

where d = diameter of inner face of stator in cm.

[N.B.—The values per cm. may be referred either to the inner face of the stator or the outer face of the rotor, but whichever is used must be kept to for both stator and rotor.]

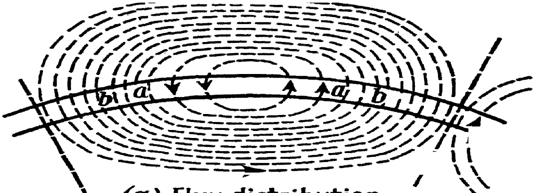
Either of the last two expressions within brackets { } gives the equivalent rotor resistance per cm.

23. Calculation of Magnetising Current

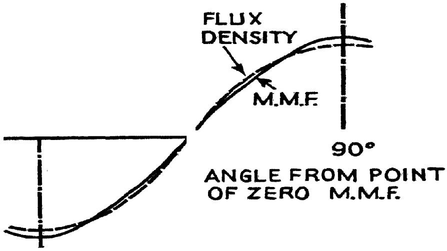
The calculation of the synchronous reactance of an induction motor, or of the true magnetising current which amounts to the same thing, follows the same general lines as the calculation of the magnetising current of a D.C. dynamo (see Vol. I., Chapter IX., Arts. 6-11).

The main difference is that whereas in the D.C. dynamo the flux is assumed to be uniform over any cross-section and the m.m.f. the same for all paths, in the induction motor the flux is distributed approximately sinusoidally and the m.m.f. varies similarly.

If the permeability were constant this would cause no difficulty, and the value of the magnetising current could be obtained accurately by considering the mean m.m.f. and the line crossing the air gaps where this mean value occurs. For sinusoidal distribution



(a) **Flux distribution**



(b) **M.M.F. & Flux-density**

Fig. 6.23.—MAGNETIC FLUX OF INDUCTION MOTOR

the mean value is $2/\pi = \cdot 637 \times$ maximum, and occurs 40° (electrical) from the point of zero m.m.f. [*aa*, Fig. 6.23 (*a*)].

Owing to the variations in permeability of the iron, particularly the stator and rotor teeth, a sinusoidal m.m.f. will produce a flattened distribution curve for the flux-density [Fig. 6.23 (*b*)]. If the value of B is calculated for the mean m.m.f., and a sinusoidal distribution of flux is assumed, the total flux obtained will be larger than the true value; or conversely, the calculated magnetising current for a given total sinusoidal flux will be smaller than the true value.

If maximum values are taken instead the opposite result is obtained, *i.e.* flux too small, or magnetising current too large.

Some intermediate position therefore will give a correct value. Though this position may differ somewhat in different motors, a line [*bb*, Fig. 6.23 (*a*)] crossing the air-gaps 60° (electrical) from the point of zero m.m.f. will always give a good approximation.

The reason for this is that the flux-density distribution curve can be approximated to closely by a sine-wave with a third harmonic (see Chapter II.), but is usually still more flattened. The 60° value is the same whether the third harmonic is present or not. The mean value of the sine curve is, however, a little smaller, and this makes some allowance for the additional flattening.

A more accurate method is to calculate the flux-density at a number of points and hence obtain the mean value. This is much slower, and as the m.m.f. wave is only approximately sinusoidal, the extra labour is not warranted by the gain in accuracy.

24. Magneto-motive Force

The m.m.f. of the stator winding is the same as the m.m.f. of armature reaction in an alternator (see Chapter IV., Art. 12). As, however, the stator has a considerable number of slots per pole per phase it is worth while working out the case of a uniformly distributed winding, which is the limiting case of a single-layer winding.

In Fig. 6.24, (*a*) shows the windings of the three phases, and (*b*) the m.m.f.s at the instant when the current in phase I is at its positive maximum, as indicated in the vector diagram.

The full line shows the m.m.f. of phase I. The shape of the m.m.f.s of phases II and III is similar, but they are of only half their maximum value and reversed. Adding the three gives the total m.m.f. as shown by line of circles. This has a maximum value equal to twice the maximum m.m.f. of one phase, at the point between phase II and the return side of phase III.

One-twelfth of a period (30°) later the m.m.f.s are as shown in (*c*). No current flows in II and 86·6 per cent. of maximum in I and III, that in the latter being negative. The maximum m.m.f. is 1·73 times maximum of one phase, and the m.m.f. wave is now flat-topped as shown. The centre of its top is 30° to the right of the peak of the wave at the instant shown in (*b*).

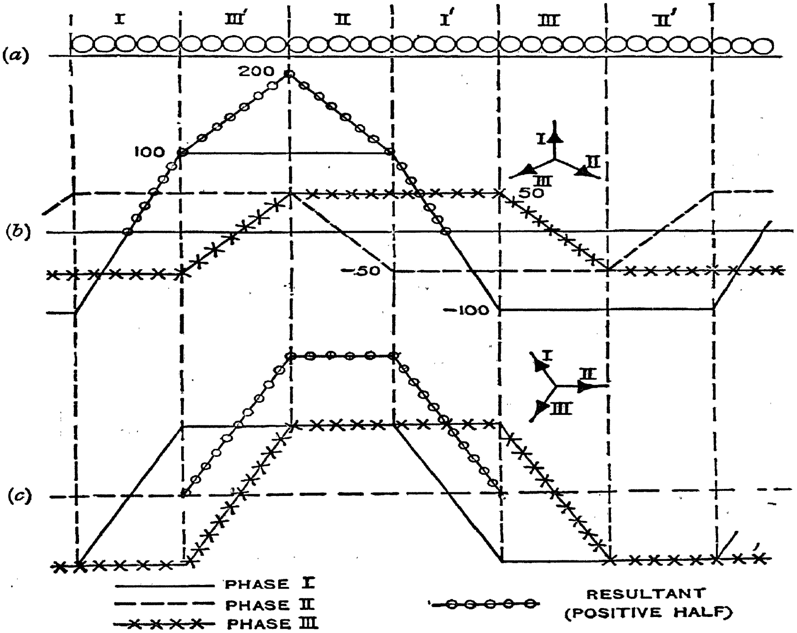


Fig. 6.24.—RESULTANT M.M.F. OF STATOR.

Another $\frac{1}{12}$th period later the m.m.f. wave will have returned to its original form, but its peak will be between II and I′, *i.e.* 60° to right of original position. It changes back again to shape (*b*) in the next $\frac{1}{12}$th period, and continues to alternate between the two shapes, meanwhile moving continually to the right, *i.e.* rotating at synchronous speed.

If wave (*b*) be analysed into its various harmonics (see Chapter II.), it will be found that the fundamental sine-wave has an amplitude of ($18/\pi^2 =$) 1·82 times the maximum m.m.f. of one phase. If wave (*c*) is similarly analysed, it will be found that the fundamental has this same value, the difference in the wave-forms being due to the differences in the harmonics, particularly the fifth and seventh.

It is, therefore, reasonable to replace the actual m.m.f. waves by a sine-wave of this value, rotating without change of shape. The three half-waves are plotted in Fig. 6.25.

Hence, if I_m = R.M.S. magnetising current per phase,

and $\mathfrak{T}_1$ = stator turns per pole per phase.

$$\text{Maximum equivalent m.m.f.} = (18/\pi^2)\,(4\pi/10)\,\sqrt{2}\,.\,I_m\,.\,\mathfrak{T}_1$$
$$= 3{\cdot}24\ I_m\mathfrak{T}_1 = 1{\cdot}62\ I_m N_1,$$

where N_1 = *number of stator conductors per pole per phase;*

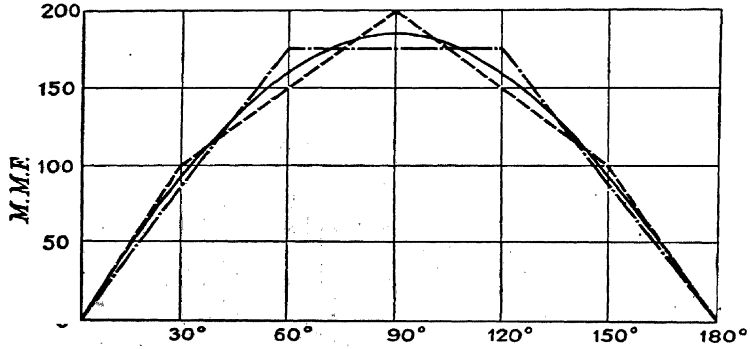


Fig. 6.25.—M.M.F. WAVES.

- - - - Wave (*b*) of Fig. 6.24. — - — - Wave (*c*) of Fig. 6.24. —— Common fundamental.

and:—

The m.m.f. at any point $= 3{\cdot}24\ I_m\mathfrak{T}_1 \sin\theta = 1{\cdot}62\ I_m N_1 \sin\theta$.

The value at 60° (see Art. 23) is $1{\cdot}62\ I_m N_1 \,.\, \sqrt{3}/2 = 1{\cdot}40\ I_m N_1$.

Let δ = length (radial) of air-gap in cm. Add to this

$\frac{1}{2}$ length/(area × permeability)

for each portion of the iron path of the 60° line, and let δ' air-gap length corrected in this way. Then:—

$$\text{Flux-density at } 60^\circ = 1{\cdot}40\ I_m N_1/\delta'$$

$$\text{and } \mathbf{B}_{60}/\mathbf{B}_{\text{mean}} = (\sqrt{3}/2)/(2/\pi) = 1{\cdot}36.$$

The relation between B_{mean}, N_1, and applied p.d., after allowing for resistance, can be found just as in the case of an alternator (Chapter IV., Arts. 4-6) for any particular winding, and thence the magnetising current can be obtained.

It is more usual to work in ampere-turns. If this is done, the above formulae are modified as follows:—

$$\text{Maximum m.m.f. per phase} = \sqrt{2}\,.\,I_m\mathfrak{T}_1 \text{ amp.-turns per pole}$$
$$= 1{\cdot}41\ I_m\mathfrak{T}_1 \quad ,, \quad ,, \quad ,,$$
$$\text{Max. equivalent m.m.f. of 3 phases} = (18/\pi^2)\,.\,\sqrt{2}\,I_m\mathfrak{T}_1 \text{ ampere-turns per pole}$$
$$= 2{\cdot}58\ I_m\mathfrak{T}_1 \text{ amp.-turns per pole.}$$
$$\text{M.m.f. of 3 phases along } 60^\circ\text{-line} = \sqrt{3}/2 \times \text{maximum}$$
$$= 2{\cdot}23\ I_m\mathfrak{T}_1 \text{ amp.-turns per pole.}$$

Example 5. *Calculate the magnetising current of a 6-pole three-phase 500-volt 50 cyc. motor with star-connected stator having 4 slots per pole per phase and 9 conductors per slot. The rotor has 53 slots. The following further particulars are given :—*

Nett length of iron, 12·5 cm. ; gross length, 14 cm.
Stator teeth : height, 2·8 cm. ; mean width, 0·68 cm.
Rotor teeth : height, 1·1 cm. ; mean width, 1·10 cm.
Air-gap : depth, ·08 cm. ; contraction coefficient (see Volume I., Chapter IX., Art. 11), 0·83 ; mean diameter, 40 cm.
Stator core : nett section, 70 sq. cm. ; mean magnetic length, 22 cm.
Rotor core : nett section, 70 sq. cm. ; mean magnetic length, 15 cm.

Allowing 10 volts for resistance losses, etc., and taking the breadth coefficient (Chapter IV., Art. 4) as ·958;

$$(490/\sqrt{3}) \times 10^8 = {\cdot}958 \times 4{\cdot}44 \times \Phi \times (9/8) \times 4 \times 6 \times 50.$$
$$\text{Whence } \Phi = 1{\cdot}23 \times 10^6 \text{ lines per pole,}$$
$$\text{Area of air-gap per pole} = 14 \times \pi \times 40{\cdot}6 \times 0{\cdot}83 = 244 \text{ sq. cm.,}$$
$$\text{Area of stator teeth per pole} = 12{\cdot}5 \times 4 \times 3 \times 0{\cdot}68 = 102 \text{ sq. cm.,}$$
$$\text{Area of rotor teeth per pole} = 12{\cdot}5 \times (53/6) \times 1{\cdot}10 = 121 \text{ sq. cm.}$$

The following table can now be drawn up:—

PORTION	AREA OF SECTION SQ. CM.	B_{MEAN}	B_{60°	LENGTH CM.	AMPERE-TURNS	
					PER CM.	PER POLE
Stator core	70	$9{\cdot}8 \times 10^3$	$12{\cdot}0 \times 10^3$	11	7	77
Stator teeth	102	$12{\cdot}1 \times 10^3$	$16{\cdot}4 \times 10^3$	2·8	32	90
Air-gap ..	244	$5{\cdot}04 \times 10^3$	$6{\cdot}85 \times 10^3$	·080	5450	436
Rotor teeth	121	$10{\cdot}2 \times 10^3$	$13{\cdot}8 \times 10^3$	1·1	10	11
Rotor core	70	$8{\cdot}8 \times 10^3$	$12{\cdot}0 \times 10^3$	7·5	7	53
				Total per pole ..		667

Note (*a*) the cores carry only half the flux of one pole; (*b*) only half the mean (60°) magnetic length of the cores is to be taken.

The number of turns per pole per phase $= 4 \times 9/2 = 18$;

$\therefore$ Magnetising current $= 667/(18 \times 2{\cdot}23) = 16{\cdot}6$ amperes.

25. Equivalent Reactance

The experimental method of finding the total equivalent reactance has been explained in connexion with the circle diagram (Art. 19). If it is desired to express this per centimetre of periphery the same procedure is used as for resistance; *i.e.* the reactance per cm. = (actual reactance per conductor) ÷ (conductors per cm.).

As in the case of resistance, the reactance referred to rotor = (reactance referred to stator) $\times (N_2/N_1)^2$. The reactance per cm. has the same value whether referred to stator or to rotor (cf. Art. 21).

In calculating the equivalent reactance from the dimensions of a motor, the leakage is divided into *three* parts, viz. stator leakage, rotor leakage, and zig-zag leakage. This last is the flux which passes from a stator tooth into a rotor tooth, and then back into a stator tooth, and so on. It therefore does not link with both sets of conductors although it passes through both the stator and the rotor. The paths of these three components of the leakage flux are indicated in Fig. 6.26.

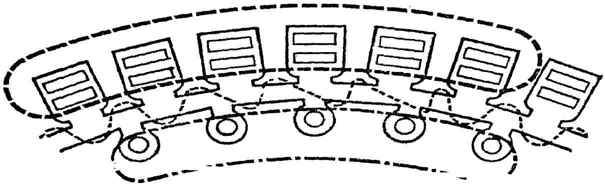


Fig. 6.26.—LEAKAGE FLUXES.

--- Stator. ····· Rotor. ------ Zig-zag.

The calculation of the stator and rotor leakage fluxes is similar to the calculation of the magnetic leakage in transformers (Volume I., Chapter XVII., Art. 20). It is, however, complicated by the current at any instant being different in different conductors.

Consider a single slot, and simplify its section as shown in Fig. 6.27 (*a*), in which the section-lined portion represents the conductors, the insulation between them being omitted. In (*b*) the leakage m.m.f. is shown. In the upper portion, where this m.m.f. varies as the distance below the top of the coil, the number of turns linked by the resulting flux varies likewise. Hence the linkages per cm.

of height vary as the square of the distance below the top of the coil, as shown in (*d*), the curve being a parabola.

Since the mean width of such a parabolic segment is $\frac{1}{3}$rd of its maximum width, it follows that the equivalent leakage and equivalent reactance are the same as if the maximum m.m.f. produced a flux in a path only *one-third* as broad as the actual path (h_1), which flux linked with *all* the conductors in the slot (see Fig. 6.27). Thus since—

$$\text{Total flux} = (\text{M.m.f.})/\text{Reluctance}$$
$$= \text{M.m.f.} \times \text{Permeance};$$

and, for paths in parallel, total permeance equals the sum of permeances: hence the total equivalent permeance per cm. of axial length

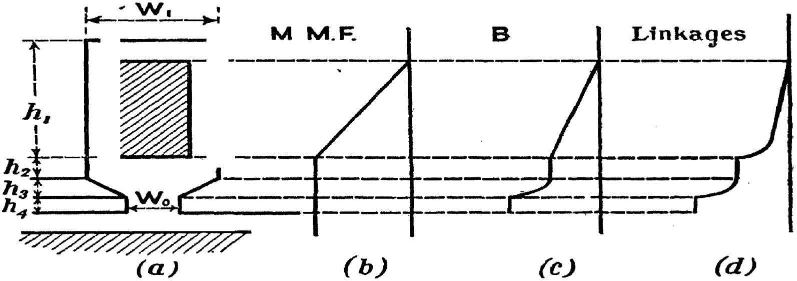


Fig. 6.27.—LEAKAGE FLUX OF ONE SLOT.

$$= (\tfrac{1}{3}h_1 + h_2)/w_1$$
$$+ h_3 \log_e (w_1/w_0)/(w_1 - w_0)$$
$$+ h_4/w_0.$$

The middle term may be replaced by the simpler approximation, $2h_3/(w_1 + w_0)$, when this term is not a large fraction of the total.

Let y = pitch of slots, and l_a = width of air-gap.

Then the permeance of the leakage path along air-gap per axial cm. = l_a/y per slot.

Half of this may be assigned to the stator and the other half to the rotor when the leakages are being separated.

Let N_1 = total no. of stator conductors,
S_1 = total no. of stator slots,
λ_1 = equivalent permeance per cm., including air-gap allowance,
L = nett length of stator, *i.e.* core-length less duct-widths.

Then linkages per ampere for one slot = flux per amp. × turns

$$= (4\pi/10)\,(N_1/S_1)^2 \times \lambda_1 L,$$

and the inductance in henries is the same divided by 10^8.

If $2p$ is the number of poles, then in a three-phase motor there are $(S_1/6p)$ slots per pole per phase. The leakage m.m.f. per pole is therefore increased in this proportion. But the reluctance is increased in the same proportion since the leakage flux has to pass across this number of slots, and so the flux is unchanged. The linkages (and the inductance) are therefore increased in the ratio of the turns, *i.e.* $(S_1/6p)$.

If the windings of the poles of each phase are in series (as is usual) the total inductance per phase will be $2p$ times the value per pole, *i.e.* $(S_1/3)$ times the value per slot.

For the zig-zag leakage the flux may be supposed to travel as shown in Fig. 6.28. When the centres of two slots are opposite, there is no zig-zag leakage flux, and it reaches a maximum when the centre of a rotor tooth is opposite the centre of the stator slot under consideration. This occurs when the value of x is $\frac{1}{2}(y' - w_0 - w_0')$ which may be denoted by x_m.

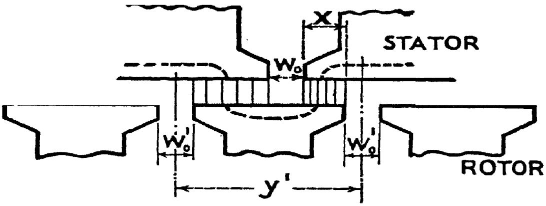


Fig. 6.28.—ZIG-ZAG LEAKAGE.

In the position shown in Fig. 6.28, the width of the leakage path on the left is $(2x_m - x)$. Hence:—

$$\text{Total permeance per cm. axial length} = 1/\{l_a/x + l_a/(2x_m - x)\}$$

$$= (1/l_a)\left\{\frac{x\,(2x_m - x)}{2x_m}\right\} = (1/l_a)\,\{x - x^2/2x_m\}.$$

By integration, the mean value for a movement of half a rotor slot-pitch is found to be*—

$$2x^2_m/3l_a y' = (y' - w_0 - w_0')^2/6l_a y'.$$

Since x_m cannot exceed half the width of the stator tooth, the formula must be modified when $(y' - w_0')$ is greater than the stator slot pitch y, and it then becomes:—

$$\text{Mean permeance per cm.} = (y - w_0)^2/6l_a y'.$$

---

* See *Alternating Current Machinery* (Hawkins, Smith and Neville, p. 343).

The calculation of the equivalent inductance per phase then proceeds just as for the slot leakage, the permeance calculated from one of the above formulae taking the place of λ_1. A further allowance must be made for leakage at the ends of the core. One method of doing this is given in the paper referred to above.

A similar set of calculations can then be carried out for the rotor. The formulae are the same, but the dimensions of the rotor slots must be inserted in place of those of the stator slots and vice versa.

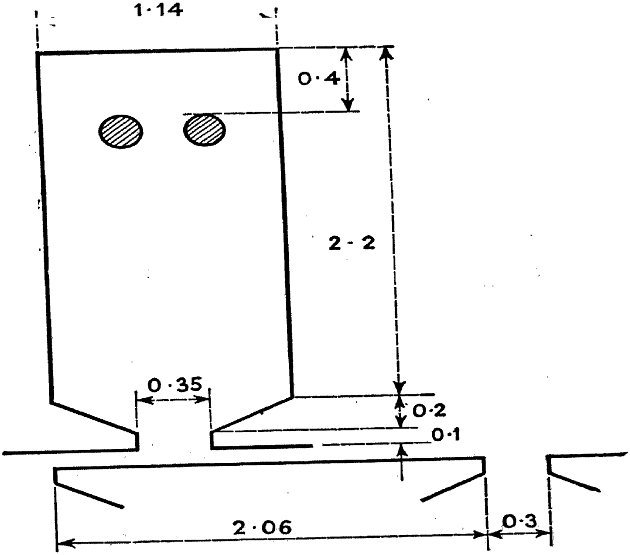


Fig. 6.29.—CALCULATION OF LEAKAGE INDUCTANCES. (Dimensions in cm.)

Example 6. *Calculate the inductance per slot for stator slot and zig-zag leakages for the cores shown in Fig. 6.29, if they are 14 cm. long with no ducts, and with 8 stator conductors per slot, and a squirrel cage rotor. The depth of the stator coils is 1·6 cm. over the copper. The pitch of the stator slots is 1·74 cm., and the air-gap is ·08 cm.*

(a) $(\frac{1}{3}h_1 + h_2)/w_1 = (\frac{1}{3} \times 1{\cdot}6 + 0{\cdot}2)/1{\cdot}14 = 0{\cdot}64.$

(b) $h_3 \log_e (w_1/w_0)/(w_1 - w_0) = 0{\cdot}2 \log_e (1{\cdot}14/0{\cdot}35)/(1{\cdot}14 - 0{\cdot}35)$

$= 0{\cdot}2 \times 1{\cdot}181/0{\cdot}79 = 0{\cdot}30,$

or approximately, $2h_3/(w_1 + w_0) = 2 \times 0{\cdot}2/(1{\cdot}14 + 0{\cdot}35) = 0{\cdot}27.$

(c) $h_4/w_0 = 0{\cdot}1/0{\cdot}35 = 0{\cdot}29.$

Total slot permeance per cm. $= 0{\cdot}64 + 0{\cdot}30 + 0{\cdot}29 = 1{\cdot}23.$

Permeance of half air-gap per slot per axial cm. $= (\cdot 08/2)/1{\cdot}74 = \cdot 02$, making the total $1{\cdot}25$ per cm.;

$\therefore$ Total permeance per slot $= 1{\cdot}25 \times 14 = 17{\cdot}5$;

$\therefore$ Leakage inductance per slot $= (4\pi/10) \times 8^2 \times 17{\cdot}5/10^8$
$= 14{\cdot}1 \times 10^{-6}$ henries.

Slot leakage inductance per cm. $= 14{\cdot}1/(1{\cdot}74 \times 10^6) = 8{\cdot}1 \times 10^{-6}$ henries.

Since the top of the rotor tooth is wider than the slot pitch, the second formula must be used in calculating the zig-zag leakage. Thus:—

Mean zig-zag permeance per stator slot per axial cm.
$= (y - w_0)^2/6l_a y' = (1{\cdot}74 - 0{\cdot}35)^2/(6 \times \cdot 08 \times 2{\cdot}36) = 1{\cdot}71$;

$\therefore$ Total mean zig-zag permeance per slot $= 1{\cdot}71 \times 14 = 23{\cdot}8$;

$\therefore$ Zig-zag leakage inductance per slot $= (4\pi/10) \times 8^2 \times 23{\cdot}8/10^8$,
$= 19{\cdot}1 \times 10^{-6}$ henries.

Dividing by $1{\cdot}74$,

Zig-zag leakage inductance per cm. $= 11{\cdot}0 \times 10^{-6}$ henries.

26. Output Coefficient

The output of an induction motor is settled roughly by its size and speed, much as in the case of D.C. dynamos (Volume I., Chapter X., Art. 14). Conversely, to obtain a given output at a given speed requires a motor of a fairly definite size: and as the speed is determined approximately by the frequency and the number of poles, the output alone settles the size roughly. This can be shown as follows.

The mean flux-density is settled within fairly wide limits by magnetic considerations, and generally lies between 3 and 6 kilolines per sq. cm. If it is taken as constant then the torque (see Art. 9) is proportional to $l . I_2N_2r_2 \cos \phi_2$.

Now $\cos \phi_2$ is larger than $\cos \phi_1$, and its value at full load will not vary much in different motors. Taking this too as constant, and putting in the diameter of the rotor in place of the radius, the torque becomes proportional to $l\, I_2N_2d$.

The product I_2N_2, *i.e.* the ampere-conductors, on the rotor is limited at full load by heating considerations, together with the provision of sufficient cross-section in the teeth to carry the flux. A change in the length has no effect as, apart from the end connexions and surface, both the resistance and the surface for dissipating heat are altered in the same proportion. An increase in the diameter likewise alters the surface for heat dissipation in proportion, but permits more conductors of the same current-carrying capacity to be used in direct proportion to the diameter.

If the number of conductors is altered, their cross-section must be changed in approximately inverse proportion to fit the same slots.

The resistance per conductor, therefore, is changed in the same proportion as their number, and the same applies to the copper loss per ampere. The total copper loss per ampere thus varies as N^2, and as the loss depends on I^2, the full-load value of I must vary inversely as N, *i.e.* NI remains constant.

Therefore, the permissible full-load value of NI varies directly as the diameter, and when this is inserted in the above expression for the torque, it reduces to:—Full-load torque is proportional to ld^2.

As the output depends on (torque) × (r.p.m.), the output is proportional to ld^2n, *i.e.*:—

$$P = k\,.\,ld^2n\,; \text{ or } P/n = k.ld^2$$

where k is the output coefficient; P = power developed by motor.

The method of derivation shows that k is only roughly constant. It increases with the size of motor, and the following table gives average values for various sizes.

OUTPUT COEFFICIENTS OF INDUCTION MOTORS

P = b.h.p. developed by motor; d = diam. of rotor;
n = r.p.m.; l = axial length of rotor core;
$k' = 10^3 \times P/ld^2n$ (cm. units); k'' = the same with inch units.

$10^3 \times P/n$	k'	k''	$10^3 \times P/n$	k'	k''
0·4	·0005	·008	40	·0017	·028
1·0	·00075	·012	100	·0019	·031
2·0	·0009	·015	200	·0021	·034
4·0	·0011	·018	400	·0023	·038
10	·0013	·021	1000	·00255	·042
20	·0015	·025	2000	·0028	·046

27. Approximate Values of Leakage Coefficient

Since the greater part of the reluctance in the circuit of the main flux is in the air-gaps, the magnetising current (I_m) varies approximately as δ, the radial length of the air-gap. It likewise varies directly as the flux-density and inversely as the number of stator-turns per pole.

Similarly the reluctance of the leakage paths is nearly proportional to the pole-pitch on the assumption that the effect of the teeth in reducing the reluctance along and parallel to the periphery (see Fig. 6.26, Art. 25) is a constant percentage. The equivalent ideal standstill current in the rotor, I_s, therefore varies approximately as τ, the pole-pitch. And, like I_m, it varies as the flux-density and inversely as the number of *stator* turns per pole (stator not rotor turns as it is the *equivalent* rotor current).

Moreover, the flux-density in the two cases is the same, since the flux rotates at synchronous speed whether it is leakage flux or not, and in both cases it must produce in the stator a voltage equal to the applied p.d. The effects of resistance are excluded since I_m is the true magnetising current and I_s the ideal standstill current. Now:—

$$\text{The leakage coefficient } \sigma = I_m/I_s \text{ (Art. 15)};$$

$$\therefore \ \sigma \text{ is proportional to } \delta/\tau;$$

$$\textit{i.e.} \quad \sigma = c\delta/\tau.$$

The value of c (Behrend's constant) lies between 12 and 15 for partially closed stator slots and completely closed rotor slots (tunnels).

A closer approximation can be obtained by taking other factors into account. Dr. A. S. McAllister (*Alternating Current Motors*, pp. 309-311) has developed the following formula from H. M. Hobart's experimental results:—

$$c = (10{\cdot}5 - 5{\cdot}5\,S_0 + 2{\cdot}9\tau/l)\,(0{\cdot}54 + {\cdot}247/\delta h),$$

where S_0 = average of stator and rotor fractional slot openings (*i.e.* opening/slot width),

l = core length — ducts,

h = average number of stator and rotor slots per pole per phase.

If dimensions are in inches the constant ·247 must be replaced by ·097, but otherwise the formula is unaltered.

This formula compared with Behrend's simpler one has the advantages of closer accuracy, and of being applicable to open, closed, and partially closed slots.

Dr. Hans Behn-Eschenburg has developed a different formula which is equally applicable to slots of any form. This is as follows:—

$$\sigma/(1 - \sigma) = (1/3h^2) + (\delta/w_0 h\tau) + (6\delta/l),$$

where w_0 = average width of slot openings in cm., and the other symbols have the same meanings as before, all dimensions being in centimetres.

28. Deductions from Leakage Coefficient

Behrend's approximate value for the leakage coefficient shows the importance of keeping the air-gap small. This is limited partly by mechanical considerations, and partly by the fact that with very small gaps slight wear of the bearings makes a large percentage difference between the gaps at top and bottom. This results in unequal flux-densities and consequently unequal currents and mechanical forces. By the use of ball-bearings air-gaps of 0·4 mm. can be employed in small motors.

The drawback of a large air-gap is that it reduces the maximum power-factor (see Table, Art. 15). This reduces the output for a given current, and as the heating limit fixes the maximum current, the maximum output is reduced (see Example 6 and Fig. 6.30).

Another case readily dealt with on the same lines is the winding of a given frame for different numbers of poles. The greater the number of poles the shorter the pole-pitch, and the leakage coefficient therefore varies approximately as the number of poles.

But an increase in the air-gap has a different effect from an increase in the number of poles. For the former increases the magnetising current and leaves the standstill current practically unaffected. The change in the poles on the other hand does not affect the magnetising current since the flux-density will not be altered. As the flux rotates at a speed inversely proportional to the number of poles, the total number of stator conductors in series must vary directly as the number of poles to produce the necessary induced e.m.f.

The number of stator conductors in series per pole is therefore unchanged. Consequently, the standstill current varies as the reluctance of the leakage path, *i.e.* as the pole-pitch approximately, and so varies inversely as the number of poles. Thus the diameter *of the circle diagram varies inversely as the number of poles* (see Example 6 (*b*) and Fig. 6.31), and consequently the maximum input varies in this way too. The full-load input likewise varies approximately inversely as the number of poles, owing to the increase in the number of stator conductors necessitating a decrease in their section and therefore in their current-carrying capacity.

The above may be proved also by noting that the torque is unchanged since it depends on flux-density and rotor current. The speed varies inversely as number of poles. The output depends on torque × speed, and so varies inversely as number of poles. Reference to the formula for output (Art. 26) furnishes another alternative proof.

A large motor cannot be built with a very small number of poles, as the peripheral speed would be too high with the best proportion of diameter to length. The number of poles used for motors for supplies at 50 cycles per sec. seldom exceeds four below 40 b.h.p., unless a lower speed is desirable for the required drive.

Example 7.—*A four-pole motor has its maximum power-factor of* 0·85 *at its normal full load of* 10 *h.p. Examine the effect of* (*a*) *increasing the air-gap by* 50 *per cent.,* (*b*) *winding the original frame for six poles.*

Since $0{\cdot}85 = \cos\phi_m = 1/(1 + 2\sigma)$ (see Art. 15)

$$\sigma = 0{\cdot}15/(2 \times 0{\cdot}85) = {\cdot}088.$$

Each of the changes increases σ to $1{\cdot}5 \times {\cdot}088 = {\cdot}132$. Hence:—

New value of $\cos\phi_m = 1/(1 + 2 \times {\cdot}132) = 0{\cdot}79$.

(*a*) The new full load is approximately $(0{\cdot}79/{\cdot}85) \times 10 = 9{\cdot}3$ h.p., but is rather less than this for two reasons.

The new full load does not occur at the new maximum power-factor, but nearer to the same stator current as before. Thus, if OP is the original full-load current, touching the circle in the circle diagram (Fig. 6.30), the new position of O is O where O′B = 1½ OB. If O′P′ = OP, O′P′ will not touch the circle, and so gives less than the maximum power-factor for the new conditions.

If the diagram is drawn to scale as in the figure, it will be found that the power-factor for O′P′ is 0·78. This, however, gives a smaller rotor current than in the original case, and so the new full-load power-factor may be taken as 0·79. And as the new stator current is somewhat increased it is nearly correct to take the new input as 93 per cent. of the original output, *i.e.* to neglect the first reason for reducing the new full-load below the value calculated above.

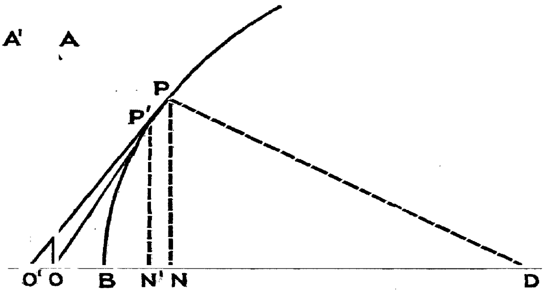


Fig. 6.30.—EFFECT OF INCREASE OF AIR-GAP.

The second reason is that the losses are the same as before. Thus, if the original efficiency is 88 per cent., the losses are still 12 per cent. of the *original* input. Therefore, the new output is (93 per cent. − 12 per cent. =) 81 per cent. of original input, and therefore is $(81/88) \times 10 = 9{\cdot}2$ h.p.

The new efficiency is $(81/93) \times 100$ per cent. = 87 per cent.

The maximum torque is unaltered since the radius of the circle diagram is unchanged.

(*b*) In this case, if BPC is the original circle diagram and BP′C′ the new one, BC′ = ⅔ BC, and the position of O is unchanged.

The maximum *torque* is again unaltered because the length representing a given torque for the 6-pole is only ⅔ of that for the 4-pole, since the synchronous speed varies inversely as (No. of poles).

The full-load current must be reduced to about ⅔ of its original value, and so the full-load input is $(\tfrac{2}{3}) \times (0{\cdot}79/0{\cdot}85) \times 100$ per cent. = 62 per cent. of original value.

The losses still remain nearly at the original value since the stator resistance is increased in the ratio $(\tfrac{3}{2})^2$, as there are 50 per cent. more conductors, and

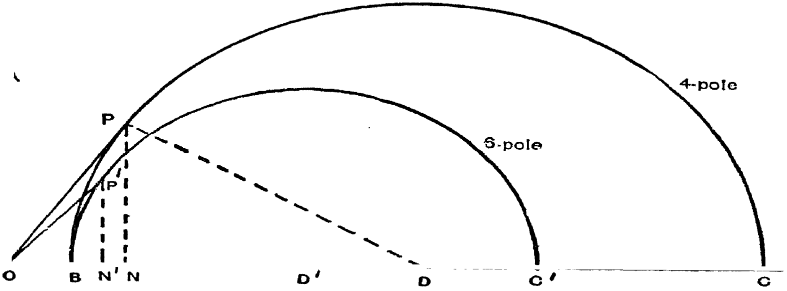


Fig. 6.31.—EFFECT OF INCREASING NUMBER OF POLES.

their cross-section must therefore be $\frac{2}{3}$ of original value. The rotor loss is likewise unchanged since the torque and flux-density, and therefore the rotor current, are unaltered. The friction loss is only $\frac{2}{3}$ as great as before, but 1 per cent. is ample allowance for this reduction.

Thus, new output = (62 per cent. − 11 per cent.) = 51 per cent. of original input, and is therefore equal to (51/88) × 10 = 5·8 h.p.

The fact that the new full-load current (OP′ in Fig. 6.31) does not occur at maximum power-factor has been neglected for the reasons given under (*a*).

The new efficiency is (51/62) × 100 per cent. = 82 per cent.

29. Effect of Altering the Frequency

If the frequency is changed without altering the number of poles it leaves the leakage coefficient unchanged. As the speed of rotation of the flux varies as the frequency the number of stator conductor must vary inversely as the frequency to produce the same e.m.f. with the same flux.

The result of lowering the frequency is, therefore, to reduce the output in a manner similar to that caused by an increase in the number of poles. The effect is not so great, however, since the power-factor is not lowered; *e.g.* if the frequency is reduced from 60 cyc. to 40 cyc. and the same number of poles used, the full-load input is reduced to $\frac{2}{3}$ of the original value.

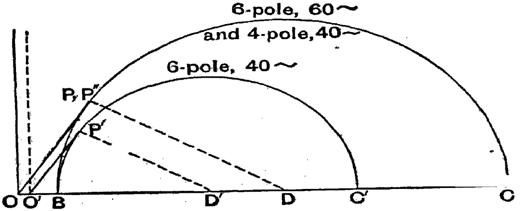


Fig. 6.32.—EFFECT OF REDUCING FREQUENCY.

OP. Full load current, and D centre of semicircle for 6-pole 60 cyc. motor. O′P′ and D′ ditto for 6-pole 40 cyc. O′P″ and D ditto for 4-pole 40 cyc. O′B = $\frac{2}{3}$ O B.

The copper losses are unaltered, but the friction losses are reduced to $\frac{2}{3}$ of previous value, and the iron losses to less than this. The efficiency is, therefore, reduced but slightly.

On the other hand, unless the original motor is a 2-pole one, the reduction of frequency may allow the use of fewer poles without causing excessive speed. In this case, the leakage coefficient will be reduced and the power-factor improved.

If the number of poles is reduced in the same ratio as the frequency, the speed is unchanged and so is the total number of stator conductors. Hence, the magnetising current is reduced in proportion to the frequency, as there are more turns per pole. The diameter of the circle is unchanged: and so the effect is the same as reducing the air-gap in proportion to frequency. Both cases are illustrated in Fig. 6.32.

30. Starting Methods

The best method of starting an induction motor is by increasing the resistance in the rotor circuits. This enables any torque up to the maximum to be obtained at the start (see Art. 11). If the resistance is variable the maximum torque can be obtained in addition at intermediate speeds as the motor's speed increases. By the same means the current during starting can be kept below any required value.

The drawback to this method is the extra cost and complication of providing the motor with brushes, slip-rings and short-circuiting switch, and gear for lifting the brushes after the slip-rings have been short-circuited.

It is important that the added resistance be as nearly non-inductive as possible, since any addition to the reactance diminishes the torque. This can be ensured by using a cast-iron grid or a stamped metal grid [see Fig. 6.33 (*a*)]. The central corrugation and the three-point suspension of the latter produce a rigid rheostat.

Another type is shown in Fig. 6.34. It is made by bending a strip of resistance metal. This gives a low reactance, and permits of expansion when heated by the starting current without risk of short-circuits.

For large motors liquid resistances are used frequently (cf. Volume I., Chapter IX., Art. 8). A rheostat of this sort is shown in Fig. 6.35.* The variation of the resistance is effected by a centrifugal pump driven by an electric motor. This pumps the liquid from the lower tank into the upper one. The acceleration is

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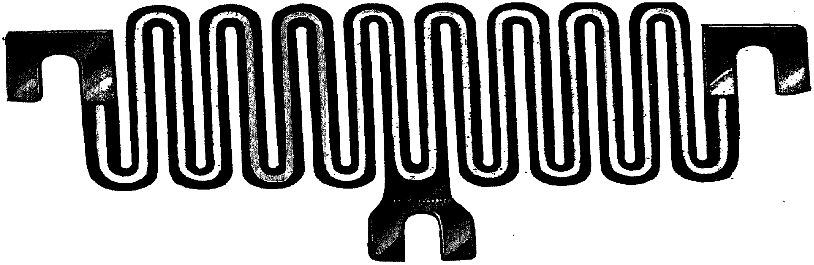
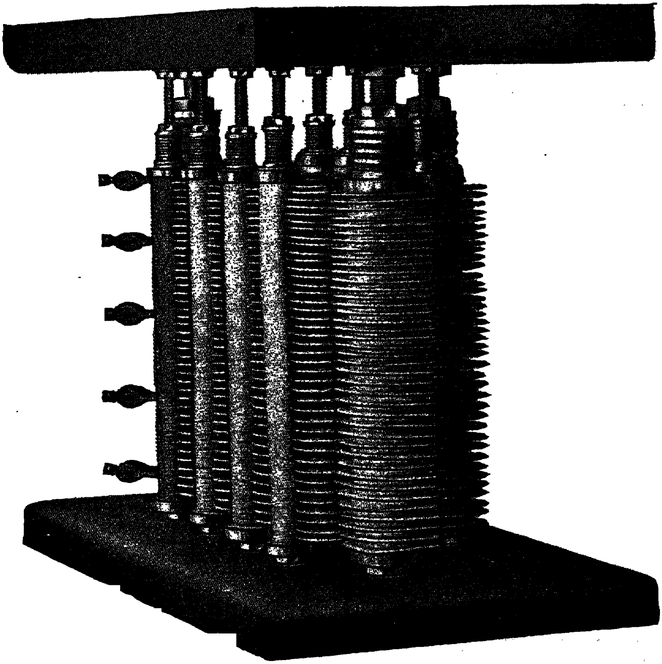
* From *J.I.E.E.*, vol. 60, p. 205.



(*a*) ALLEN WEST & CO.'S STAMPED GRID.



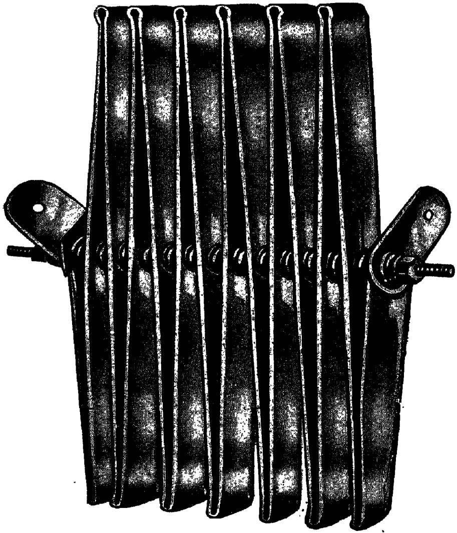
(*b*) ALLEN WEST & CO.'S ASSEMBLED RESISTANCE.

Fig. 6.33.—ROTOR STARTING RHEOSTATS.

graded by using three different lengths of fixed electrodes. When the upper tank is full the starting lever is thrown into the running position. This short-circuits the rheostat by an oil switch and stops the pump motor. The liquid then runs back into the lower tank ready for a fresh start. The lever is locked in the starting position until the motor has reached a certain speed.

When a short-circuited rotor is used, the following methods of starting are available:—

(*a*) direct switching;
(*b*) stator resistance;
(*c*) auto-transformer;
(*d*) star-delta or series-parallel switch.



CLARKE CHAPMAN & CO.'S "CRACKER" GRID

Fig. 6.34.—ROTOR STARTING RHEOSTATS.

Direct switching onto the mains is more feasible than with D.C. motors because the induction motor's starting current is relatively smaller. This is due to the equivalent reactance of the induction motor being much greater at the start than when it is running. Nevertheless, the method is suitable only for small motors, up to about 2 h.p.

In the remaining three methods the p.d. applied to the stator is reduced during starting. The stator current is thereby reduced in the same ratio as the p.d. But since the field strength is reduced nearly in the same ratio the torque is reduced approximately as the square of the p.d. Hence, in order to obtain sufficient starting torque, particularly if the motor has to start under load, the rotor resistance must be larger than in a similar wound rotor. This results in a lower efficiency, and a greater variation of running speed with change of load.

The advantages of the stator resistance method are that it is simple and cheaper than an auto-transformer for small motors. Moreover, as the speed rises the stator current falls, and so the p.d. on the stator increases. This gives smoother starting than methods (*c*) and (*d*).

On the contrary, with an auto-transformer the line current is less than the stator current, and, as it is chiefly the former that must be limited, the auto-transformer enables a larger starting torque to be developed by the same motor.

E.g. if the line current is to be limited to half the value that direct switching gives, a stator resistance must reduce the p.d. to 50 per cent. of full value, and the torque to 25 per cent. of value for full p.d. With an auto-transformer the stator p.d. need only be reduced to 71 per cent. ($100/\sqrt{2}$) of full value, and so the torque to half its value with full p.d. The stator current will be 71 per cent. of full p.d. value, but the line current will be only 71 per cent. of the stator current, *i.e.* 50 per cent. of line value with full p.d.

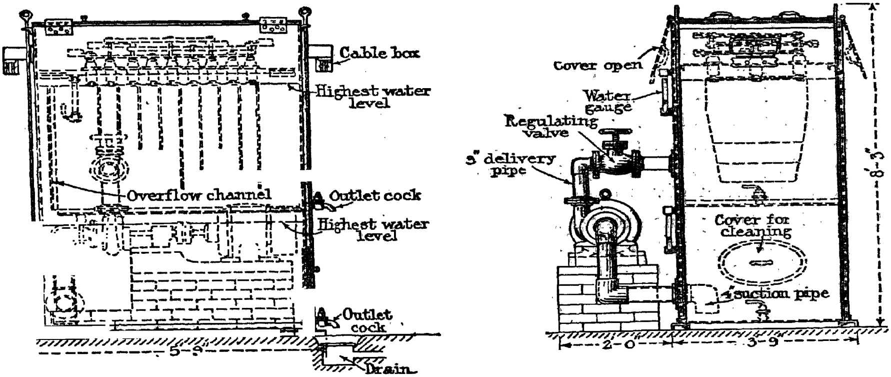


Fig. 6.35.—LIQUID STARTING RHEOSTAT FOR 3,000 B.H.P. MOTOR.

The efficiency during starting is likewise improved, but this is of less importance, as starting occupies only a small fraction of the time the motor is in use.

The last method consists of a switch, by means of which the windings of a three-phase motor can be connected in Y (star) for starting, and in Δ for running. The effect of this is to reduce the p.d. per phase at the start to 58 per cent. ($100/\sqrt{3}$) of the normal value, and the torque to one-third of its value with full p.d.

The current per phase is reduced to 58 per cent. of its value with full p.d. With Δ-connexions the line current is 1·73 ($= \sqrt{3}$) times the phase current, and with Y-connexions the line and phase currents are equal, hence the line current with Y-connexions is only one-third of the current with Δ-connexions.

The effect of the star-delta switch is, therefore, the same as that of an auto-transformer with its secondary voltage 58 per cent. of the full p.d. The switch has the added advantages of being cheaper, and of avoiding the losses which occur in an auto-transformer.

The disadvantages of the switch are:—

(*a*) It can give only 58 per cent. of full p.d., whereas the auto-transformer can give any ratio. A smaller ratio may be desirable to effect a greater reduction in starting current, or a greater ratio to give more starting torque for starting under load.

(*b*) It requires six terminals on the motor instead of three, and six leads between motor and switch.

(*c*) The motor must be connected in Δ for running, and this means more stator conductors, though of smaller cross-section. Their insulation will occupy more space than that of the fewer larger conductors, and so the output of the frame has to be diminished somewhat, and the cost of winding is increased. This is of less importance in large motors, particularly when so large that the Y-connexion gives conductors too large for single bars to be used

For two-phase motors a method giving similar effects is to wind each phase in two halves. These are connected in series for starting and in parallel for running.

Another way of starting, which is not often available, is to connect the motor to the alternator before the latter is started. If the excitation is on during the running up to speed of the alternator, the motor will start while the alternator's speed is low. The two will rise in speed together, and the motor current will never be excessive.

31. Connexions of Star-Delta Switch

The changes of connexions necessary for this method of starting could evidently be effected by a six-pole two-way switch, one " pole " for each terminal of the motor. The number of " poles " can be reduced to four, as is shown in Fig. 6.36 (*c*).

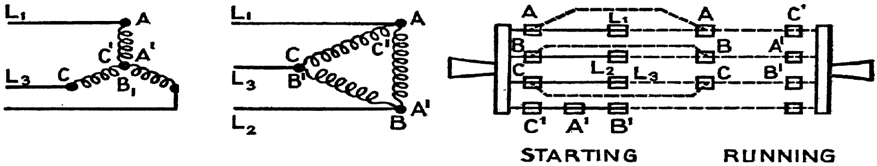
This represents a four-pole throw-over switch. The lines are connected to three of the hinge connexions, and the remaining connexions are indicated by letters. It will be seen that in the starting position the switch connects a line to one end of each phase, and the fourth blade of the switch connects the other three ends together, thus forming the Y-connexion shown at (*a*).

On throwing the switch over to the running position each line is connected to two terminals forming the Δ-connexion, as shown at (*b*). A dummy contact is provided as a guide to the fourth blade.

Each terminal of the motor is connected to two contacts of the switch. These pairs are connected together, so that it is necessary to run only six leads from switch to motor. It is important to have the connexions such that the phase rotation is correct on changing over; and to keep the change in phase of the p.d. across each winding to the minimum of 30°.

If the fuses are connected between pairs of contacts AA, BB, CC, and the leads to the stator are run from the three left-hand ones, the fuses are out of circuit during the starting period.

The switch is often made in the controller pattern, but the connexions are similar. It is preferable to place the star position between the off and running positions, so as to avoid the possibility of switching direct to the latter by mistake; or an interlock may be used which prevents the occurrence of this error. A magnetic catch and a spring to return the switch to the off position if the supply fails further increase the safety.



(*a*) Y-CONNECTED. (*b*) Δ-CONNECTED. (*c*) SWITCH CONNEXIONS.

Fig. 6.36.—CONNEXIONS OF STAR-DELTA STARTING SWITCH.

N.B.—The two points marked C′ in (*c*) are permanently connected together; the same applies to the two marked A′, and the two marked B′. By placing fuses between A and A, B and B, C and C, and running the motor leads from the left-hand three, the motor is protected when running, and there is no liability of the fuses melting during starting.

32. Connexions of Auto-Transformer Starter

The necessary operations in using this type of starter are:—

(*a*) Connect primary windings to lines.

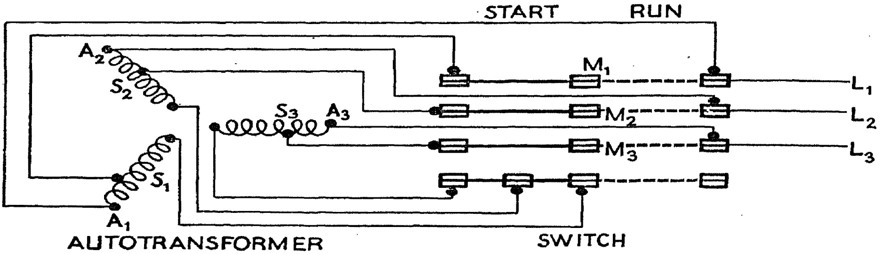
(*b*) Connect secondary portions of windings to motors.

The above two operations give the starting connexions.

(*c*) Disconnect transformer windings from lines.

(*d*) Connect motor to the lines.

These two operations give the running connexions.



A_1 A_2 A_3, Auto-transformer terminals.
M_1 M_2 M_3, Terminals connected to motor.
L_1 L_2 L_3 Supply lines.
S_1 S_2 S_3 Secondary tappings.

Fig. 6.37.—AUTO-TRANSFORMER STARTER CONNEXIONS.

In order to simplify the switch for effecting the above *either* the connexion of the primary windings (A) to the lines, *or* the motor connexions to the secondaries (S), are not broken, but current is prevented from flowing in the auto-transformer windings in the running position by breaking their star-point connexions.

The connexions are then as shown in Fig. 6.37 for the first of the above methods. The four-pole throw-over switch makes all the required connexions, and, as before, there is a dummy contact as a guide for the fourth blade in the running position.

For the second method, the M and L connexions are interchanged, and so are the A and S connexions. This has the advantage that in the off position the auto-transformer is disconnected from the lines.

33. Speed Regulation

Speed regulation can be effected by using a rotor resistance capable of carrying the full-load rotor current continuously. As shown in Art. 11, the effect of this is to increase the slip in proportion to the resistance for any given torque. Thus if the full-load slip is five per cent. (*i.e.* full-load speed 95 per cent. of synchronous speed), a reduction of 20 per cent. in speed (*i.e.* to 76 per cent.) requires a slip of 24 per cent., and so an increase of rotor circuit resistance in the ratio 4·8 (*i.e.* five per cent. to 24 per cent.).

Since the rotor current remains the same for the same torque, the stator current and input are unaltered. Therefore the efficiency is reduced in the same ratio as the speed, since the h.p. varies as the latter.

Moreover the variation of speed with load is increased; *e.g.* in the above case at a torque requiring normally three per cent. slip, the same torque is produced with the resistance in circuit at a slip of $4 \cdot 8 \times 3$ per cent., *i.e.* 14·4 per cent. Between these loads, therefore, the change of speed is 9·6 per cent. of synchronous speed against a normal change of 2 per cent. Or stated generally, the variation of speed with load increases in the same ratio as the rotor resistance.

These results are similar to those obtained with resistance in series with the armature of a shunt-wound D.C. motor (see Volume I., Chapter XI., Art. 9). An effect similar to that obtained with a field rheostat in a shunt-wound motor, cannot be obtained in the induction motor except by altering the number of poles (see Chapter VII., Art. 10), and even then only partially.

34. Cogging Action

If the stator and rotor had the same number of teeth the magnetic attractions between these would tend to bring them into line. Consequently when starting at reduced p.d. the torque might be insufficient to overcome the magnetic attractions and the frictional resistance. As the p.d. is increased the torque and magnetic attraction increase as $\mathbf{B}^2$, but the friction does not change, and so the motor may start, but only sluggishly.

To avoid this, the numbers of teeth in stator and rotor should have no common factor. The relative position of the teeth at different points of the periphery will then vary. Consequently the magnetic attractions will vary both in strength and direction in such a way that there is no resultant torque due to them, the pulls in one direction being balanced by others in the opposite direction.

In a short-circuited rotor the number of slots is often made a prime number. This can then be used with various numbers of stator slots without any cogging action.

In a wound rotor the number of slots must be divisible by the number of phases, which is usually three. It is generally divisible also by the number of poles. The number of stator slots in the usual type of winding is likewise divisible by (No. of phases) × (No. of poles). This product is, therefore, a common factor of the numbers of stator and rotor slots.

But if the slots per phase per pole in the stator and in the rotor respectively differ by 1 (or, more generally, have no common factor), very little cogging action will take place. Moreover, the starting torque of a slip-ring rotor can be made so much higher than that of a short-circuited one that this action is less troublesome in motors with slip-ring rotors.

35. Crawling

Occasionally a motor will start, but fail to rise to full-speed. This is due to harmonics in the field distribution, caused by the arrangement of the stator coils, and by the irregularities introduced by the teeth in the magnetic circuit. The most usual " crawling " speed of a three-phase motor is approximately $\frac{1}{7}$th of synchronous speed. The explanation of this phenomenon is as follows.

If the graph of m.m.f. due to one phase against electrical angle (cf. Fig. 6.24) contains harmonics, as it always does, they produce the following effects. The fifth harmonic by itself produces five times as many poles as the fundamental. But the electrical angle between two successive windings is 5 × 120°, *i.e.* 600°, referred to this harmonic. This amounts to a double pole-pitch plus 240°; and two such steps give a phase difference in space of 480°, which is equivalent to 120°.

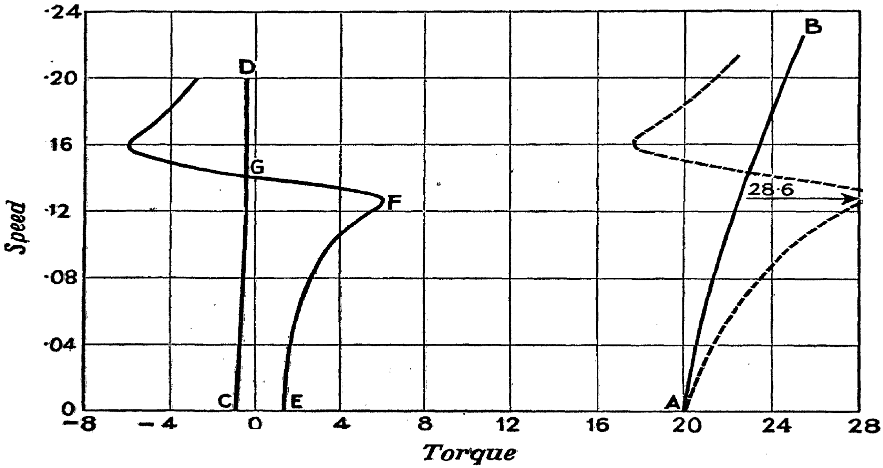


Fig. 6.38.—FUNDAMENTAL AND HARMONIC TORQUES.

Thus the relative space relation of the three phases for the fifth harmonic is the reverse of that for the fundamental, and so the rotating field produced by the fifth harmonic components of the three m.m.f.s rotates in the opposite direction to that of the main field, and at a speed one-fifth of the main field's speed.

In the case of the seventh harmonic components the angle becomes 7 × 120°, *i.e.* 840°, which is equivalent to two double pole-pitches plus 120°. So the seventh harmonic field rotates in the same direction as the main field, but at only one-seventh of its speed.

With the rotor at rest the fifth harmonic field produces a torque opposing that due to the main field. As the rotor speeds up in the direction of the main field's rotation the slip relative to the fifth harmonic field *increases*, and so its torque diminishes, becoming approximately half its initial value at one-fifth of fundamental synchronous speed, since this gives 200 per cent. slip relative to fifth harmonic speed.

The initial torque due to the seventh harmonic field assists the main torque, and increases at first as the rotor speed rises since the slip is diminishing. It reaches a maximum, and then diminishes to zero at one-seventh of fundamental synchronous speed, *i.e.* at its own synchronous speed. Above this speed its torque becomes negative (see Chapter VII., Art. 9).

The combined effect of three such torques at low speeds is shown in Fig. 6.38. The torque due to the fundamental is shown by AB, its maximum value is 100 at a slip of 0·1; and its starting value just under 20. The negative torque due to the 5th harmonic is shown by CD diminishing from 0·8 at the start to 0·4 at speed of 0·2. The 7th harmonic torque starts at a value of 1·2 at E, increases to 6 at a speed of nearly 0·13 (F), becomes zero at $\frac{1}{7}$th of synchronous speed (G), and reaches its maximum negative value of 6 at a speed of nearly 0·16. The sum of the three torques is shown by the broken line. It will be seen that if the resisting torque is between 18 and 20 this motor will start, but will settle down to a speed round about 0·15. If the resisting torque increases somewhat with speed the same result will follow even with a smaller 7th harmonic torque, but the crawling speed will be nearer to $\frac{1}{7}$th of synchronous speed.

The third harmonic has no such effect since its phase difference is 3 × 120°, *i.e.* 360°. This means that the poles coincide, and as the vector sum of three 3-phase currents is zero there is no resultant field. The same applies to all triple harmonics.

The other higher harmonics produce results similar to those of the 5th and 7th harmonics: viz. 11th, 17th, etc., produce negative torques diminishing with speed; and 13th, 19th, etc., produce positive torques increasing to a maximum and then reversing. But their effects are generally very small.

QUESTIONS ON CHAPTER VI

1. Show that three-phase currents can produce a rotating magnetic field of constant magnitude, and of maximum intensity equal to 1·5 times that of the maximum of each phase.

2. Plot the torque-speed curve of an induction motor taking

$$R_2 = 5 \times 10^{-3}, \text{ and } X_0 = 4 \times 10^{-2}.$$

Find the speed (expressed as a fraction of the synchronous speed) at which the torque is a maximum.

In what ratio must R_2 be increased to obtain the maximum starting torque? Draw a second torque-speed curve with the new value of R_2.

3. Prove from first principles that the torque of an induction motor is proportional to the slip near synchronism, and nearly inversely proportional to it at starting.

4. A 50 h.p. three-phase induction motor has a no-load speed of nearly 750 r.p.m. and a full-load slip of 2·5 per cent. The maximum torque is 2½ times the full-load torque. Draw to scale a torque-speed curve for all speeds from stand-still to 750 r.p.m.

5. Draw to scale torque-speed and current-speed curves for an induction motor with an equivalent rotor resistance of 0·4 and rotor reactance of 1·7. Draw two other curves showing these relations when the resistance in the rotor circuit has been brought up to 1·2 ohms.

6. Explain what is meant by the circle diagram for an induction motor. What tests must be made in order to enable the circle diagram for a motor to be drawn? Show how (*a*) torque, (*b*) maximum power-factor, (*c*) slip, for such a motor may be found from the diagram. [C. & G., II.

7. From the following test results on a 5 h.p. three-phase 4-pole motor for 200 line volts 50 cyc., draw its circle diagram and deduce from it the maximum power-factor, the efficiency with this power-factor, and the maximum torque and its ratio to full-load torque.

	Amp. per phase	Line volts	Cos ϕ
At no load	5·0	200	0·2
With rotor fixed ..	14	35	0·6
	22	55	
	33	80	

Stator resistance = 0·88 ohm, between terminals.

8. A 50 b.h.p., 500 volts, three-phase induction motor took:—

At no-load: 500 volts, 16 amperes, 1 800 watts.
At standstill: 91 volts, 53 amperes, 2 400 watts.

Draw its circle diagram and find its maximum output, and the ratio of torque at this output to full-load torque. Assume the copper losses to be divided equally between stator and rotor.

9. The following readings were taken on a 16 h.p., 6-pole, three-phase induction motor for a supply at 400 line volts, 50 cycles per sec.:—

	Line volts	Line amperes	Power-factor
No load	400	10·8	0·24
Standstill	100	22·4	0·52
	150	35·3	0·55

The stator is Y-connected and has a resistance of 0·7 ohm per phase. Construct the circle diagram, and from it find the current, power-factor and torque at full-load, and the ratio of the maximum torque to this torque.

10. A 450 h.p., 8-pole, three-phase 2000-volt 50 cyc. induction motor gives on test the following figures:—

Running light at 2 000 volts:—
Current per phase 48 amperes; total power 17 300 watts.
Standstill test at 500 volts:—
Current per phase 160 amperes; total power 12 700 watts.

The resistance per phase of the star-connected winding is 0·086 ohm.

Draw the circle diagram of this motor; and calculate how much resistance must be connected to the rotor to make it give full-load torque at the start, if the rotor has one-third as many bars as the stator.

11. What is meant by the equivalent resistance of a loaded motor? Find that of a three-phase induction motor in which the line current is 55 amperes at 290 line volts when developing 30 brake horse-power. Hence find the value of the rotor starting resistance to give full-load torque at starting, the rotor volts being 200 across the slip-rings on open circuit.

12. A three-phase induction motor with a Y-connected rotor winding produces 55 volts between its slip-rings on open-circuit with the rotor stationary and normal pressure applied to the stator terminals. Each phase of the rotor winding has a resistance of 0·7 ohm and a standstill reactance of 5 ohms.

Calculate the current per phase in the rotor:—

(*a*) when stationary and joined to a Y-connected rheostat having 4 ohms resistance per phase;

(*b*) ditto with 4 ohms resistance and 2 ohms reactance per phase;

(*c*) when running with 5 per cent. slip, and slip-rings short-circuited.

13. The no-load current of a three-phase induction motor is 1·35 amperes, and the stand-still current 16·0 amperes. The power-factor at no-load is 0·4 and at stand-still 0·3. Find by drawing curves of torque and slip the slip at which the torque is a maximum.

(Make a reasonable assumption as to the division of copper losses between the stator and the rotor.)

14. Draw diagrams of the flux distribution in the air gap of a three-phase induction motor, assuming a sinusoidal applied voltage. What is the effect of three phases as compared with one? Calculate the ratio of the maximum to the average value of the resultant flux density, also the percentage variation of the maximum flux density throughout a cycle.

15. Give a reliable formula for the dispersion co-efficient of an induction motor and show how it can be applied in the predetermination of a set of performance curves for the motor. [C. & G., Final, A.

16. Upon what factors does the leakage coefficient of an induction motor mainly depend?

If the leakage coefficient of a certain motor is ·04, and a similar motor is built with double its air-gap, estimate the maximum power-factors of the two motors, the relative values of their full-load currents and their overload capacities.

Assume that each motor has its maximum power-factor at full-load.

17. A 6-pole $7\frac{1}{2}$ h.p. motor has its maximum power-factor of 0·81 at full-load, when its efficiency is 85 per cent.

Estimate the maximum power-factor, the full-load, and the full-load efficiency:—

(*a*) if the air-gap is reduced by a quarter;

(*b*) if with the original air-gap the motor is wound for 4 poles.

18. A 6-pole 10 h.p. 60 cyc. motor has its maximum power-factor of 0·83 at full-load, when its efficiency is 85 per cent.

Estimate its maximum power-factor, and full-load output and efficiency, if it is rewound for 40 cyc.;

(*a*) with 6 poles;
(*b*) with 4 poles.

19. Discuss the relative advantages for starting a three-phase squirrel-cage induction motor of (*a*) a stator rheostat, (*b*) a star-delta switch, (*c*) an auto-transformer.

20. Give diagrams of connexions of switches for starting induction motors by the star-delta and auto-transformer methods.

State their relative advantages, and also those of the rotor resistance method.

21. Describe a method of starting suitable for each of the following three-phase induction motors:—

(*a*) one of 20 h.p. driving a line of shafting;
(*b*) one of 35 h.p. with a squirrel-cage rotor driving a D.C. generator;
(*c*) one of 50 h.p. driving a fan.

22. If the open delta or V-connexion is used for an auto-transformer starter, show by a diagram how the starting switch should be connected.

23. Calculate the rotor leakage inductances (slot and zig-zag) for the motor of Example 5 (Art. 25) given the following further dimensions:—

Width of rotor slot = 1·1 cm.
Depth of parallel portion = 1·1 cm.
,, ,, tapered portion = 0·2 cm.
,, ,, roof portion = 0·1 cm.

Distance from bottom of slot to nearest point of conductor 0·1 cm.

24. Draw the torque-speed graph for a three-phase induction motor whose standstill rotor reactance is eight times its resistance, and which has a seventh harmonic in field distribution producing a maximum torque = 4 per cent. of maximum torque due to fundamental.

Can crawling occur with this motor if the resisting torque is constant?

If the resisting torque at the start is 23 per cent. of maximum torque due to fundamental, and increases to 28 per cent. at 20 per cent. of synchronous speed, find the crawling speed.

CHAPTER VII

SINGLE-PHASE INDUCTION MOTORS, ETC.

1. Single-Phase Induction Motors

If one lead of a three-phase induction motor is disconnected while the motor is running, it will continue to run ; it will then be operating as a single-phase, or monophase motor. If the stator is Y-connected it will carry current in two of the three phase windings. These phases are in series and so carry the same current, and since a single-phase current is incapable of producing a rotating field by itself the method of action is not very evident. It is actually due to the combined action of the alternating stator currents and the induced currents in the rotor producing a field which is partly a rotating one.

The action can be connected up with that described in the case of polyphase induction motors by analysing the alternating field into two oppositely rotating ones (see Art. 2).

The monophase motors are, however, incapable of starting without the use of a second (starting) winding on the stator. This has to be supplied with a current out of phase with the main stator current (see Art. 7).

The construction is similar to that of polyphase induction motors, except that the rotor-winding is nearly always a short-circuited one. The reason for this is that increase of rotor resistance during starting is of much less advantage than in polyphase motors (see Art. 4).

2. Analysis of Alternating Field

A field whose direction is fixed and whose magnitude changes sinusoidally with time is equivalent to two fields, of equal constant magnitude, rotating in opposite directions at equal speeds. The strength of each of the rotating fields is half the maximum value of the alternating field.

For let OA and OB in Fig. 7.01 represent two fields of equal strength rotating in opposite directions at equal speeds. And let OC be the direction in which OA and OB acted when they coincided. Since their speeds of

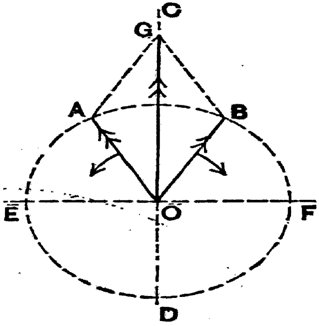


Fig. 7.01.—Analysis of Field.

rotation are equal they will reach OC again at the same instant. They will likewise both reach OD at the same instant, where OD is CO produced.

Draw EOF at right angles to COD. Owing to the equality of the speeds of OA and OB, $\angle$ COA = $\angle$ COB; therefore $\angle$ EOA = $\angle$ FOB = a (say). And the resultant of OA and OB bisects the angle AOB, *i.e.* it lies along OC or OD, because OA and OB are equal.

The magnitude of this resultant = 2 OA cos AOC = 2 OA sin EOA = 2 OA sin a, and a changes uniformly with time; *i.e.* the resultant field is in the constant direction OC, or its opposite OD, and its magnitude varies sinusoidally with time.

Naturally the actual field in a single-phase motor does not act merely along a single line. But if its space-distribution in the air-gap is sinusoidal it may still be represented by a line of varying length in the fixed direction COD where it is always at its momentary maximum. Similarly, OA and OB represent two fields, each of which has a constant strength, but whose point of maximum strength rotates round the air-gap at constant speed (see Chapter VI., Art. 5).

The two component rotating fields evidently rotate once for each complete cycle of the alternating field.

3. Application to Single-Phase Motor

If the alternating field due to the stator is split into two equal rotating fields as described, the rotor currents may be considered as made up of two components each due to one of the rotating fields. The resulting reactions are then the same as in polyphase motors, and the results of the investigations of Chapter VI. may be applied to the monophase motor.

When the rotor is running with a fractional slip s relative to one of the rotating fields it is rotating in the opposite direction to that of the second rotating field. Its fractional slip (s') relative to this is $(2 - s)$, *i.e.* except when $s = 1$, the second slip is greater than the synchronous speed. In particular, when the rotor is running at synchronous speed ($s = 0$) the value of s' is 2.

If the speed-torque curve of a polyphase induction motor is continued for values of s greater than 1 the torque continues to decrease, approximately inversely as slip (see Chapter VI., Art. 9). A slip greater than 1 means a negative speed, *i.e.* rotation in the opposite direction to that of the field. As the torque remains positive it will now oppose the motion, *i.e.* the motor must be driven against it by some other torque.

Thus in Fig. 7.02, ABC represents the torque-speed curve due to one rotating field, and CD its continuation for slips between 1 and 2. If CE is drawn similar to CD, but upwards instead of downwards, it will represent the retarding torque due to the oppositely rotating field. For at any slip s the torque given by CE is the same as the torque given by CD at a slip $(2 - s)$, i.e. s'.

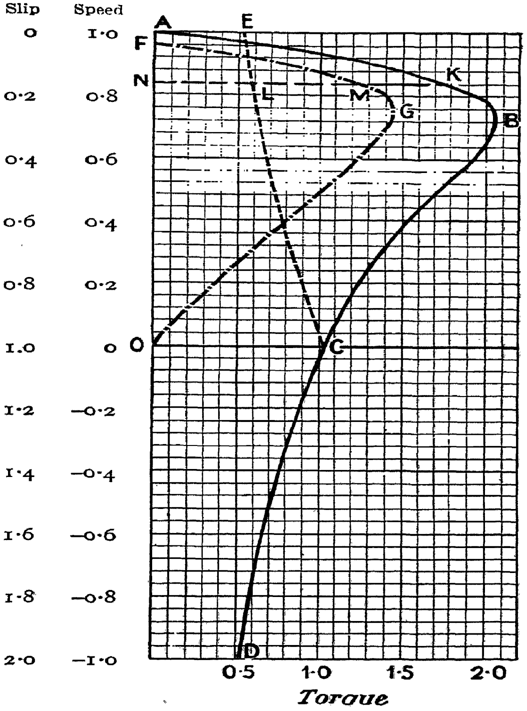


Fig. 7.02.—TORQUE-SPEED CURVES.

The nett torque is therefore given by FGO, which is obtained by subtracting from the forward torque given by ABC, *e.g.* KN, the backward torque, LN, given by the point where CE cuts KN;

i.e. MN = KN − LN.

The following are the main points to be noted as to this single-phase induction motor torque-speed curve:—

(*a*) The torque is zero at standstill. This is otherwise evident from the consideration that there is nothing to cause the rotor to move in the direction of one rotating field in preference to that of the oppositely rotating one.

(*b*) The torque is again zero at a speed slightly below synchronous speed. The amount of the reduction increasing if the ratio of OC to the maximum forward torque is increased.

(*c*) The maximum torque is less than for a similar polyphase motor, and occurs at a speed slightly higher than that which gives maximum torque in the latter (see further Art. 6). This effect is due to the backward torque decreasing as the speed rises.

4. Effects of Varying Rotor Resistance

It is at once evident that no change in the rotor resistance will alter the value of the standstill torque from zero. But it is of interest to examine the effect at other speeds and compare the results with those found for a polyphase motor in Chapter VI., Art. 11.

Using these results it will be seen that while an increase of rotor resistance increases the forward torque (given by ABC, Fig. 7.02) at low speeds without altering its maximum value, the backward torque is increased for *all* speeds.

The result is that the maximum resultant torque is diminished as the rotor resistance is increased. The speed at which the maximum torque is produced is diminished at the same time. The total result is that even at speeds near that for the new maximum torque there is little, if any, increase of torque, and that at the higher speeds there is a much greater reduction of torque than in a polyphase motor. If the original resistance is low, however, the torque at low speeds will be increased appreciably.

Example 1. *Using the particulars and results of Example 1, Chapter VI., Art. 9, find the torque-speed curve for the corresponding single-phase motor (a) with the original rotor resistance, (b) with the rotor resistance doubled.*

(*a*) For the curve ABC in Fig. 7.02 the following points were calculated in Chapter VI.:—

s = ·05; 0·1; 0·2; 0·25; 0·3; 0·4; 0·5; 0·6; 0·7; 0·8; 0·9; 1·0.
T = ·755; 1·37; 2·00; 2·074; 2·07; 1·92; 1·73; 1·546; 1·387; 1·250; 1·135; 1·037.

For the curve CD the calculation is continued thus:—

s	$100s^2$	$900s^2$	$64 + 900s^2$	$T = 1000s/(64 + 900s^2)$
1·1	121	1089	1153	0·954
1·2	144	1296	1360	0·882
1·3	169	1521	1585	0·820
1·4	196	1764	1828	0·766
1·5	225	2025	2089	0·718
1·6	256	2304	2368	0·676
1·7	289	2601	2665	0·638
1·75	306	2754	2818	0·622
1·8	324	2916	2980	0·604
1·9	361	3249	3313	0·573
1·95	380	3420	3484	0·560

Reversing this to obtain CE, and subtracting corresponding torques, viz. those for which the two values of s add up to 2, *e.g.* ·560 is subtracted from ·755; ·573 from 1·37, etc., the following results are obtained:—

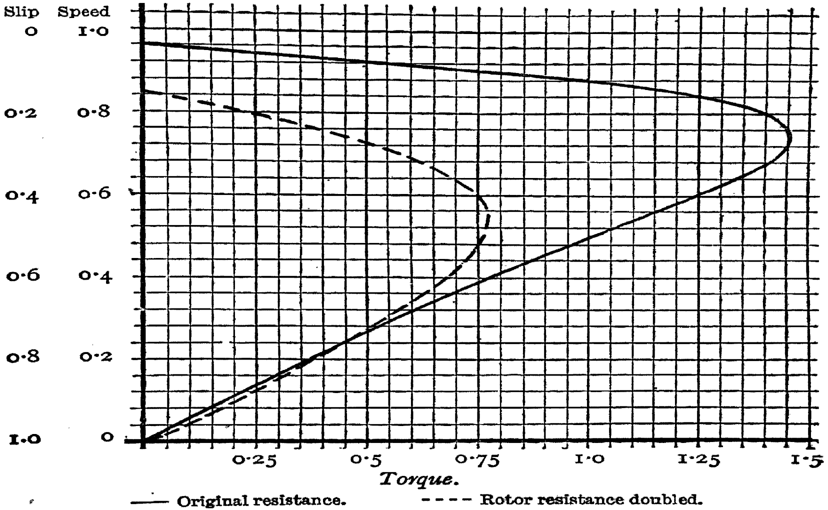


Fig. 7.03.—EFFECT OF ROTOR RESISTANCE ON TORQUE.

$s =$ ·05; 0·1; 0·2; 0·25; 0·3; 0·4; 0·5; 0·6; 0·7; 0·8; 0·9; 1·0
T = 0·195; 0·80; 1·40; 1·45; 1·43; 1·24; 1·01; 0·780; 0·567; 0·368; 0·138; 0.

These are plotted in the full line curve in Fig. 7.03.

(*b*) When the rotor resistance is doubled the same polyphase torque occurs at double previous value of slip (Chapter VI., Art. 11, and Example 2), hence the monophase torque can be obtained as follows:—

s	POSITIVE TORQUE	NEGATIVE TORQUE	NETT TORQUE
0·1	0·755	1·084	— 0·329
0·2	1·37	1·135	0·235
0·4	2·00	1·250	0·75
0·5	2·074	1·317	0·76
0·6	2·07	1·387	0·68
0·8	1·92	1·546	0·37
1·0	1·73	1·73	0

The figures for positive torque are those obtained in Example 2, Chapter VI., Art. 11.

Those for negative torque are mainly those obtained in Example 1, Chapter VI., Art. 9; *e.g.* negative torque for $s = 0{\cdot}6$ is positive torque for $s = 1{\cdot}4$, which is equal to the original positive torque for $s = (1{\cdot}4)/2 = 0{\cdot}7$.

To find the negative torque for $s = 0{\cdot}5$, substitute in the original formula $T = 1\,000s/(64 + 900s^2)$, the value $s = (2 - 0{\cdot}5)/2 = 0{\cdot}75$. This gives

$$T = 750/(64 + 506) = 1{\cdot}317.$$

Similarly the negative torque for $s = 0{\cdot}1$ equals original positive torque for

$$s = (2 - 0{\cdot}1)/2 = 0{\cdot}95;$$

$$\text{whence } T = 950/(64 + 812) = 1{\cdot}084.$$

The results are plotted in the broken line curve in Fig. 7.03.

It will be seen that the maximum torque is only a little over half its value with the original resistance.

Further, that the torque is reduced at all speeds, except for a slight increase at speeds in the neighbourhood of $\frac{1}{10}$th of synchronous speed.

5. Further Points in Theory of Action

When the rotor is running at synchronous speed the forward rotating component of the field due to the stator produces no e.m.f. in the rotor since there is no slip. The other rotating component has a slip of double the synchronous speed and so produces a comparatively large current. This lags behind the e.m.f. by nearly 90° owing to the large reactance ($2X_0$), and its magnetic effect is therefore to weaken the backward rotating component. Consequently the resultant field becomes a rotating one of a magnitude which fluctuates only slightly.

A somewhat different way of regarding the action is to consider that the stator current produces an alternating m.m.f. in one direction, and that the currents induced in the rotor produce another in a direction nearly at right angles (electrically), because the maximum field is produced at points 90° from the maximum currents. The combination of the two produces a rotating field as in a two-phase motor (Chapter VI., Art. 2). The field is not of constant magnitude because the two m.m.f.s are not quite the same as those in the two-phase motor unless the rotor resistance is negligible.

When the motor is loaded the rotor current increases, and the stator current increases so as to balance the m.m.f. due to the increase. Thus the field initially caused by the stator current is maintained constant, except for the small change due to stator resistance. The component of the rotor current which sets up the m.m.f. at right angles to that of stator still exists, but its magnitude is diminished. The resultant field is therefore a rotating one, but its magnitude fluctuates more than before. Finally, when the load has increased so much that the rotor is stopped the second m.m.f. disappears, and the resultant field becomes purely alternating.

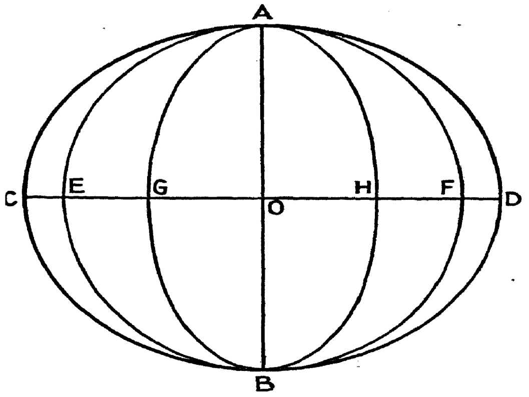


Fig. 7.04.—RESULTANT FIELDS AT DIFFERENT SPEEDS.

These changes are illustrated in Fig. 7.04. The vectors OA and OB represent the positive and negative maxima of the stator field. In a two-phase motor the second phase produces a m.m.f. of the same magnitude at right angles to AOB, represented by COD. The resultant field is given by the circle ACBD. A monophase motor with zero rotor resistance produces the same result at no load.

When the effect of rotor resistance is included the resultant field of the single-phase motor at no load is an ellipse such as AEBF. At increased load the minor axis is reduced so that the field becomes like AGBH. This shrinkage continues until at zero speed the resultant field is reduced to the straight line AOB.

6. Relative Magnetising Currents of 1-Phase and 3-Phase Motors

If the same frame is wound for single-phase and for three-phase, the input of the former for the same losses (and therefore the output too) is considerably less. For if the same field is produced in the two cases and this rotates at the same speed the average voltage induced per turn varies as the winding factor (Chapter IV., Art. 5). If the total number of turns and the current are kept the same in both windings the value of (input — stator losses) varies as the induced e.m.f., *i.e.* as the winding factor.

Comparing a uniformly distributed single-phase winding with a similar three-phase one, the breadth of each side of the former is π radians (180°), and of the latter is $\pi/3$ radians (60°). Hence the breadth coefficients (Chapter IV, Art. 5) are respectively $\sin\frac{\pi}{2}\Big/\frac{\pi}{2}$ and $\sin\frac{\pi}{6}\Big/\frac{\pi}{6}$.

Therefore the ratio of the inputs $= \frac{\pi}{6}\sin\frac{\pi}{2}\Big/\left(\frac{\pi}{2}\sin\frac{\pi}{6}\right)$

$$= \sin\frac{\pi}{2}\Big/3\sin\frac{\pi}{6} = 1/1\cdot5 = \tfrac{2}{3}.$$

If, however, only two-thirds of the stator slots are used for the 1-phase stator winding, so as to leave the rest available for a starting winding (see Art. 8), its input is reduced further. With the same size of conductor as before their number is reduced to 2/3rds, but the breadth coefficient is increased, so that the ratio of inputs in this case is $\left(\frac{2}{3}\sin\frac{\pi}{3}\Big/\frac{\pi}{3}\right) \div \left(\sin\frac{\pi}{6}\Big/\frac{\pi}{6}\right)$, which equals $\sin\frac{\pi}{3}\Big/3\sin\frac{\pi}{6} = 1/\sqrt{3} = 0\cdot577$.

This can be obtained otherwise by noting that 1-phase volt-amperes $= V_LI_L$, and 3-phase volt-amperes $= \sqrt{3}V_LI^L$, where $V_L =$ line volts, and $I_L =$ line amperes; so that if two phases of a 3-phase stator are used as a single-phase stator the input ratio is $1/\sqrt{3}$.

The above assumes that the power-factors are equal. Actually the 1-phase power-factor will be lower owing to its relatively greater magnetising current, caused by its having two oppositely rotating fields. The relationship can be obtained by considering a 3-phase stator with alternate conductors connected together to form a winding A, and the other alternate conductors to form a similar winding B. If the A and B parts of each phase are joined in series the ordinary 3-phase winding is obtained.

If, however, the B parts of two phases are interchanged the result is as shown in Fig. 7.05. I_A, II_A and III_A indicate the positions of the three parts of a two-pole winding A which are connected to the corresponding phases, each occupying one-sixth of the stator circumference. I_A', II_A' and III_A' show the positions of the corresponding return conductors. The positions of the parts of winding B are marked in a similar way. I_A, including I_A', is connected in series with I_B, including I_B', to form phase winding I. The other two phase windings are made up in the same way, viz. II_A in series with II_B, and III_A in series with III_B.

If the supply phase sequence is I, II, III, winding A by itself produces a field rotating anticlockwise; but winding B by itself produces a field rotating clockwise.

Since all the conductors between P and Q are connected to phase I their m.m.f. is in phase with, and proportional to the current in phase I. Between Q and R half the conductors carry the current of phase III reversed, and the other half carry the current of phase II reversed, so their m.m.f. is in phase with, and proportional to half the vector sum of these two currents. This vector sum is equal to half the current in phase I, and is in phase with it. The same result applies to the conductors between P and S.

Consequently the two windings A and B when carrying 3-phase currents are equivalent to a winding carrying the same 1-phase current, but with the number of conductors in PS and QR halved. This is nearly the same as retaining the original number of conductors per cm. but confining the winding to the part from the centre of SP to the centre of QR, with the return conductors opposite.

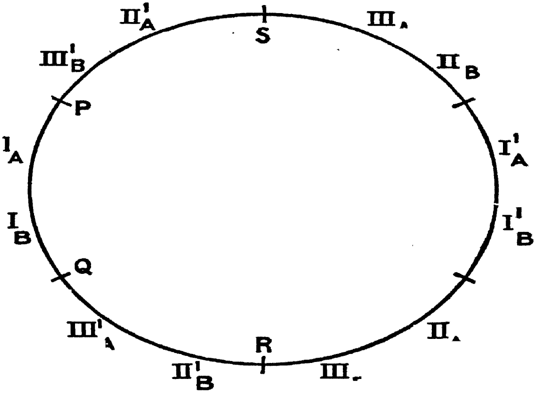


Fig. 7.05.

This gives a concrete meaning to the analysis of an alternating field into two oppositely rotating ones (Art. 2), and further enables the relation between the circle diagrams to be determined. If the same voltage (V) is applied to A and B in series as to the corresponding normal 3-phase winding the voltage across A (V_A) is to the voltage across B (V_B) in the ratio of the respective equivalent impedances Z_A and Z_B.

With the rotor at rest these impedances are equal; and each is half the impedance of the normal motor, since the number of conductors is halved. Thus $V_A = V_B = \frac{1}{2}V$, and the standstill current is the same as for the normal motor.

When the motor is running under no load in the direction of rotation of winding A's field, the impedance of A is increased in the ratio (standstill current)/(no load current). Neglecting resistance effects this ratio is $(1 + \sigma)/\sigma$ (see Chapter VI., Art. 15). But for B the slip is nearly 2, so the end of its current vector has moved beyond the standstill point to a point about half-way towards the torque line, so that RS/PS in Fig. 6.17 is nearly 2 (see Chapter VI., Art. 17). The current under these conditions is not much greater than the standstill current, so the impedance of B is only slightly less than at standstill. Therefore $V_A/V_B = Z_A/Z_B \doteq (1 + \sigma)/\sigma$.

The approximation is close, since both impedances are less than the approximate values used.

Both impedances are mainly reactive, so V is nearly the *arithmetical* sum of V_A and V_B. Therefore $V_A/V \doteq (1 + \sigma)/(1 + 2\sigma)$. Again Z_A is now half the no-load impedance of the normal motor. Therefore (magnetising current of A and B in series)/(magnetising current of normal motor) $= 2\,(1 + \sigma)/(1 + 2\sigma)$.

7. Relative Outputs of 1-Phase and 3-Phase Motors

If ATS is the simple circle diagram of a 3-phase motor, OS is likewise the standstill current of the corresponding 1-phase motor (see Art. 6). If BRS (see Fig. 7.06) is the circle diagram of the latter motor the following relations hold good. Taking AS as unity, $OA = \sigma$, so from Art. 6, $OB = 2\sigma\,(1 + \sigma)/(1 + 2\sigma)$. $BS = OS - OB = 1 + \sigma - 2\sigma\,(1 + \sigma)/(1 + 2\sigma) = (1 + \sigma)/(1 + 2\sigma)$; ∴ leakage coefficient of 1-phase motor $= OB/BS = 2\sigma =$ twice leakage coefficient of 3-phase motor.

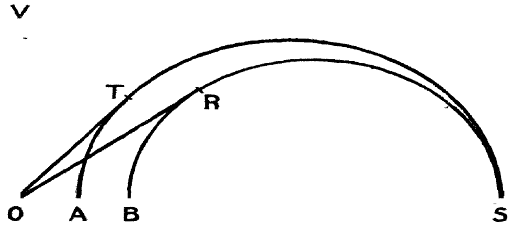


Fig. 7.06.—COMPARISON OF OUTPUTS OF 1-PHASE AND 3-PHASE MOTORS.

From the relation of power-factor to σ (Chapter VI., Art. 15) it follows that (maximum p.f. of 1-phase motor)/(max. p.f. of 3-phase motor) $= (1 + 2\sigma)/(1 + 4\sigma)$. As the current determines the heating, the lower power-factor means a corresponding reduction of full-load input.

So the input of a given frame wound monophase is reduced to a smaller fraction of its three-phase input than the ratios obtained in Art. 6. And the amount of this further reduction is greater the larger the value of the leakage coefficient of the motor.

The effect of the losses is to reduce the output to a still smaller fraction of the three-phase output, in a way depending on the original losses, and so the efficiency is lower by a considerable, but variable amount (see Example 2). If the flux-density is unchanged the iron and friction losses remain nearly the same, though the former are somewhat increased by the fluctuation in the single-phase flux.

With the usual two-thirds wound stator the copper losses in the stator are reduced in the ratio 2/3, since the current is unchanged if the line voltage remains the same. The rotor copper losses are reduced too, but not to the same extent, a ratio of 3/4 being approximately correct.

Example 2. *A 3-phase, 20 b.h.p. motor has at full load an efficiency of 89 per cent. and a power-factor of 0·90. If it is re-wound as a 1-phase motor for the same line voltage, using 2/3rds of the stator slots, estimate its full load output, power-factor, and efficiency.*

From Chapter VI., Art. 15, $1/(1 + 2\sigma) = 0{\cdot}90$; $\therefore\ 2\sigma = 1/9$;

$\therefore$ Max. power-factor of 1-phase motor $= 1/(1 + 2/9) = 9/11 = 0{\cdot}82$.

Output of 3-phase motor $= 20 \times 746 = 14\ 920$ watts.

Input of 3-phase motor $= 14\ 920/0{\cdot}89 = 16\ 760$ watts;

$\therefore$ Input of 1-phase motor $= (16\ 760/\sqrt{3}) \times (0{\cdot}82/0{\cdot}90) = 8\ 820$ watts.

Losses in 3-phase motor $= 16\ 760 - 14\ 920 = 1\ 840$ watts.

Assuming that the copper losses are 960 watts, and the other losses 880 watts, and that the copper losses are divided equally between stator and rotor, the losses in 1-phase motor are:—

Stator copper 2/3 of 480, *i.e.* 320 watts;
Rotor copper 3/4 of 480, *i.e.* 360 watts;
Other losses .. .. .. 880 watts;

Total .. 1560 watts;

$\therefore$ Output of 1-phase motor $= 8\ 820 - 1\ 560 = 7\ 260$ watts $= 9{\cdot}7$ b.h.p.

Efficiency of 1-phase motor $= 1 - 1\ 560/8\ 820 = 82{\cdot}3$ per cent.

8. Circle Diagram

The locus of the end of the stator current vector is an arc of a circle as in the case of the polyphase motor, and the allowances for the various losses can be made in a similar way. Thus in Fig. 7.07,

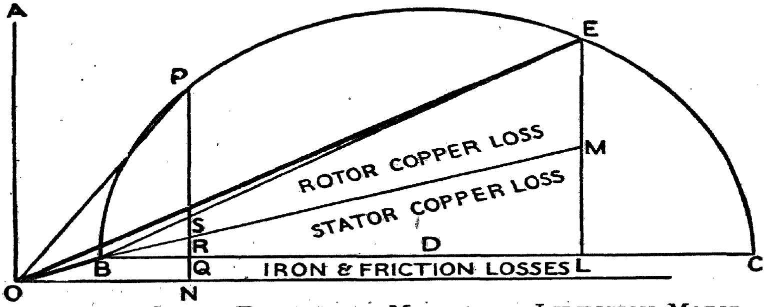


Fig. 7.07.—CIRCLE DIAGRAM OF MONOPHASE INDUCTION MOTOR.

OA represents the applied p.d.,
OB the no-load current,
OE the standstill current,
BC is perpendicular to OA and is the diameter of the semicircle (centre D) passing through B and E,
EL is perpendicular to BC,
ML = standstill stator copper loss,
EM/ML = rotor copper loss/(stator copper loss),
OP is the stator current under load,
PB is the corresponding equivalent rotor current,
PN is parallel to AO, and represents the input,
PN is cut by BDC in Q, by BM in R, and by BE in S,
PS represents the output, and PS/PN equals the efficiency,
SR represents the rotor copper loss,
RQ represents the stator copper loss,
QN represents the iron and friction losses,
$\sqrt{(PS \times PR)}$ represents the torque.

Compare this last with the polyphase motor in which PR represents the torque.

The speed of the motor is given by (synchronous speed) $\times \sqrt{PS/PR}$; and so the fractional slip equals $(1 - \sqrt{PS/PR})$.

Note that to obtain the input scale the number of amperes represented by unit length is multiplied by V (not by $\sqrt{3}V$ as for the three-phase motor in Example 4, Chapter VI., Art. 19). The result is the number of watts represented by unit length.

9. Starting Methods

Since the standstill torque is zero, special methods have to be used for starting. With very small motors the current may be switched on to the stator and the rotor started by hand. If given sufficient speed the torque produced will cause it to accelerate to normal speed and the load can then be applied.

With larger motors a starting winding is required. This is wound in the slots not occupied by the main winding but is of smaller cross-section, as it is in use only while starting. It is supplied from the mains but with an inductance (or a capacitance) in series with it so that its current is out of phase with that in the main winding. The stator currents are, therefore, roughly two-phase, and the motor starts up like a two-phase induction motor. When the speed has risen to about half synchronous speed the starting winding is disconnected leaving the main winding connected to the supply.

When a series inductance is used its effect can be increased by having twice as many turns in the starting winding as in the main one. This method is carried a step further in the Heyland motor (Fig. 7.08). In this the series inductance is abolished and the phase difference obtained by making the inductance of the starting winding high. This is effected (*a*) by concentrating the starting winding into one large slot per pole instead of spreading it over a number of small slots; (*b*) by making the starting slots with *wide* partial roofs so as to give an easy path for the flux linking with the starting coils.

The use of capacitance instead of inductance has increased for small-sized motors owing to the reduction in price and increase in reliability of capacitors. These have the advantage that the two starting currents can be made to differ in phase by 90°; whereas with inductance the lag of the main current causes the phase difference to be much less. In some cases capacitance is used when running too, so as to improve the power-factor.

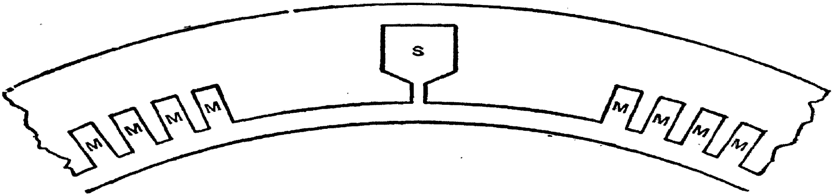


Fig. 7.08.—HEYLAND SINGLE-PHASE MOTOR.

M M. Slots for main winding. S. Slot for starting winding.

Even with these methods the starting of monophase induction motors is inferior to that of squirrel-cage polyphase ones, and so the 1-phase motors are not satisfactory when starting has to be done under load. Fast and loose pulleys or a clutch are used when possible so that the load need not be thrown on until the motor has run up to normal speed.

10. Shaded-Pole Motors

For small single-phase motors, *e.g.* for driving small fans, the " shaded-pole " method of construction is frequently employed.

The stationary portion consists of an electro-magnet with a laminated core, the winding of which is supplied with alternating current. The rotating part is a disk of copper or aluminium passing

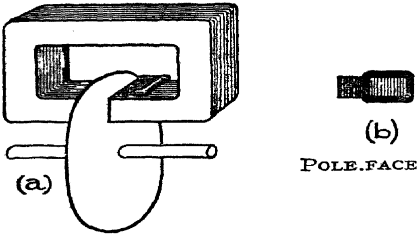


Fig. 7.09.—SHADED-POLE MOTOR.

through the narrow gap between the poles of the electro-magnet [see Fig. 7.09 (*a*)].

The shading consists in surrounding about half of the pole-faces with stout rings of copper [see Fig. 7.09 (*b*)]. The alternating flux set up by the current in the magnet winding induces currents in the copper ring by transformer action. These currents have the effect of making the resultant flux in the shaded portion lag behind the flux in the unshaded part of the pole.

The result is that a torque is produced by the interaction of these fluxes and of the currents thereby induced in the copper disk. The action is similar to that of a two-phase induction motor, or of a monophase one when its starting winding is in use (see Art. 9).

The torque can be increased by using a number of such shaded-pole magnets acting on different parts of the disk.

11. Induction Generators

Just as the torque-speed curve of a polyphase induction motor can be continued down to slips greater than the synchronous speed

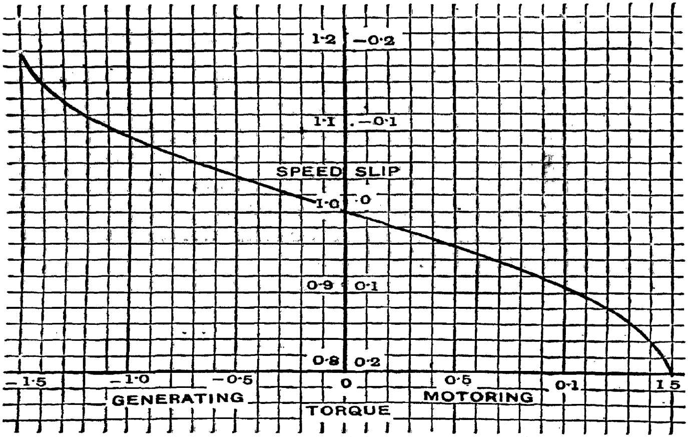


Fig. 7.10.—TORQUE-SPEED CURVE OF INDUCTION GENERATOR.

(see Art. 3), so it can be continued upwards to negative values of the slip, *i.e.* to speeds above synchronism. A portion of this curve is shown in Fig. 7.10.

The curve shows, as does the formula for torque (Chapter VI., Art. 9), that at speeds above synchronism the torque becomes negative, *i.e.* the motor must now be driven mechanically. The electrical input likewise becomes negative, *i.e.* the machine gives out electrical energy and thus acts as a generator of alternating current. This is because the reversal of slip reverses the rotor e.m.f. The rotor current is thus changed in phase by approximately half a period, and the stator current is altered correspondingly.

The change is well shown by the circle diagram (Fig. 7.11) in which OA is the terminal p.d., OA′ the e.m.f. generated, and OB the no-load current. At speeds below synchronism (motoring) the rotor current is PB and the stator current OP. At speeds above synchronism the rotor current changes to some such value as QB, and therefore the stator current becomes OQ.

The electrical *output* is OA . OQ cos A′OQ, *i.e.* OA . QN, and so is represented to some scale by QN (cf. Chapter VI., Art. 19). To obtain the mechanical input there must be added to QN, (*a*) friction and iron losses, NM; (*b*) stator copper loss, ML; (*c*) rotor copper loss, LK. Thus the mechanical input is QK; and the efficiency is QN/QK. ML and LK are obtained from the standstill current in the same way as for a motor (Chapter VI., Art. 16).

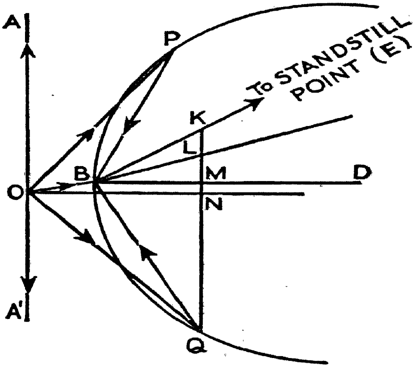


Fig. 7.11.—CIRCLE DIAGRAM FOR MOTOR AND GENERATOR.

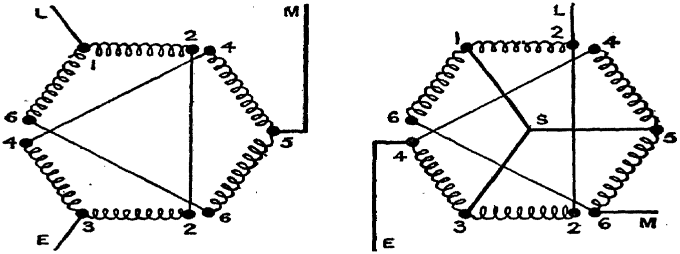
Such generators are sometimes called **asynchronous generators**, since when acting thus their speed must be above synchronism. The frequency and the voltage are settled by those of the line. It is therefore necessary to have one or more synchronous machines connected in parallel with the induction generators. One at least of the synchronous machines must be a generator to fix the frequency.

The synchronous machines must be of sufficient size (kVA) to supply the magnetising currents of the induction generators. Moreover, when the load on the latter increases, the synchronous machines must supply the whole of the wattless components (ON) of the currents in the induction generators, together with the lagging wattless component of the load current (if this leads, only the difference need be supplied). Thus the total kVA of the synchronous machines must be a large proportion of the total load, particularly when this has a low lagging power-factor.

The main advantage of the induction generator is that it does not require synchronising. Further, its frictional losses are low owing to the absence of brushes, and its construction is simple.

12. Pole-Changing and Cascade Control

One of the disadvantages of polyphase induction motors is that their running speed cannot be varied economically. With wound rotors the insertion of additional resistance in the rotor circuit lowers the speed under a given load. This method has the disadvantages of increasing the losses and of making the variation of speed with load greater (cf. Chapter VI., Art. 33).



(*a*) STARTING. (*b*) FULL SPEED.

Fig. 7.12.—SIMPLON POLE-CHANGING DEVICE.

One method of overcoming this difficulty is to wind the stator so that it can be connected to give two different numbers of poles. This cannot be done easily (see further Art. 13) except when one number is double the other, giving a low speed of half the full speed. Moreover, even in this case the efficiency and the output are reduced somewhat.

An example of this method of speed control is given by the motors for the Simplon Railway, which embody a pole-changing device. The principle of this is shown diagrammatically in Fig. 7.12. The stator winding is tapped at six equidistant points (Nos. 1 to 6). For starting the connexions are as in (*a*), L and M being the two lines, and E the earth connexion. This causes the two halves of each phase (*e.g.* 1-2, 2-3) to produce similar poles, the flux returning by the intervening spaces. For full speed the connexions are altered to those shown in (*b*). Each phase now consists of two parts in parallel (*e.g.* 2-1, and 2-3), which produce opposite poles. The phases are now Y-connected instead of Δ-connected, so that the p.d. across each of the sections in the windings is only $2/\sqrt{3}$, *i.e.* $1{\cdot}155$ times its former value.

The connexions of the switch to effect these changes are shown in Fig. 7.13. The stator winding is, in this case, shown as a continuous line for simplicity. When the right-hand line of moving contacts is brought into contact with the fingers, F, F, the resulting connexions are as shown in Fig. 7.12 (*a*). When the barrel of the switch is moved so that the left-hand line of contacts touches the fingers, the connexions become those shown in Fig. 7.12 (*b*), halving the number of poles.

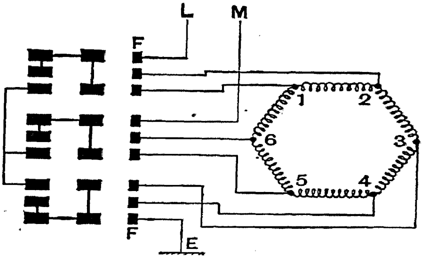


Fig. 7.13.—POLE-CHANGING CONTROLLER.

The rotor of the motor has a six-phase winding brought to six slip-rings, and the rheostats have six similar sections which are altered simultaneously. The result is that no change is necessary in the rotor connexions when the number of poles is changed. The resistance is re-introduced before switching over to the smaller number of poles, just as in the case of the change-over from cascade to single working in that type of equipment (see below).

The actual numbers of poles used in the Simplon motors are eight and sixteen, and the supply frequency is 16 cycles per sec. The maximum train speed is about 67 kilometres per hour, and the second economical speed is half this.

A better method is to use two motors connected together " in cascade." By this is meant that the supply mains are connected to the stator of one motor, and that the rotor of this is used to supply power to the second stator. The result is that the synchronous speed of the combination is that of a motor whose number of poles is equal to the sum of the numbers of poles of the two motors.

For example, if a six-pole and a four-pole motor are connected in cascade to a 50 cyc. supply their synchronous speed will be that of a ten-pole motor, viz. 600 r.p.m. The field of the six-pole motor will rotate at 1 000 r.p.m., and so when its rotor is running at 600 r.p.m. the slip is 400 r.p.m. or 40 per cent., and the frequency of the rotor currents is 50 × 40/100, *i.e.* 20 cyc. These currents, flowing in the stator of the second motor (4-pole), will produce a field rotating at 600 r.p.m., *i.e.* there will be zero slip.

When a load comes on there will be a small slip, and therefore current and a torque are produced in the second rotor. At the same time its stator current, which is the first motor's rotor current, must increase. This it can do owing to its increased slip. The first motor therefore likewise exerts a torque, and the line current is increased.

If p_1 and p_2 are the numbers of pole-pairs of the first and second motors respectively, the ratio (cascade speed)/(speed of first motor alone) equals $p_1/(p_1 + p_2)$. So for a given torque exerted by the first motor its output is reduced in the same ratio. A fixed torque means the same slip and same current in both cases; and when the motors are working in cascade the *excess* power is supplied to the second motor, and converted into mechanical power apart from losses. Thus the ratio of the powers developed is p_1/p_2 (see further Chapter VIII., Art. 20).

A higher synchronous speed can be obtained by short-circuiting the rotor of the first motor so that it acts alone. By using the second motor alone a still higher speed could be obtained if it has fewer poles than the first as in the example. This latter is not usually done because the second stator and the first rotor are generally wound for a lower voltage than that of the supply.

By making the "rotor" of the second motor stationary, and allowing its "stator" to rotate, the latter can be connected to the stator of the first motor without the use of slip-rings: if there is a bearing between the two, the connexion is made by cables run through a hollow inside the shaft. One set of slip-rings is still necessary for short-circuiting the first rotor, or when it is desired to use resistances in this rotor's circuits to obtain intermediate speeds, so that the saving is not great.

13. Cascade Motor

In these the two motors used in cascade control (Art. 12) are replaced by a single motor. The currents in the main stator windings from the supply produce a rotating field having (say) eight poles. This induces currents in the rotor in two circuits, one having eight poles and the other half as many. The eight-pole circuit reacts with the stator winding in the normal way. The four-pole circuit induces currents in the stator. This action is like that in the second motor when cascade control is used, with its rotating part connected to the rotor of the first motor as described in Art. 12.

The method by which the supply currents and the induced currents can both flow through the same stator winding is shown in Fig. 7.14, supplied by Messrs. Sandycroft, Ltd. The supply terminals are T, T, T, and the supply is three-phase. The winding is of the usual hemitropic type for four pole-pairs, except that each phase has two halves in parallel instead of having all its coils in series. The arrows *inside* the phase windings show the directions of the currents when the bottom terminal is negative.

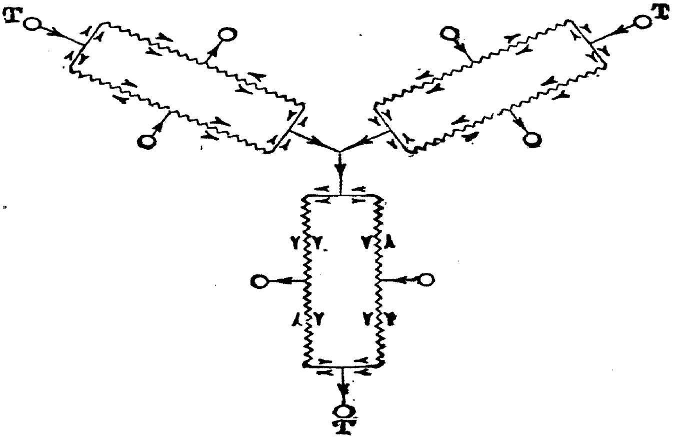


Fig. 7.14.—STATOR CONNEXIONS OF CASCADE MOTOR.

If the tapping points, O, O, are connected together in pairs, induced currents can flow in the directions shown by the arrows outside the phase windings. A comparison of the two sets of arrows shows that alternate pole-pairs are reversed. Thus the induced currents produce only four poles, *i.e.* half as many as the supply currents.

The two rotor circuits are connected respectively in Δ, and in Y with the ends connected to the corners of the mesh. With the stator tapping points short-circuited, the rotor four-pole circuit induces currents in the four-pole stator circuits. The motor, therefore, behaves like two motors, one with eight poles the other with four, connected in cascade, *i.e.* its speed is that corresponding with twelve poles.

On the other hand, by short-circuiting the rotor slip-rings the motor acts in the normal way, *i.e.* as an eight-pole one. Thus its cascade speed is two-thirds of full speed. Starting or controlling rheostats can be used with both connexions. By sub-dividing the stator winding its connexions can be changed so as to give half as many poles as the normal. This gives three speeds in all, in the ratios 2 : 3 : 6. In another type the normal winding has the smaller number of poles and so the lower speed is one-third of the higher.

A later development is the use of a second winding on the stator and a modified rotor winding. This gives four speeds with one motor; *e.g.* those corresponding with four poles, six poles, eight poles and twelve poles, together with rheostatic control on three of the four speeds.

A still larger number of speeds can be obtained by connecting a motor with a pole-changing switch in cascade with a cascade motor; *e.g.* if the cascade motor gives speeds corresponding with 12 poles and 18 poles, and the other motor can be connected to give either two or four poles then six speeds in all are possible, *i.e.* those corresponding with every number of pole-pairs from six to eleven inclusive.

For fuller details of these motors, which have been developed by L. J. Hunt, M.Inst.C.E., M.I.E.E., see *J.I.E.E.*, Vol. 52, p. 406.

14. Creedy Motors

F. Creedy* has shown how windings can be produced for other ratios of speeds than those developed by Hunt (Art. 13), and in particular for cases in which the two numbers of pole-pairs have no common factor. The general method is to wind the rotor for more than three phases. This can then be used for any two numbers of pole-pairs whose sum, after dividing by any common factor, is equal to the number of phases; *e.g.* with five phases the lower speed corresponds with five pole-pairs, but the upper speed may correspond with any smaller number, *i.e.* it may be made a cascade motor with eight poles and two poles, *or* with six poles and four poles in the two windings.

Actually double these numbers of poles are used so as to have magnetic balance. Moreover by suitably dividing the winding and connecting it to a number of slip-rings its connexions can be altered from one type of winding to the other. It is thus possible to make the rotor in the above case suitable for any even number of pole-pairs from two to ten inclusive. Usually, only some of these possible arrangements are required, and this reduces the complication of the rotor winding and the number of the slip-rings.

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* *J.I.E.E.*, vol. 59, p. 511.

The stator windings are arranged to give two or more numbers of poles; *e.g.* for six poles and two poles, three similar coils 120° (actual) apart may be connected in series. Their e.m.f.s will be in phase on six poles, and any desired number of such sets of coils may be used as one phase of the six-pole winding. The other two phases will be similar, but displaced 40° actual, *i.e.* 120° electrical. Considered as a two-pole winding the e.m.f. of each set of three coils is zero. By connecting two, or preferably three, branches in parallel in each phase of the six-pole winding a short-circuited two-pole winding is provided for cascade working (see also *J.I.E.E.*, Vol. 61).

Cascade motors of both types have an advantage over ordinary induction motors even when only one speed is required, if this is low. This is because the former can be wound for a smaller number of poles and so give a higher power-factor (see Chapter VI., Art. 28); *e.g.* for 500 r.p.m. on a 50 cyc. supply an ordinary motor requires 12 poles, whereas a cascade motor can have eight poles only, with a secondary four-pole winding.

15. Phase Advancers

One of the drawbacks of the induction motor is its somewhat low power-factor, especially at small loads. In addition to the methods of improving power-factor dealt with in Chapter V., Art. 14, induction motors may be fitted with apparatus for this purpose, suitable only for motors of this type. Their effect is to increase the slip at full load but to increase at the same time the over-load capacity of the motor.

A most important advantage is that the leading volt-amperes supplied to the rotor result in a much larger amount of leading volt-amperes in the line, in the ratio $s : 1$, where s is the fractional slip; *e.g.* if the increased slip of the motor is four per cent. the line receives 25 times as many leading volt-amperes as are supplied to the rotor.

The apparatus for producing these effects is called a (slip-ring) **phase advancer,** and there are two types, the rotary (Scherbius) and the oscillatory (Leblanc). The latter type as developed by G. Kapp, and built by Sandycroft, Ltd.,* is shown in Fig. 7.15. It consists of three armatures, one for each phase, of the usual D.C. type. The magnets are excited with direct current and have their yokes in common. The armatures are connected in series with the rotor winding by means of the slip-rings of the latter.

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* See "Phase Advancing," by G. Kapp, *J.I.E.E.*, vol. 51, and "The Improvement of Power-factor," ibid., vol. 61.

The armatures are free to move, and, therefore, when traversed by the rotor currents they vibrate at slip frequency. This pattern is consequently called a **vibrator.** A consideration of the torque produced by the current shows that, neglecting friction, the e.m.f. due to the movement is $\frac{1}{4}$ period out of phase with the current. Moreover this e.m.f. leads the current and so tends to advance its phase. Thus the current leads the applied p.d. by nearly 90°, and so the apparatus acts like a condenser of large capacitance.

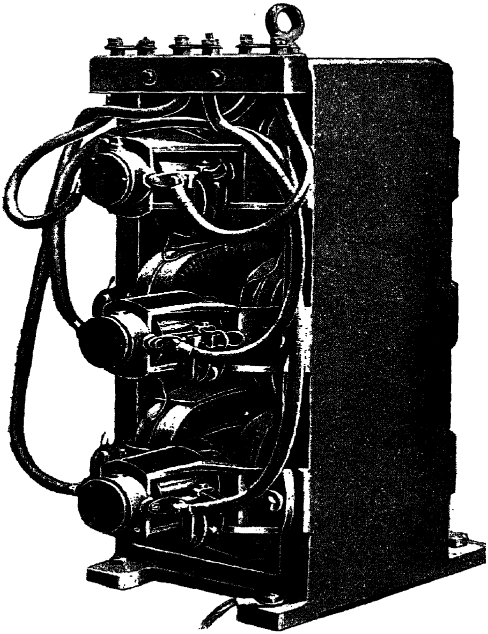
Fig. 7.15.—KAPP VIBRATOR.

The e.m.f. supplied by a given vibrator varies as (rotor current)/(slip frequency), just as for a fixed capacitance. This e.m.f. therefore diminishes with decreasing load much less rapidly than the normal e.m.f., and so the angle by which the current is advanced increases as the load diminishes. This is desirable since the normal power-factor is low at light loads.

To increase the amount of vibration, and so the vibrator e.m.f., the armatures are made of small diameter compared with their length to reduce their inertia, and the air-gaps are kept small to give a strong field. The tooth flux-density is high and so keeps the difference between the magnetic pulls small if the air-gaps become unequal.

In the rotary type no D.C. excitation is necessary but the phase advancer must be driven mechanically. It may be excited with A.C. in its stator coils, or the flux may be produced by the currents in its rotor. This latter is supplied through a commutator, so its magnetic axes are fixed (cf. Chapter IX., Art. 1). The Scherbius pattern uses the latter method and has no stator. But the rotor iron is extended outside the winding so as to provide magnetic paths as if a stator were present, with the advantage of the absence of air-gaps. The drawback is that commutation must be effected in the strongest part of the field. Miles Walker* has designed phase advancers with stator excitation.

In both cases the excitation currents produce in the phase advancer a field rotating at slip frequency. If its rotor is driven at a speed higher than this the e.m.f. is reversed. It therefore then behaves as a capacitance instead of as an inductance. The higher the speed the greater the added e.m.f.

QUESTIONS ON CHAPTER VII

1. Draw the torque-speed curve for a monophase induction motor in which the rotor has a standstill reactance eight times as large as the rotor resistance.

Plot a second curve showing the effect of doubling the rotor resistance.

2. In what respects is a single-phase induction motor inferior to a polyphase one?

A 10 h.p. three-phase induction motor has a maximum power-factor of 0·85.

About what power-factor and h.p. could be obtained from a single-phase motor of the same size and air-gap? If the efficiency of the 3-phase motor is 87 per cent., what will be the efficiency of the 1-phase motor?

3. A single-phase induction motor, 4 poles, 200 volts, 50 cyc, takes 5·4 A. with a power-factor of 0·25 at no-load. With rotor clamped and 100 volts applied it takes 16 A. at a power-factor of 0·4.

Draw its circle diagram and find the maximum power-factor, and the current, torque, output and slip at this power-factor.

Stator copper loss = 60 per cent. of total copper losses.

4. Explain the construction and action of a single-phase induction motor and describe a device used for starting such a motor. [C. & G., II.

5. If an induction generator of 100 kVA. output has a full-load power-factor of 0·8, find the least size of synchronous generator to run in parallel with it if the load is (*a*) non-inductive; (*b*) inductive, of power-factor 0·7.

Find the total output in each case.

6. What synchronous speeds are possible with two coupled induction motors having 4 poles and 8 poles respectively, when supplied at 40 cycles per sec.?

7. Describe two methods by which a single induction motor can be given two or more normal speeds.

8. Explain the action of an apparatus for improving the power-factor of an induction motor.

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* *J.I.E.E.*, vol. 50, p. 329.

CHAPTER VIII

A.C. COMMUTATOR MOTORS

1. A.C. Commutator Motors

Although one of the great advantages of A.C. motors of the induction and synchronous types, compared with D.C. motors, is the elimination of the commutator and its attendant troubles, nevertheless A.C. commutator motors have many advantages. These are particularly great in cases such as traction and crane work, where a high starting torque or variable speed or both are desired.

For single-phase supply in particular A.C. commutator motors have strong claims, owing to the weakness of induction motors as regards starting and speed control on such circuits (see Chapter VII.). The commutator motors can be made to give either a series or shunt characteristic as desired, and in the latter case the speed can be varied at will over a wide range in an economical manner.

The method of varying the speed in these motors is to alter the applied p.d., which can be done with an A.C. supply by means of transformer tappings with very little loss. In a single D.C. motor this requires a series resistance, causing a reduction of efficiency in the same ratio as the speed. The advantage of this method is, that with constant current and field strength, the torque remains constant at all speeds.

The alternative method of speed control by varying the field strength, which is the only efficient one for D.C. motors, can be used for the A.C. motors too. Its disadvantage is that the torque diminishes inversely as the increase of speed. Consequently, when the torque to be overcome is constant (or when it rises with the speed) a larger motor is needed for the same load than when the former method is used (cf. Volume I., Chapter XI., Art. 9).

The A.C. commutator motors can be made for polyphase supply too. They are not in so strong a position in this case, owing to the superiority of the polyphase induction motor over the monophase one. Their sphere of usefulness comprises those cases in which speed regulation is required to a greater extent than can be managed economically by induction motors.

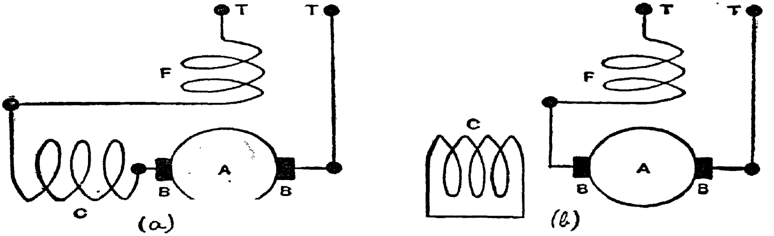
2. The Single-Phase Series-Wound Motor

If the connexions of a D.C. series-wound motor to the supply mains are reversed, the motor runs in the same direction as before, because both field and armature currents are reversed. Hence it is possible to drive a series-wound motor from an A.C. supply.

It is necessary to laminate the whole of the magnetic circuit, and not merely the armature core. Otherwise the alternating flux induces large currents in the cores entailing much heating and loss of energy. The hysteresis loss is unavoidable, but can be kept low by working at small flux-densities.

The salient-pole type of construction is unusual, however, being generally replaced by a winding similar to that of a single-phase induction motor. This produces a field, which is a closer approximation to a sinusoidal one, and so is both better in operation and easier to design. The same core readily allows the application of compensating coils, which are necessary in these A.C. motors.

Since the suppression of sparking at the commutator is the chief difficulty in the design, the armature reaction must always be balanced by a compensating winding except in small low-voltage motors. The resulting type is called the **compensated series motor.** The compensating coil may either be connected in series with the armature [see Fig. 8.01 (*a*)] as in the case of a D.C. compensating winding (see Volume I., Chapter IX., Art. 27), or short-circuited on itself [see Fig. 8.01 (*b*)]. In the latter case the alternating currents in the armature induce, by transformer action in the compensating coil, currents of values which almost exactly balance the magnetic effects of the armature currents (see Volume I., Chapter XVII., Art. 2 and Art. 10).



(a) SERIES COMPENSATION. (b) SHORT CIRCUITED COMPENSATION.

Fig. 8.01.—COMPENSATED SERIES MOTOR.

A. Armature. B, B. Brushes. C. Compensating coil. F. Field Coil.
T, T. Supply terminals.

The action will be clearer if it is remembered that the magnetic axis of an armature supplied through a commutator passes through the points (B, B in Fig. 8.01) at which current is led to and from the armature. And further, that this axis remains fixed, whether the armature itself is rotating or not. Consequently, the armature is equivalent magnetically to a fixed coil with BB as axis. As this is carrying an alternating current, it acts as the primary of a transformer, of which the compensating winding [C in Fig. 8.01 (*b*)] is the short-circuited secondary.

The speed of this motor can be regulated easily by an auto-transformer with a number of tappings, and a switch in two parts with a choking coil connected between them to avoid short-circuiting (see Art. 23).

To keep the power-factor high the field-turns should be few compared with the armature turns, since the inductance of the latter is compensated almost completely by the compensating winding (see further Art. 3).

3. Difficulties of Commutation

In all A.C. commutator motors satisfactory commutation is made much more difficult by an action which is additional to the sources of sparking in a D.C. machine. This is the transformer action of the main field on the coils when they are short-circuited during commutation.

Each armature coil while short-circuited by the brushes has its axis approximately at right angles to the brush axis, and, therefore, parallel to the field axis. Consequently it has a p.d. induced in it which depends on the rate of change of the field ampere-turns.

The general means of preventing this action from producing excessive sparking include the following:—(*a*) When there is a choice, keep the supply frequency low, *e.g.* 15 cycles/sec. The smaller the ratio, (time of commutation)/(periodic time of supply), the less is the change of field strength during commutation, and so the less is the transformer effect. A low frequency improves the power-factor in addition.

(*b*) A high speed, well above synchronism, is also helpful, since it reduces the time of commutation. But too high a speed may increase the reactance voltage more than it reduces the transformer voltage. High speed improves the power-factor, too, since with a given current it increases the p.d. across the armature (which is nearly non-inductive) without altering the p.d. across the inductive field winding.

(*c*) Use brushes which are narrow peripherally; see (*g*) also.

(*d*) Make the air-gap short and the flux-densities moderate, so as to keep the number of turns in the field winding small.

(*e*) Keep the commutator voltage low. This may necessitate using a transformer, which would not otherwise be necessary if speed regulation is not required.

(*f*) Use an interpole in addition to the compensating winding. The object of this is to produce a flux linking with the short-circuited coils, but not with the armature coils as a whole. If this flux is properly adjusted it will produce an e.m.f. in the short-circuited coils, equal but opposite to the transformer e.m.f. of the main field.

In the compensated series-wound motor the interpole is connected in series with the armature, but has its winding shunted by a non-inductive resistance. The result is that the interpole exciting current and its flux lag behind the main current by a considerable angle. Thus the e.m.f. due to the rotation of the short-circuited coils, which is proportional to the interpole flux, is large at an instant in the period when the main flux is small and consequently its rate of change is large.

These motors have been built of sizes up to 3 000 b.h.p. One pole-pair for each 100 b.h.p. to 150 b.h.p. is usual in the larger motors, which are used chiefly for traction.

(*g*) Use high resistance material for the connexions between the armature coils and the commutator segments. If the brushes are so narrow that they never touch more than two segments simultaneously there will be two high resistance connectors in series in each short-circuit path. In the main circuit there will be two connectors *in parallel* at each brush.

The performance of a motor of this type is shown in Fig. 8.02. This shows the wide range of voltage over which such a motor can be used: consequently the control is efficient because it is unnecessary to waste power in resistances, as is the case with D.C. series motors.

The cost of an A.C. equipment is about 20 per cent. greater than that of a similar D.C. one, but the cost of the overhead line is reduced by the higher voltage possible with A.C. Moreover if high pressure A.C. is used for transmission the use of A.C. motors eliminates the need for rotary convertors, and so effects a further saving both in capital cost and in conversion losses.

One important drawback of the system is that satisfactory motors cannot be made for frequencies higher than 25 cycles per sec., and a still lower value is preferable. Therefore when the power supply of a district is at a frequency higher than 25 cyc. the supply

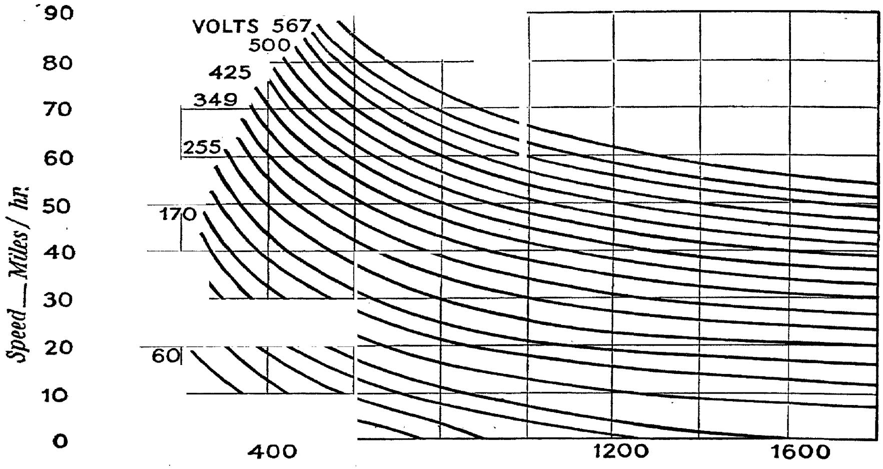


Fig. 8.02.—PERFORMANCE OF A.C. COMMUTATOR MOTOR.

to a railway cannot be taken directly from the general mains. The alternatives are the use of separate generation or the employment of some form of frequency changer. Neither of these is as economical either for the railway or for the general power supply as a common supply would be if it were feasible.

The motors of the series type can be used for D.C. without any change, all that is necessary being to design them to be suitable for the D.C. pressure. This usually means connecting two motors permanently in series when using the D.C. supply. Starting rheostats of the usual type are necessary in addition to the A.C. control equipment.

4. The Repulsion Motor

This type of motor was invented by Professor Elihu Thomson in 1887.* The armature brushes are short-circuited together so that it is an "induction" motor in the sense that the armature currents are induced by the action of the alternating flux.

It may have either a single field winding [Fig. 8.03 (*a*)], the original and usual form, or be of Atkinson's form with two separate field windings, electrically at right angles [Fig. 8.03 (*b*)]. In the former case the brushes are movable, in the latter they are fixed in the axis of one of the field windings (T). The two forms are equivalent, since the double winding produces a resultant m.m.f. whose axis is intermediate between the axes of the two windings. But for analytical purposes the action of the second form is simpler to follow.

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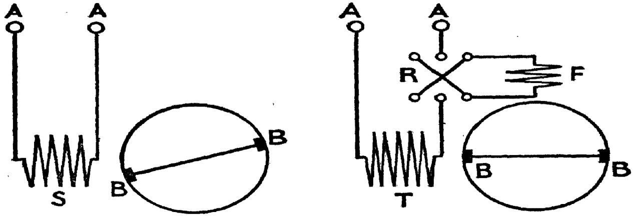
* See *Electrician*, vol. xxv., p. 35.

The winding (T), along whose axis the brushes are placed, is called the " transformer " coil. Its action is to induce current in the armature coils in the same way as the primary of a transformer induces current in its secondary. The other winding (F), the " field " coil, sets up an alternating flux along an axis at 90° (electrical) with the axis of the flux of the transformer coil, but in time-phase with this flux. The two together produce a torque by the interaction of the induced currents in the armature and the flux due to F.

When the armature rotates a " back " e.m.f. is produced, which has to be overcome by the e.m.f. induced by the transformer action. Hence, neglecting resistances, magnetic leakage, and iron losses, the p.d. across T = (back e.m.f.) × (No. of turns in T)/(No. of turns in armature); and current in T = (armature ampere-turns)/(No. of turns in T). Again, the back e.m.f. varies as (speed) × (flux due to F): and this flux depends on the current and the number of turns in F.

Thus, with constant total applied p.d., the back e.m.f. is diminished by the resistances with an increase of current, and the two causes produce a decrease of speed a little more than inversely



(*a*) WITH MOVABLE BRUSHES AND SINGLE FIELD COIL.

(*b*) WITH FIXED BRUSHES AND DOUBLE FIELD COIL.

A, A. A.C. Supply. B, B. Brushes. F. Field winding.
S. Single stator winding. T. Transformer winding.

Fig. 8.03.—REPULSION MOTOR.

as flux F. And the torque (which is proportional to current × flux F) increases more rapidly than the current; *i.e.* the motor has the same type of characteristic as a series-wound motor (see further Art. 13).

The effect of a change in the applied p.d. is to alter the speed corresponding to any particular current and torque in nearly the same ratio as the applied p.d. So that in this respect, too, the repulsion motor behaves like a series-wound motor, either D.C. or A.C.

The speed can be varied also by altering the number of active turns in one or both of the stator windings by means of tappings. The effect of reducing the number of turns in F is to reduce the flux per ampere; consequently the speed at a given current rises, but the torque is reduced inversely as the speed. This method is equivalent to using a field diverter with a D.C. series-wound motor, and is not so good as voltage variation (see Art. 1). But it avoids the need for a transformer, which is required by the latter method.

If the number of turns in T is reduced, then for the same main current the armature current is reduced. As the driving flux (*i.e.* that due to F) is unaltered the torque is diminished, and the speed falls. The effect is similar to placing resistance in parallel with the armature of a D.C. series-wound motor.

In the single-coil type the effects of changes of current and of applied p.d. are the same as long as the brushes are not moved. A movement of the brush axis away from the axis of the coil, *i.e.* anticlockwise in Fig. 8.03 (*a*), increases the motor field at the expense of the transformer field. It is thus equivalent to increasing the number of turns in F, and reducing the number in T in the double-wound stator, *i.e.* it reduces the speed and increases the torque per ampere.

To reverse the direction of rotation of the motor the brushes must be moved to the opposite side of the axis of the coil. This reverses the phase of the driving component without altering the transformer component. Similarly, the double-wound stator type can be reversed in rotation by reversing the connexions between the two stator windings by means of a reversing switch, as shown at R in Fig. 8.03 (*b*).

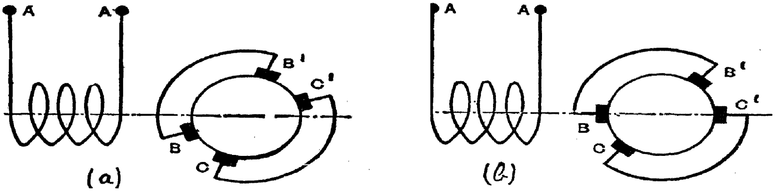
5. Other Brush Arrangements

If four brush sets are used instead of two, and these are short-circuited in pairs (see Fig. 8.04), the action is modified, though unaltered in essentials. The effect is that the portion of the armature between the pairs of short-circuited brushes (viz. BB′ and

CC′ in the figures), is short-circuited and takes part in the transformer and motor actions. The remaining portions (viz. BC and B′C′) take part in the motor action only.

All four brushes are moved together in Latour's arrangement [Fig. 8.04 (*a*)]. In this case the advantage is that the transformer voltage induced in the armature can be altered without altering the number of armature turns. Thus the same motor can be given different characteristics by altering the angle between the short-circuited brushes. The speed is adjusted by moving all four brushes together, the effect being the same as the movement of the two brushes in the original form. The resultant brush axis is the line joining the middle points of BC and of B′C′.

In Déri's arrangement [Fig. 8.04 (*b*)], two of the brushes (B, C′) are fixed in the coil axis. The other two (B′, C) can be moved together for speed regulation. The advantage of this method is that the resultant brush axis moves through only half the angle that the brushes are moved. Hence closer regulation of the speed is possible than with the other two methods.



ALL MOVABLE TOGETHER. TWO FIXED, TWO MOVABLE.

Fig. 8.04.—SPEED ADJUSTMENT BY BRUSH SHIFT.

A A, A.C. supply. B B′ C C′. Brushes.

With any of the arrangements with movable brushes the torque is zero when the resultant brush axis is in line with the coil axis. The current induced in the armature has then its maximum value, so that this is called the "short-circuit position" of the brushes. When the brush axis has been moved through 90° (electrical) the torque is again zero because no current is induced in the armature. This is called the "neutral position" of the brushes.

The repulsion motor can be started by placing the brushes in the neutral position and applying the full line p.d. The brushes are then moved away from the neutral position, and the motor starts up in the opposite direction to that in which the brushes have been moved. With Déri's brush arrangement [Fig. 8.04 (*b*)], the neutral position is when B and B′ coincide, and C and C′ coincide, consequently no coils are then short-circuited; whereas with the other brush arrangements the zero induced current in the neutral position is due to the induced e.m.f.s balancing. This is a further advantage of the Déri arrangement.

6. Modified Repulsion Motors

The main advantage of the plain repulsion motor is that, since the armature current is induced, the stator and armature voltages are independent, and can be given any desired ratio by using suitable windings. Thus the stator may be directly connected to a high-pressure supply, and yet a suitably low voltage be used in the armature without employing a transformer.

On the other hand, its commutation is unsatisfactory except in the neighbourhood of synchronous speed. The reason is that the transformer effect in the short-circuited coil (see Art. 3), due to the field coil [F, Fig. 8.03 (*b*)], is partly compensated by the rotational e.m.f. due to the flux produced by the transformer coil T. But this compensation is fairly exact only close to synchronous speed (see Art. 13), and the difference is particularly great at speeds above synchronism. Hence the repulsion motor is unsuitable for such a wide range of speed as is required for traction.

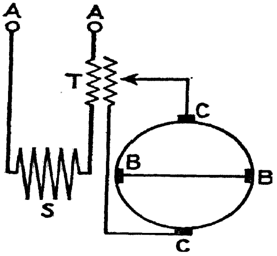


Fig. 8.05.—WINTER-EICHBERG MOTOR.

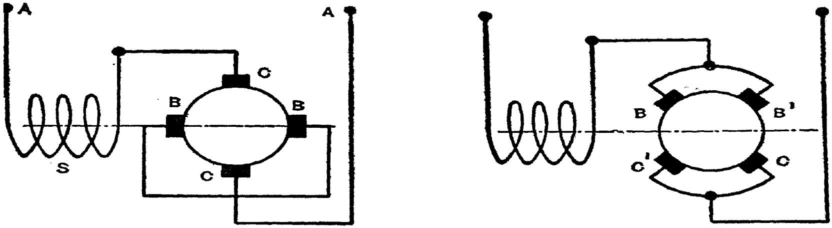
The power-factor is about the same as that of the series-wound motor, but it can be improved by the use of the connexions shown in Fig. 8.05. This motor was invented by **Winter and Eichberg,** and almost simultaneously by **Latour.** The supply terminals (A, A) are connected to the stator winding S, and to the primary of a small transformer (T) in series. The secondary of this transformer is connected to a pair of small exciting brushes (C, C). The main brushes (B, B) are placed on the axis of the stator coil. The speed is adjusted by altering the number of secondary coils connected to the exciting brushes.

The action is similar to that of a double-wound repulsion motor. The stator coil (S, Fig. 8.05) produces the transformer action. The field coil is replaced by the circuit through the armature between the exciting brushes CC. The current in this circuit produces a flux, with its axis along CC, which acts as the driving flux.

This motor can be supplied from a high-pressure circuit, and requires only a comparatively small transformer, viz. one of sufficient size to supply the exciting current to the brushes CC. At the same time its power-factor at synchronous speed, and above it, is close to unity.

By omitting the transformer and supplying the exciting current by conduction the connexions take the simpler form shown in Fig. 8.06. This is called a compensated repulsion motor. Its action is the same as that of the Winter-Eichberg motor, but it requires a low-pressure supply. Therefore when power is to be taken from high-pressure mains the transformer has to deal with the whole of it. Consequently the Winter-Eichberg motor is preferable in such cases.

The compensated repulsion motor of the Winter-Eichberg-Latour type was used for the Spindlersfeld and the Hamburg-Altona Railways, and for the London, Brighton, and South Coast Railway's suburban lines, now part of the Southern Railway Co.



(*a*) ORDINARY BRUSH ARRANGEMENT. (*b*) LATOUR BRUSH ARRANGEMENT.

Fig. 8.06.—COMPENSATED REPULSION MOTOR.

In the latter case the motors had eight poles and were of 115 h.p. (1 hour rating) at 25 cycles per sec. The line p.d. was 6 000 volts and was stepped down to 750 volts by transformers on the train. These transformers had two intermediate tappings giving 300 volts and 450 volts respectively. The former was used for the car lighting and heating and the air-pump motor, the latter and the full 750-volt connexions for the driving motors. The stator winding was tapped at three points for the variation of the p.d. applied to the armature.

This gives six (2 × 3) possible control positions for the main motors, but only five of these are utilised and the running is mainly on four of the five. The performance of the motor for positions 1, 2 and 4 of the controller is shown in Fig. 8.07.

In the Spindlersfeld equipment, too, the line p.d. is 6 000 volts. This is connected directly to the stator with the primary of the

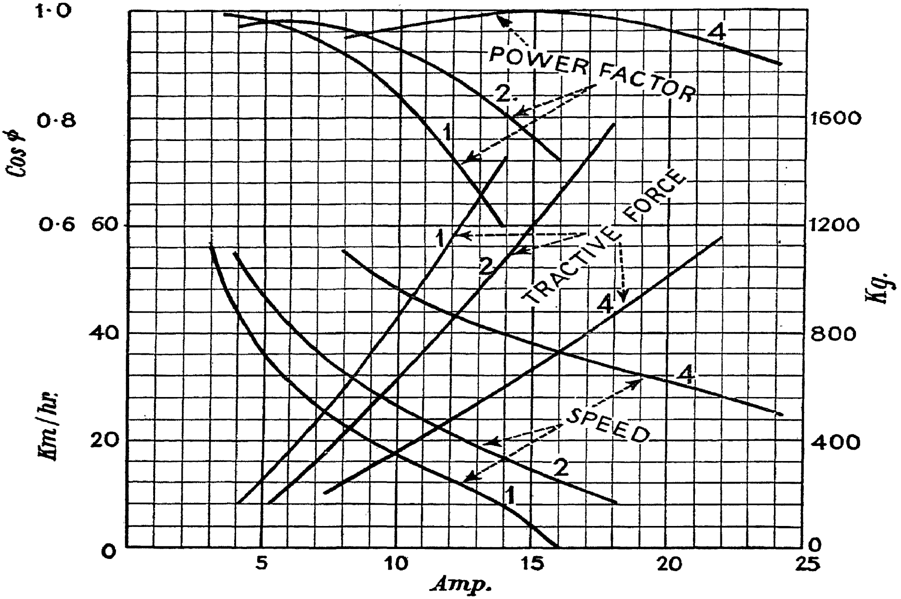


Fig. 8.07.—PERFORMANCE OF WINTER-EICHBERG-LATOUR MOTOR.

armature transformer in series (see Fig. 8.05). The secondary has five tapping points and the speed is controlled by changing the connexions of the armature to these. The characteristics are very similar to those shown in Fig. 8.07.

In the later equipment for this line, compensated series motors (see Art. 2) were used, and this type appears to be the most satisfactory for A.C. traction work.

7. Other Forms of A.C. Commutator Motors

By supplying current directly to the armature and thereby inducing currents in two coils on the stator, at right angles to each other, the **inverted repulsion motor** is obtained (Fig. 8.08). The armature winding now acts as the primary of a transformer, inducing an e.m.f. in the secondary winding (T). This causes a current to flow in the field coil (F), thereby producing a torque by the interaction of its flux and the armature current.

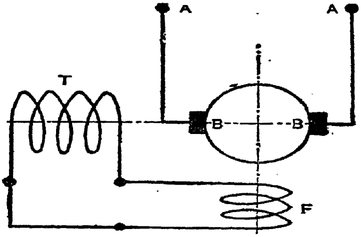


Fig. 8.08.—INVERTED REPULSION MOTOR.

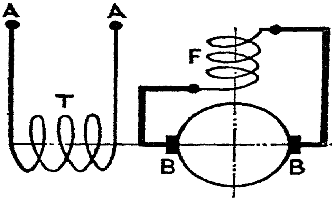


Fig. 8.09.—INDUCED SERIES MOTOR.

Another modification of the double-wound repulsion motor is produced by connecting the field-winding (F, Fig. 8.09) across the brushes (B, B), instead of in series with the transformer winding T. The latter is connected directly to the supply A, A. The circuit BFB forms a load on the transformer secondary which the armature windings form. This may be called an " induced series-wound " motor.

The series-wound and repulsion methods are sometimes combined in one motor (see Fig. 8.10). The circuit ATFBBA′ is that of a series-wound motor with T forming a conductive compensating coil [cf. Fig. 8.01 (*a*)]. The circuit ATFC is that of a doubly-wound stator for a repulsion motor, and though the brushes are not short-circuited they are connected through CA′ so that repulsion action can take place.

By connecting ACA′ to the same secondary winding as shown, the regulation of speed can be effected by moving C. Movement to the left will raise the speed by increasing the armature p.d. and reducing the flux per ampere. The advantage of the method is that the transformer e.m.f. in the short-circuited coils is more nearly balanced by the rotational e.m.f. than in the series or in the repulsion type (because at low speeds the action is mainly "repulsion" and becomes "series" at high speeds).

Various other combinations of the above methods have been suggested, together with different ways of exciting the interpoles (if any). None of these appear to have any general advantage over the types described.

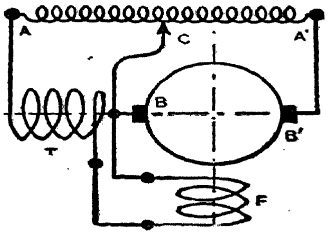
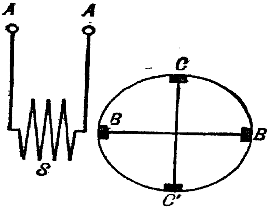


Fig. 8.10.—SERIES AND REPULSION MOTOR.

8. Shunt Type Motors

All the motors described in the preceding part of this chapter have "series" characteristics; *i.e.* their speed diminishes with increase of load, roughly in inverse proportion to the current as in a D.C. series-wound motor. It is equally possible to obtain A.C. commutator motors with "shunt" characteristics, *i.e.* of nearly constant speed at all loads, like a shunt-wound D.C. motor. Further, this speed can be adjusted at will as in the case of the D.C. motor.

A D.C. shunt-wound motor with laminated magnetic circuit would produce little torque if supplied with A.C. For the flux produced by the field current would lag behind the applied p.d. by nearly 90°, whereas the armature current would be nearly in phase with this p.d. Hence the torque, which depends on (current) × (flux) × (cosine of phase difference) (cf. Chapter VI., Art 9), would be very small; the phase-difference moreover would decrease as the armature speed rose, so the starting torque would be particularly low.



A.A. Supply terminals.

Fig. 8.11.—SHUNT MOTOR.

This difficulty could be overcome if a two-phase supply were available to produce an exciting current 90° out of phase with the brush p.d. When the supply is monophase a similar result is obtained by the method described by L. B. Atkinson in 1898.*

The armature of the motor shown in Fig. 8.11 is in the field due to the stator coil (S) which is connected to the A.C. supply (A, A). At standstill it acts merely as the secondary of a transformer producing a current in the circuit between the brushes B, B'. The resultant torque is zero since the direction of the pull is one way round in BC and B'C', and the opposite way in CB' and C'B, and the magnitudes of these pulls are equal.

But when the armature rotates an e.m.f. is produced between the brushes CC'. If these are joined, as shown in Fig. 8.11, a current flows in the circuit CC', and this produces a flux with its axis along CC'. This current and flux lag nearly 90° behind the rotational e.m.f. This in turn is in phase with the flux along BB', which is 90° out of phase with the applied p.d. Consequently the flux along CC' is nearly in phase with the applied p.d.

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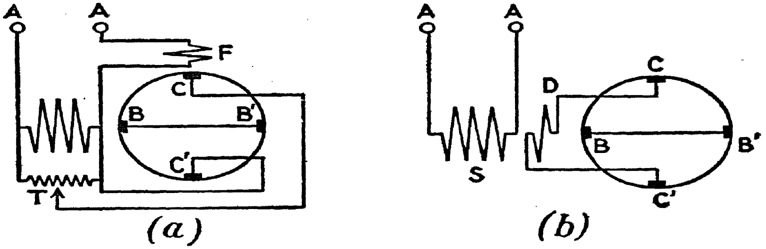
* *Proc. Inst. C.E.*, vol. cxxxiii., p. 113.

When the speed is near to synchronous value the current in circuit BB′ is nearly in phase with the applied p.d. Consequently this current and the flux along CC′ produce a driving torque.

This motor behaves like a monophase induction motor, for it has zero starting torque and always runs at a speed close to synchronism. It has no advantage over the latter type, and has the disadvantages of lower power-factor, a commutator, and lower efficiency. But by modifying the circuits in various ways improvements can be effected.

9. Modified Shunt Motors

The power-factor of the shunt type of motor can be improved by supplying the exciting brushes (CC′, Fig. 8.11) with a small voltage in phase with the supply p.d. This may be done either by means of a small separate transformer [T, Fig. 8.12 (*a*)], or more simply by a compensating coil [D, Fig. 8.12 (*b*)], wound in line with the main stator winding. The effect of these is to bring the flux along CC′ more nearly into phase with the induced current in BB′.



(*a*) WITH AUTO-TRANSFORMER. (*b*) WITH PHASE COMPENSATION.

Fig. 8.12.—MODIFIED SHUNT MOTOR.

The speed is varied in a manner analogous to that employed for D.C. shunt-wound motors, viz. by altering the field current. But this current ought to lag 90° behind the e.m.f. producing it. Hence the regulating coil [R, Fig. 8.13 (*a*)] connected in circuit CC′ ought to be highly inductive; if resistance is used it will affect injuriously the phase of the exciting current. The greater the value of the inductance inserted the weaker the field at a given speed, *i.e.* the greater the speed towards which the motor tends.

If a capacitance is inserted in place of the inductive coil the opposite effect is produced: *i.e.* the speed is lower than with the brushes CC′ short-circuited. The smaller the capacitance of the condenser (*i.e.* the greater its negative inductance) the greater the amount by which the speed is lowered.

An alternative method is to connect the exciting brushes (C, C′) to an auxiliary exciting coil [E, Fig. 8.13 (*b*)] placed on the stator. One brush is connected to the middle point of the auxiliary coil and the other to a movable contact, as shown. Thus the auxiliary coil can be made either to assist or to oppose the magnetic effect of the armature circuit CC′. In this way the driving flux can be either increased or diminished at a given speed, *i.e.* the speed can be lowered or raised.

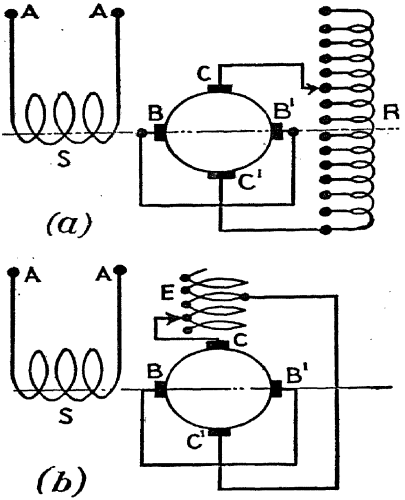


Fig. 8.13.—SPEED REGULATION OF SHUNT MOTOR.

(*a*) WITH REGULATING INDUCTANCE.
(*b*) WITH AUXILIARY FIELD COIL.

Another method of controlling the speed is shown in Fig. 8.14. This differs from the previous methods because the exciting circuit (CC′) is left unchanged. A variable ratio auto-transformer (T) has its primary connected to the supply terminals (A, A). The secondary is connected to the main brushes (B, B′). Any variation in the voltage applied to these results is a corresponding change in the speed. By this method the no-load speed may be made either higher or lower than synchronous speed according to the way the auto-transformer is connected to the brushes. The characteristics obtainable are shown in Fig. 8.15. Each line corresponds with a fixed position of the tapping on T.

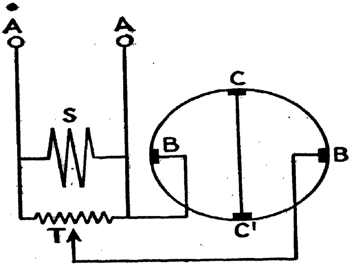


Fig. 8.14.—WITH VOLTAGE SPEED CONTROL.

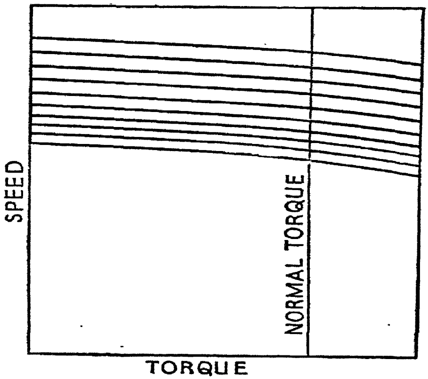


Fig. 8.15.—CHARACTERISTICS OF SHUNT MOTOR.

For starting purposes the connexions are altered so that the motor becomes a repulsion motor. When the speed has risen sufficiently the connexions are restored in order to obtain the shunt characteristic. One method is shown in Fig. 8.16. When the connexion between brush C and the starting resistance R is broken, the motor is a compensated repulsion motor (see Fig. 8.06) and will start well. The connexion between C and R is then made, and R is gradually cut out. The connexions are now those of a shunt-type motor (Fig. 8.11).

This can be combined with the methods used for improving the power-factor and for varying the speed. If the auxiliary field coil method [Fig. 8.13 (*b*)] is used for the latter purpose the starting is improved thereby. For if the speed-regulating switch is set so that the auxiliary coil assists the armature m.m.f. the field per ampere is increased. Hence the torque per ampere is increased correspondingly, and the motor can start under a heavier load.

10. Circle Diagram for Series-Wound Motor

If the resistances and iron losses of a series-wound motor are neglected it will when stationary take a current lagging 90° behind the voltage. When it is running a back e.m.f. is generated and the current is reduced and brought closer to the phase of the voltage.

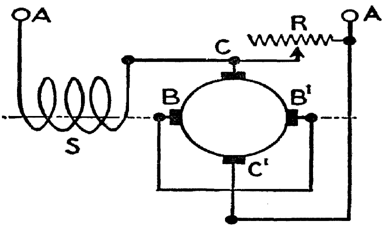


Fig. 8.16.—STARTING OF SHUNT MOTOR.

With constant applied voltage, V, let the standstill current be V/X. Let E = back e.m.f.; I = corresponding value of current; and $R_L = E/I$. Then R_L is the resistance corresponding to the load since the power developed = E.I = $I^2.R_L$. Thus variations of the load are represented by changes in R_L.

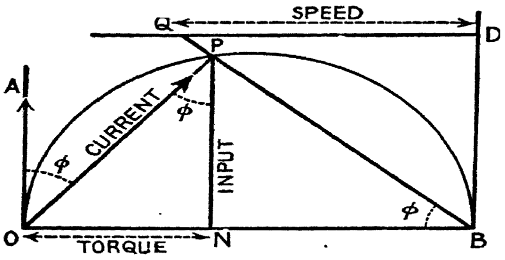


Fig. 8.17.—CIRCLE DIAGRAM OF SERIES MOTOR.

Hence, as was proved for the rotor of an induction motor (see Chapter VI., Art. 14), the locus of the end of the current vector is a semicircle with its diameter at right angles to the fixed voltage vector (OA, Fig. 8.17). The diameter of the semicircle (OB) = V/X, and OP represents on this scale the current when the angle of lag (ϕ) is AOP. The input (VI cos ϕ) is given to a certain scale by PN, since this equals I cos ϕ on the current scale. And likewise the output is given by PN, as the losses are being neglected for the present.

The speed is proportional to (back e.m.f.)/(flux) and so is proportional to (E/I) or to R_L, if the permeability is taken as constant. Draw BD perpendicular to OB, and at any convenient height draw DQ parallel to BO. Produce BP to cut DQ in Q. Then DQ represents the speed to some scale.

For $\angle$ BPO = 90°, since it is the angle in a semicircle.

Therefore $\angle \text{DQB} = \angle \text{QBN} = \phi$.

Hence $\text{DQ} = \text{BD} \cot \text{DQB} = \text{constant} \times \cot \phi$.

And $\cot \phi = R_L/X = \text{constant} \times R_L = \text{constant} \times \text{speed}$.

Therefore DQ is proportional to the speed.

The torque is proportional to (output)/(speed), *i.e.* neglecting losses, to PN/speed, *i.e.* to PN/cot ϕ, *i.e.* to ON.

11. Corrections of Simple Circle Diagram

The effect of the resistances of the windings on the output can be represented by a straight line (ORS, Fig. 8.18) through O. For (see Chapter VI., Art. 16) the length (RN, Fig. 8.18) which this

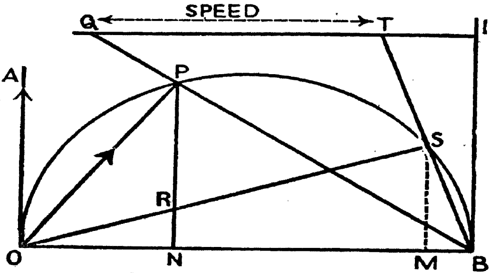


Fig. 8.18—CIRCLE DIAGRAM CORRECTED FOR RESISTANCE

cuts off from PN is proportional to OP^2, *i.e.* to the square of the current (I^2). Hence, by drawing the line at the proper slope, RN represents the I^2R loss, and PR the output, to the scale on which PN represents the input.

The line OS now represents the standstill current, the whole input at this point going in copper losses. If BS is produced to cut DQ in T, then DT represents the reduction in speed due to resistance, *i.e.* TQ represents the speed corresponding to a current OP.

The effect of the iron and friction losses is more difficult to represent exactly. They both increase with the speed, but in more than direct proportion in the former case at least. The iron losses depend further on the flux, again not in proportion, more nearly as its square. But, since low speeds are associated with large fluxes, and vice versa, it is fairly accurate to assume the sum of the iron and friction losses constant.

The construction of the diagram may then proceed as follows:—Let OS be the standstill current plotted at its proper angle of lag behind the p.d. vector OA (Fig. 8.19). Calculate OS^2 × resistance of motor windings, *i.e.* copper losses at standstill in watts. Draw SM, equal to this on the *input* scale, parallel to AO. Draw MO′ perpendicular to AO. Describe a circular arc with centre on MO′ and passing through S and O, and let this meet O′M produced in B. Join O′S. Carry out construction for speed as described above.

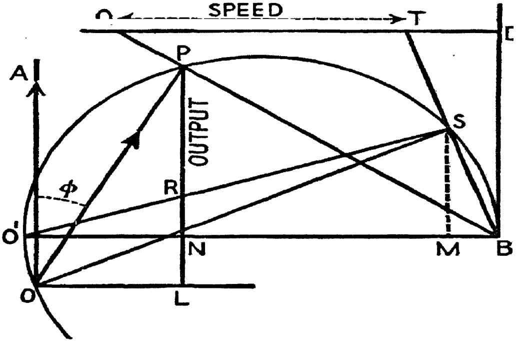


Fig. 8.19.—COMPLETE CIRCLE DIAGRAM.

For any current represented by OP draw PL parallel to AO, and OL perpendicular to AO. Let PL cut O′S in R, and O′B in N.

Then $\angle AOP = \phi$;

$$PL = \text{input},$$

$$\left.\begin{aligned} PR &= \text{output} \\ QT &= \text{speed} \end{aligned}\right\} \therefore \frac{PR}{QT} = \text{torque (to some scale)}.$$

The method of determining the scales is similar to that used for the induction motor (Chapter VI., Art. 19). The scale for speed cannot be found in this way. Either the speed at one particular current or load must be known, or else one such speed must be calculated from the magnetic circuit and other constants of the motor.

This circle diagram is not as accurate as that of the induction motor, but it is useful for showing the approximate performance of an A.C. series-wound motor It should not be used for determining the efficiency.

12. Application of Circle Diagram to Compensated Motor

The circle diagram described in Art. 11 applies to series-wound motors, whether they are compensated or not. But the proportions of the diagram are altered greatly by the introduction of compensation. The effect of compensation is to wipe out almost completely the inductance of the armature, leaving only that of the field winding. An additional resistance is introduced at the same time, viz. that of the compensating winding itself if series-connected.

The diameter of the circle, being equal to V/X, is increased by compensation. Thus the curvature of OO′P (Fig. 8.19) is reduced. Consequently, as shown in Fig. 8.20, the power-factor for equal inputs is increased, the current being changed from OP_1 to OP_2.

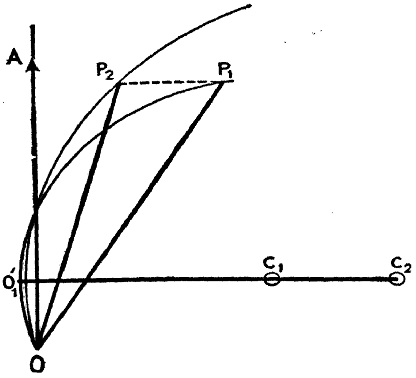


Fig. 8.20.—EFFECT OF COMPENSATION.

C_1 = Centre for uncompensated motor.
C_2 = ,, ,, compensated ,,
OP_1 = Current (uncompensated).
OP_2 = ,, (compensated).

The increased resistance of the compensated motor is counterbalanced by its current being smaller, due to the improvement of the power-factor. At small loads the total copper loss is greater in the compensated motor, but the loss is then small. At large loads the opposite is the case, and so, on the whole, the compensated motor has a considerable advantage in efficiency.

Since the maximum torque is approximately equal to the radius of the semicircle, the compensation has a very good effect on this, too.

When the compensating winding is a short-circuited one, the effect is very much the same. The inductance of the armature is not eliminated quite so completely, but nearly so. The additional resistance introduced is that of the short-circuited winding multiplied by the square of the ratio (No. of armature turn)/(No. of turns in compensating winding) (cf. the equivalent resistance of transformers —see Volume I., Chapter XVII., Art. 4).

13. Circle Diagram for Repulsion Motor

If the resistances of the windings and all other losses and magnetic leakage are neglected, the following relations hold between the quantities in a repulsion motor.

Let V_T = the voltage across the transformer winding (T, Fig. 8.21),

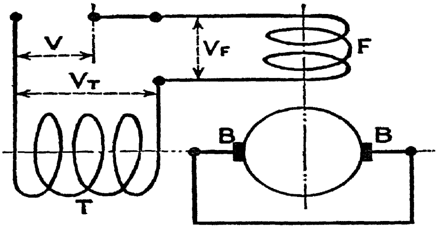


Fig. 8.21.—VOLTAGE REGULATION IN REPULSION MOTOR.

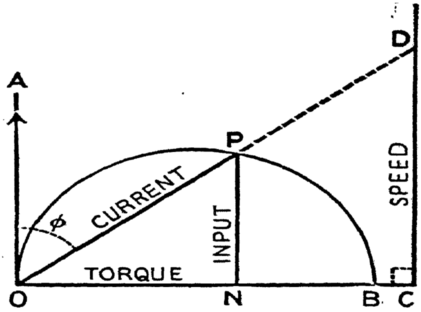


Fig. 8.22.—CIRCLE DIAGRAM OF REPULSION MOTOR.

V_F = the voltage across the field winding (F, Fig. 8.21),
Φ_T = flux along axis of T, *i.e.* BB,
Φ_F = flux perpendicular to BB.

Under the assumed conditions the e.m.f. produced by the transformer action of T in the armature circuit closed by BB, must be equal to the back e.m.f. produced by the rotation of the armature in the field due to F;

$\therefore$ each of these e.m.f.s is proportional to ($\Phi_F \times$ speed).

But the transformer e.m.f. = $V_T \times$ ratio of turns in armature to those in T; and V_T is proportional to Φ_T;

$\therefore$ Φ_T is proportional to ($\Phi_F \times$ speed).

At synchronous speed the inducing effects of rotation in a field and of the alternation of an equally strong field are equal. Therefore at this speed $\phi_T = \phi_F$, in order to make the e.m.f.s equal. Therefore at any speed $\phi_T = S \times \phi_F$, where S = speed expressed as a fraction of synchronous speed;

$$\therefore\ V_T/V_F = S \times r,$$

where r = ratio of (No. of turns in T) to (No. of turns in F). When a single stator winding is used, r = cot (electrical angle of displacement of brushes).

Again, V_T is in phase with the current, and may be put equal to I_1R_L, where I_1 = motor stator current, and R_L = a resistance equivalent to the load. Whereas V_F leads the current by 90°, and equals I_1X, where X = reactance of field winding F.

Thus the current I_1 is that sent by a constant p.d., V, through a constant reactance, X, and a variable resistance, R_L. Therefore the end of the current vector lies on a semicircle whose diameter is OB (Fig. 8.22), equal to V/X, and at right angles to the p.d. vector, OA. The proof is the same as for the rotor current of an induction motor (Chapter VI., Art. 14). The speed is proportional to V_T/V_F, *i.e.* to R_L/X, which is equal to cot ϕ. It can, therefore, be represented in the same way as for the series-wound motor (see *Fig. 8.17*). *An alternative method, applicable to the series-wound motor too,* is shown in Fig. 8.22. A line, CD, is drawn in any convenient position perpendicular to OB. The current vector is produced to cut this line in D. Then CD represents the speed to some scale, for—

$$CD = OC \cot CDO = \text{constant} \times \cot \phi.$$

In the case of the repulsion motor, however, the speed scale can be found more easily than for a series-wound motor. For

$$\cot \phi = R_L/X = V_T/V_F = S \times r.$$

Therefore at synchronous speed cot $\phi = r$, and so CD = r.OC which determines the speed scale. The same relation determines the speed scale if the other method is used for representing speed.

The torque is represented by ON, as in the case of the series-wound motor. But, since the speed scale can be found, the torque scale follows from this in a repulsion motor.

14. Armature Current in Repulsion Motor

The armature current is made up of two components. One is opposite in phase to the main current and produces an armature m.m.f. equal (but opposite) to the m.m.f. of the transformer winding (T, Fig. 8.21). The second component produces the flux ϕ_T along the transformer axis.

The armature turns are uniformly distributed, and all carry the same current. Therefore they produce a triangular space-wave of m.m.f. The fundamental sine component of this is only about 0·8 of the maximum value (see Chapter II., Example 1). Hence, the number of armature turns effective in producing an m.m.f. is 0·8 of the total number of armature turns.

Let a = (effective number of armature turns)/(number of turns in transformer winding). If the transformer winding is a distributed one, giving approximately a sinusoidal space-wave of m.m.f., the full number of turns is taken in finding the value of a. In other cases an allowance must be made according to the arrangement of the winding.

When a has been found, this determines the first component of the armature current which is equal to I_1/a, opposite in phase to the main current (I_1).

The magnetising component is proportional to and in phase with the transformer flux, Φ_T. The phase of Φ_T is $\frac{1}{4}$-period (*i.e.* 90°) ahead of the armature current component (I_1/a), and the same angle behind the main current. This phase relationship must hold because the e.m.f. induced by Φ_T is opposite in phase to the main current, forming the " back e.m.f." of the repulsion motor.

If, as is nearly always the case, the reluctance of the magnetic circuit is the same along the transformer axis and along the field axis, the fluxes are proportional to the ampere-turns producing them.

Let I_m = magnetising component of armature current. Then—

$$\frac{\Phi_T}{\Phi_F} = \frac{I_m \times (\text{No. of effective armature turns})}{I_1 \times (\text{No. of effective field turns})} = \frac{I_m}{I_1} \times a.r$$

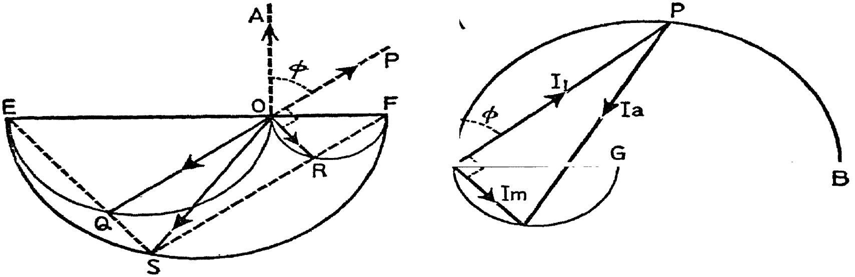
$$\text{But } \Phi_T/\Phi_F = S = \frac{\cot\phi}{r};$$

$$\therefore\ I_m = (I_1 \cot\phi)/ar^2.$$

The total armature current at any load can be obtained graphically from this equation, as shown in Fig. 8.23 (*a*).

Draw OE, leading the applied p.d., OA, by 90°. Make OE equal to V/aX; *i.e.* to $1/a$th of OB in Fig. 8.22. On OE as diameter describe a semicircle on the side away from A. Then if the main current vector, OP, is produced backwards to meet this semicircle in Q; OQ = I_1/a, and so OQ represents the balancing component of the armature current.

Produce EO to F and make OF = EO/r^2. On OF as diameter describe a semicircle on the side away from A. Draw OR perpendicular to POQ, cutting this semicircle in R. Then OR represents



(*a*) ARMATURE CURRENT. (*b*) STATOR AND ARMATURE CURRENTS.

Fig. 8.23.—ARMATURE CURRENT IN REPULSION MOTOR.

the magnetising component (I_m) of the armature current. For it leads OQ by 90°, and so is correct in phase. And its length equals FR cot FOR, *i.e.* FR cot ϕ. But the triangles FOR, OEQ, have their corresponding sides parallel;

$$\therefore \text{ FR/OQ} = \text{FO/OE} = 1/r^2;$$

$$\therefore \text{ OR} = \text{FR} \cot \phi = \text{OQ} (\cot \phi)/r^2,$$

$$i.e. \text{ OR} = (I_1 \cot \phi)/ar^2 \text{ (since OQ} = I_1/a).$$

Thus OR represents I_m in phase and magnitude.

The total armature current I_a is the vector sum of its two components OR, OQ; *i.e.* it is the diagonal OS of the *rectangle* OQSR. Therefore the end of the armature current vector lies on a semicircle having EF as its diameter.

This diagram can be combined with the circle diagram for the main current (Fig. 8.22). A simplification is obtained by drawing the armature currents to a scale *a* times as large as the scale for the main current, OE then becomes equal to OB, and OQ to OP. OF becomes equal to BO/r^2, *i.e.* OG in Fig. 8.23 (*b*); and OR becomes OT.

Then the armature current is equal to the vector sum of OP reversed and OT, *i.e.* it is given by PT on its own scale.

15. Corrections to Circle Diagram

The effect of the resistances of the windings on the output and the speed can be found in the same way as for the series-wound motor (see Art. 11) if the standstill current is known. The method is less accurate in this case because the armature current does not alter in the same proportion as the main current. When greater accuracy is required, the following method may be used.

Let R_1 = resistance of stator winding, *i.e.* the sum of the resistances of the "transformer" and "field" windings when these are separate; and R_a = resistance of armature between the brushes, including the short-circuiting lead. Then at standstill the equivalent resistance of the motor is $R_1 + (R_a/a^2)$, as in the case of a transformer.

As the speed rises, the ratio of the armature current to the main current rises, and it follows from Art. 14 and Fig. 8.23; that: $I_a^2 = OS^2 = OQ^2 + OR^2 = (I_1/a)^2 + (I_1 \cot \phi/ar^2)^2 = (I_1/a)^2 \{1 + (\cot \phi/r^2)^2\} = (I_1/a)^2 \{1 + (s/r)^2\}$ (see Art. 13). Thus at any speed = S × synchronous speed, the equivalent resistance of the motor (apart from the resistance representing the output) is equal to—

$$R_s + \left\{ 1 + \left(\frac{S}{r}\right)^2 \right\} (R_a/a)^2.$$

This can be represented graphically as follows in Fig. 8.24, in which OA is the applied p.d., OB the diameter of the current semicircle, and OC the standstill current (I_s). CD, perpendicular to OB, gives to some scale the input at standstill. Make DE equal to I_sR_1 on this scale. Join OE.

When the main current is OP, draw PN perpendicular to OB, cutting OE in M. Then MN gives the copper loss in the stator windings (cf. Art. 11).

On EC mark off EF equal to $(I_s/a)^2 . R_a$ and join OF. Make OG equal to OB/r [as in Fig. 8.23 (*b*)] and draw GH perpendicular to OG, meeting OF in H. Draw HK parallel to GO, meeting OA in K

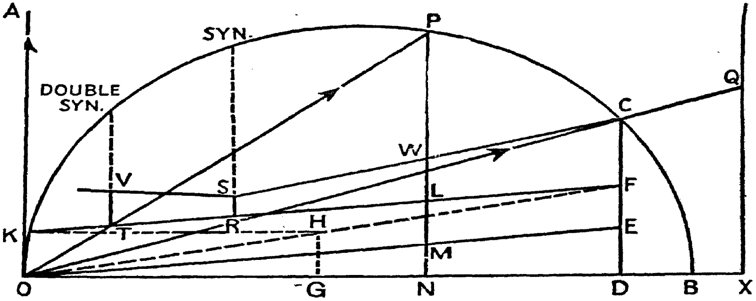


Fig. 8.24.—CORRECTED CIRCLE DIAGRAM FOR REPULSION MOTOR.

(or make OK = GH). Join FK, cutting PN in L. Then LM gives the armature copper loss.

The leakage reactances can be taken into account in a similar way. But as the equivalent leakage reactance of the armature varies as $\{1 + (S/r)^2\}$, just as its resistance does, the assumption of constant reactance is seen to be erroneous. This means that the end of the current vector does not follow exactly a semicircle as assumed. The error increases with the speed, and so when accuracy at high speeds is required the circle diagram must not be used. In these cases its place can be taken by calculation from the formulae developed above.

The speed scale is corrected by producing OC (if necessary) to cut the speed line in Q (see Fig. 8.24), and measuring speeds from Q instead of from X, the bottom end of the speed line.

An allowance for friction can be made by subtracting a constant amount from the torque. The torque requires a further correction for the armature core loss.

At synchronous speed the armature core loss is zero. For the two fields, Φ_T and Φ_F, are then equal and in quadrature both in time and in space (this last is always the case). Consequently the resultant field is, as in a two-phase induction motor, a rotating one of constant magnitude. And the armature rotates at the same speed and so has no iron loss. At other speeds the two fluxes are unequal, and the resulting field both rotates and varies in magnitude. Consequently the core loss per cycle is different for different portions of the armature, and exact calculation becomes laborious.

The iron losses in the stator depend on the fluxes and the supply frequency in the usual way.

An approximate method for allowing for the iron losses is to calculate them for the stator at synchronous speed and to plot this above KLF, as shown by RS in Fig. 8.24. Then join CS. At twice synchronous speed the armature core loss may be assumed to be the same fraction of the total as at standstill, viz. (CF — SR)/CF. Hence if the stator core loss at this speed is calculated as L_{2s}, the total loss is then L_{2s} (CF/SR). Plot this at the corresponding position of PN as shown by TV. Join SV and produce it. Let CSV cut PN in W. Then WL gives the core loss approximately. And (PW/PN) gives the efficiency.

As in the case of the series-wound motor (Art. 11) this circle diagram does not give great accuracy, but it shows the behaviour of the motor in a general way. It cannot be applied to a compensated repulsion motor (see next Article).

16. The Compensated Repulsion Motor

This motor differs in behaviour from the plain type owing chiefly to the fact that its equivalent reactance is not approximately constant. This difference is due to the e.m.f. set up between the brushes CC (Fig. 8.25) by the rotation of the armature in the field produced by the transformer winding, *s*.

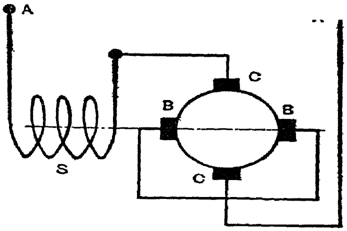


Fig. 8.25.—COMPENSATED REPULSION MOTOR.

At standstill the brushes BB are equivalent to a short circuit of the transformer winding, and therefore the ideal standstill current (*i.e.* neglecting resistance and magnetic leakage) is given by:—

$$I_s = V/X_A \dots\dots (1)$$

where V = applied p.d.; X_a = armature reactance.

When running under a load equivalent to R_L, the p.d. across the transformer winding (V_T) is given by—

$$V_T = I_1R_L \text{ (see Art. 13)} \dots\dots (2)$$

$$\text{and } \Phi_T = S\Phi_F \dots\dots (3)$$

The field flux (Φ_F) depends only on the current between the brushes CC, being unaffected in magnitude by the currents in the other circuits;

$$\therefore \text{ Voltage required to produce the field flux} = c\,.\,\Phi_F \quad \dots (4)$$

(where c is a constant for a particular motor), and this leads the flux, and therefore the current, by a quarter of a period.

For the rotational *voltage* between the brushes CC we have—

$$\text{Rotational voltage} = c\,.\,S\Phi_T = cS^2\Phi_F \dots\dots (5)$$

This is in phase with Φ_T and therefore lags a quarter of a period behind the current. It is thus opposite in phase to the magnetising voltage. So the net voltage (V_A) between the brushes CC is equal to the difference between the two (see 4 and 5).

$$\text{That is } V_A = c\Phi_F - cS^2\Phi_F\text{; or } V_A = c\Phi_F(1 - S^2) \dots\dots (6)$$

At standstill this reduces to $c\Phi_{FO}$;

$$\therefore c\Phi_{FO} = V_A = I_SX_A \dots\dots (7)$$

where Φ_{FO} = field flux at standstill.

But, neglecting changes of reluctance, Φ_F is proportional to I_1; therefore for any corresponding values:—

$$c\Phi_F = I_1X_A \dots\dots (8)$$

Substituting this in equation (6), the voltage between brushes CC is given by—

$$V_A = I_1 X_A (1 - S^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

i.e. Apparent reactance of armature $= V_A/I_1 = X_A (1 - S^2)$ ..(10)

The p.d. across the transformer winding (V_T) is $(1/a)$th of the transformer voltage in circuit BB;

$$\begin{aligned} \therefore\ V_T &= (1/a)\, c\,.\,\Phi_T \\ &= (1/a)\, c\,.\,S\Phi_F \quad \text{from (4)} \\ &= (S/a)\,.\,I_1 X_A \quad \text{from (8)} \\ \therefore\ R_L &= V_T/I_1 \text{ (see equation 2)} \\ &= (S/a)\, X_A \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11) \end{aligned}$$

Some of these relations are shown graphically in Fig. 8.26, for $a = 0{\cdot}6$. The load resistance (R_L) plotted against the speed is a straight line, whereas the reactance is a parabola (see 10). Combining these gives the apparent impedance, and thence the current and the power-factor can be calculated.

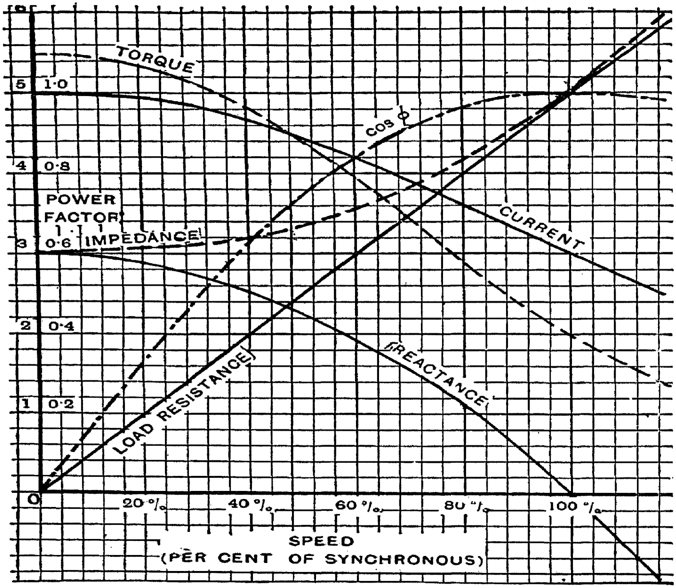


Fig. 8.26.—PERFORMANCE CURVES OF COMPENSATED REPULSION MOTOR.

Since the losses are neglected the torque is proportional to (input/speed), *i.e.* to $(I_1 \cos \phi/S)$, *i.e.* to (R_L/Z^2S), *i.e.* to $(1/Z^2)$, since R_L is proportional to S.

17. Modifications in Formulae of Compensated Repulsion Motors

The function of the field winding in an ordinary repulsion motor is fulfilled by the armature winding itself in the compensated form. Hence this behaves to some extent like a plain repulsion motor with the same number of effective turns in these two windings. Thus since—

$r =$ (No. of transformer turns)/(No. of field turns); see Art. 13: and $a =$ (No. of armature turns)/(No. of transformer turns); see Art. 14.

$$a . r = 1, \text{ in a compensated repulsion motor.}$$

Hence the expression for the magnetising component of the armature current (Art. 14) becomes:—

$$I_m = (I_1 \cot \phi)/r = SI_1.$$

The transformer component of the armature current is still I_1/a and the two components are, as before, 90° out of phase;

$\therefore$ Total armature current in circuit BB (Fig. 8.25), $= I_A$,

$$= \sqrt{\{(SI_1)^2 + (I_1/a)^2\}} = I_1\sqrt{\{S^2 + (1/a)^2\}}.$$

Hence if $R_T =$ actual resistance of transformer winding,

and $R_B =$ resistance of armature and connexions in circuit BB;

Equivalent resistance of transformer winding

$$= R_T + R_B \{S^2 + (1/a)^2\}.$$

To this must be added the actual armature resistance (R) including brushes and connexions to CC. The total is then added to the load resistance R_L to give the total equivalent resistance of the motor.

In the same way the reactance of the ideal motor must be increased by $X_T + X_B \{S^2 + (1/a)^2\}$; where X_T, X_B are the leakage reactances of the transformer winding and the armature (circuit BB) respectively.

The iron losses can be calculated by obtaining the respective fluxes (Art. 16) and combining them. But the calculation is long because different portions of the armature core have different cycles of magnetisation.

The friction can be taken as a constant torque.

Up to synchronous speed these modifications do not much affect the behaviour of the motor. But at higher speeds their effects increase rapidly.

18. Mixed Action Motors

Though commutator motors can be given shunt characteristics, it is more usual to employ for constant speed drives motors using two different actions. These act as repulsion motors during starting, and are converted into induction motors when the speed has risen sufficiently. The commutator is, therefore, in use only while starting and so it is unnecessary to suppress sparking to the same extent as in the purely commutator type. Their one drawback is that they give only one running speed, whereas the commutator type can readily give a range of speeds up to 50 per cent. above the minimum without loss of efficiency.

One method used in the mixed type is to employ a centrifugal device. The motor starts as a repulsion motor. When the speed has reached about three-quarters of synchronous value the centrifugal mechanism comes into play. It presses a flexible ring against the commutator segments, short-circuiting them. The motor then acts as a monophase induction motor. The poor starting of the latter type is thus overcome.

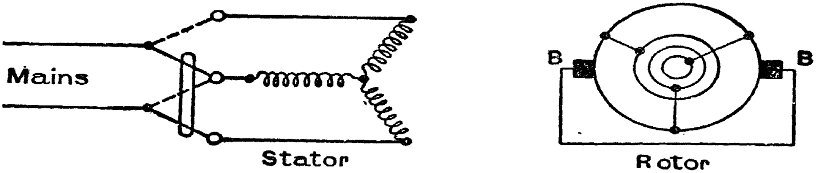


Fig. 8.27.—SCHÜLER MOTOR.

At the same time as the commutator is short-circuited the brushes are removed from it. This action is facilitated by making the surface of the commutator perpendicular to the shaft, in the form of an annular ring. The brush rocker, therefore, only has to be moved parallel to the shaft to remove the brushes from the commutator.

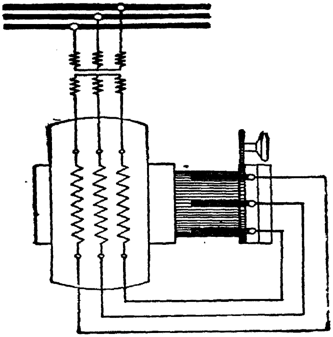
The Schüler single-phase motor (Fig. 8.27) has its stator wound like a star-connected three-phase induction motor. Its rotor has a D.C. armature winding connected at one end to a commutator, and at the other to three slip-rings. The brushes, B, B, on the commutator are connected together.

When starting, the brushes on the commutator cause the ordinary repulsion action, since the axis of the resultant stator field is 30° (electrical) from their axis. By throwing over the switch (as shown - - -) the direction of the starting torque is reversed, the idle phase being changed. The slip-rings are not connected together at first.

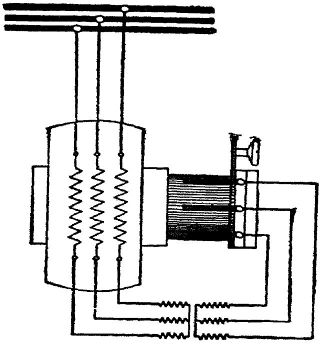
When the motor has started the slip-rings are connected to a Y- or V-connected rheostat just as in the case of a polyphase wound-rotor induction motor. The resistance of this is reduced step by step and it is finally short-circuited. The action of the motor is thus changed gradually from repulsion to single-phase induction. The slip-rings are then short-circuited and *all* the brushes are then lifted.

19. Polyphase Commutator Motors

The principles of action of polyphase commutator motors are in general the same as those of the corresponding single-phase types.



With main transformer.
* Fig. 8.28.—THREE-PHASE SERIES MOTOR.



* Fig. 8.29.—THREE-PHASE SERIES MOTOR WITH ROTOR TRANSFORMER.

Some modifications are caused by the difference in the supply, particularly in the methods of speed control as noted below.

The series type of motor can be used on a polyphase supply. If this is three-phase, three brush sets per pole-pair are required placed 120° (electrical) apart (see Fig. 8.28). On a two-phase supply there are four brush sets at 90°.

The voltage on the armature must be kept low, and if the supply voltage is too high a transformer is necessary. The size of the transformer can be reduced by supplying the stator from the mains and connecting the Y-connected primary windings in series with the stator windings (see Fig. 8.29). Thus the transformer has

* Reproduced from *J.I.E.E.*, vol. 60, p. 311.

to deal with the armature power only, instead of with the total power. The reduction is, however, a small percentage, except in small size motors.

The usual method of controlling the speed of these motors is by shifting all the brushes through the same angle. This is necessary only when different speeds for the same load torque are required. There is a likelihood of instability* at the lower speeds with this method unless the power-factor is allowed to be low (see Fig. 8.30).

By using one set of fixed brushes and another movable set instability can be avoided and a high power-factor obtained. Each secondary of the transformer is then connected between a fixed brush and a movable brush (see Fig. 8.31).

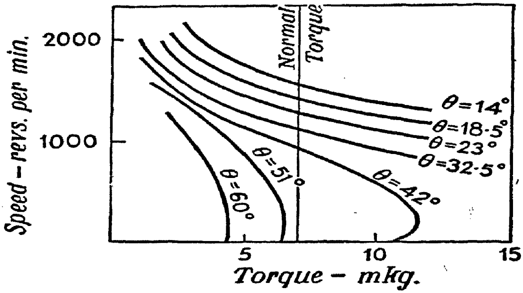
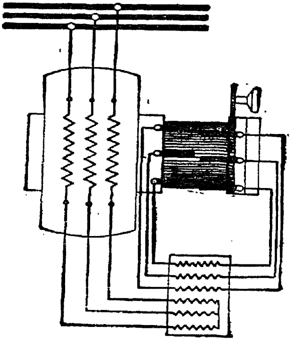


Fig. 8.30.—THREE-PHASE SERIES MOTOR CHARACTERISTICS.



* Fig. 8.31.—SERIES MOTOR WITH FIXED AND MOVABLE BRUSHES.

20. Polyphase Shunt Commutator Motors

A shunt characteristic can be given to this type of motor by connecting the rotor transformer primaries directly to the mains (see Fig. 8.32), instead of in series with the stator as in Fig. 8.29. A simpler alternative is to connect the brushes to tapping points on the Y-connected stator windings.

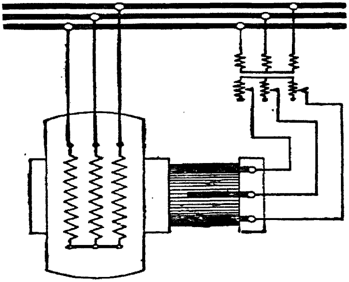
In both these cases the action is similar to that of an induction motor. But the addition of the commutator to the rotor makes the frequency in the circuits connected to the rotor brushes equal to the supply frequency. This enables the rotor to take power

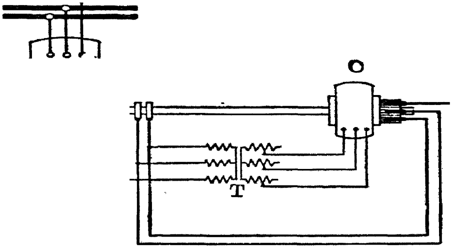
* See "Single and 3-phase A.C. commutator motors, etc." S. P. Smith, *J.I.E.E.*, vol. 60, p. 308.

directly from the mains in addition to that supplied by way of the stator, or to return part of the latter power to the mains.

If the speed is below synchronism the e.m.f. generated in the rotor is stepped up in voltage by the transformer and fed back into the mains. The greater the ratio of the number of turns on the commutator side of the transformer to those on the rotor side the greater the slip. If the brushes are moved through 180° (electrical) the action is reversed. The rotor is now supplied with power from the mains, and the speed rises above synchronism by an amount varying with the number of turns connected to the commutator.

When the power is too large to be dealt with by a motor of this type (say over 250 kW) it may be combined with an induction motor (see Fig. 8.33). The commutator motor then has to deal only with the slip power, which is the same fraction of total power as the slip is of synchronous speed. By making the synchronous speed about the middle of the range required the size of the commutator motor is kept down. The speed is set by the number of turns connected in circuit in the variable ratio transformer.


* Fig. 8.32.—THREE-PHASE SHUNT MOTOR.



* Fig. 8.33.—VARIABLE SPEED COMBINATION.

C. Commutator Motor. I. Induction Motor. T. Transformer.

21. Schrage Type Motor

In this type the arrangement of two motors described in the preceding Art. and shown in Fig. 8.33 is combined in a single motor in the following way.

The "stator" of the induction motor is made the rotating portion. Consequently it is fed from the 3-phase supply mains through slip-rings (R_1, R_2, R_3, Fig. 8.34), instead of by fixed terminals. The other ends of these windings are connected to a star point (S).

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* Reproduced from *J.I.E.E.*, vol. 60, pp. 311-13.

This rotating part carries in addition an independent auxiliary winding, similar to that of a D.C. armature and connected to a commutator in the same way. Six sets of brushes are connected in pairs to opposite ends of the three-phase windings of the "rotor", or secondary of the induction motor. This "rotor" is the stationary part of the motor.

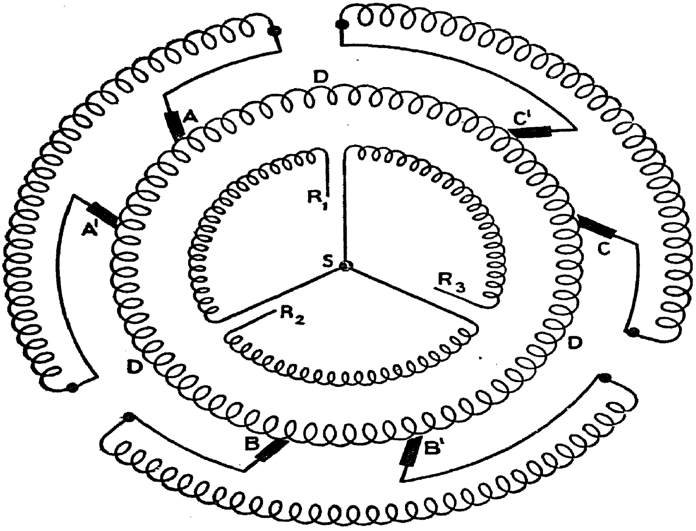

Fig. 8.34.—SCHRAGE MOTOR.

The three brushes (A, B, C, Fig. 8.34) joined to corresponding points of the "rotor" phases are 120° (electrical) apart; and the other three (A′, B′, C′) joined to the other ends of these phases are likewise 120° apart. Each set of three brushes can be moved for speed control. Thus the e.m.f.s applied to the "rotor" phases by the commutator are always 120° apart in phase.

As far as induction motor action is concerned the interchange of movement of "stator" and "rotor" makes no difference. The e.m.f. set up, by the auxiliary winding D, between a pair of brushes, such as AA′, is always of *slip* frequency. For the field due to the "stator" currents rotates at synchronous speed relative to the rotating "stator." And the induction motor action drives the "stator" round in the opposite direction. Thus the speed of field relative to the stationary "rotor" is the slip speed, and this is its speed relative to the fixed brushes on the commutator. The frequency of the e.m.f. between a pair of brushes depends on this speed, and so is the slip frequency, as stated above.

In particular, at synchronous speed the field becomes stationary, and at higher speeds the rotation of the field is reversed. Hence the e.m.f. is reversed, but it is still of slip frequency though the slip is now negative.

The magnitude of this e.m.f. can be adjusted by altering the distance between the brushes. The farther apart they are the slower the motor will run, so as to increase the slip and generate in the rotor windings enough e.m.f. to balance that generated in the winding D. When the load on the motor is increased the rotor current must increase, and so the speed must fall. But this fall in speed is only that needed to overcome the extra drop in the impedance of the rotor circuit since the e.m.f. of D is almost unaffected. Whereas with rotor resistance speed control the increased current causes more drop in the regulating resistance, and so the speed drops much more at heavy loads (see Chapter VI., Art. 33).

If the positions of the brushes of each pair are interchanged the injected e.m.f. is reversed. Consequently the rotor must now run above synchronous speed so as to generate an e.m.f. to balance that of D. apart from the small impedance drop.

22. Power-Factor Control in Schrage Type Motors

The results obtainable with this type of motor with the midpoint of the pairs of brushes fixed are shown in Fig. 8.35. In this case the arrangement is similar to that shown in Fig. 8.31, except that the two sets of the brushes are moved the same distance in opposite directions by the one hand-wheel. It is a 6-pole motor run on a 50 cyc. supply, and the curve marked "normal speed" shows its behaviour as an ordinary induction motor, viz. with the pairs of brushes set level. At full load (9·2 Kg.-M.) it has a speed 65 r.p.m. less than at no load, and develops 12 b.h.p.

At the lowest speed setting the fall of speed from no load to full load has increased to 80 r.p.m.; and the horse-power has diminished in proportion to the speed (since torque is unchanged), *i.e.* in this case it develops 6 b.h.p. At the highest speed setting the speed variation has become 120 r.p.m. (8 per cent.) and the full load 18 b.h.p. A fourth speed graph is given, for a setting which gives synchronous speed at full load.

As the speed is increased the power-factor is raised, though in a smaller ratio, and at full speed and load unity power-factor is very nearly reached.

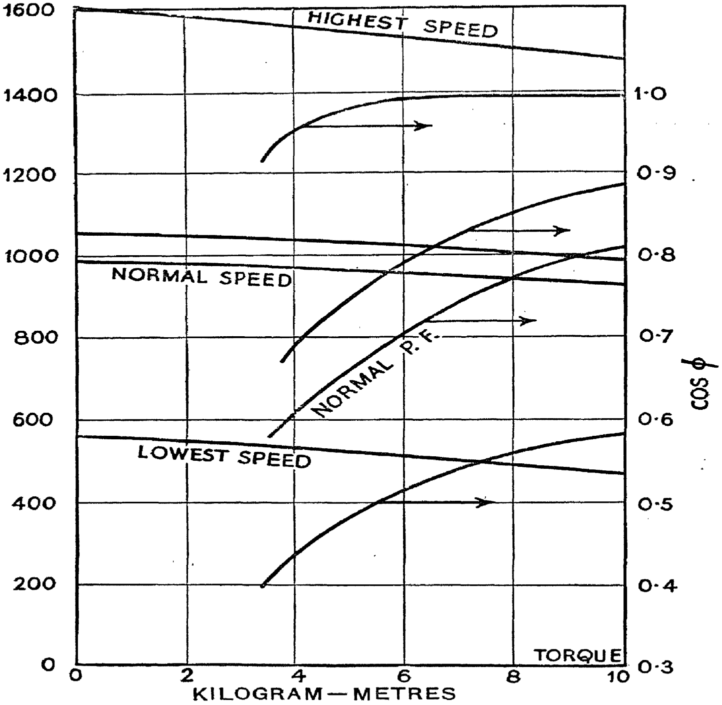

Fig. 8.35.—CHARACTERISTICS OF SCHRAGE MOTOR.

If all the brushes are moved, without altering their distances apart, the phase of the injected e.m.f. is altered without much change in its magnitude. The main effect of this is to alter the phase of the rotor current, though its magnitude is affected to some extent. Consequently the stator current is altered in a similar way, and so the power-factor of the motor can be improved. This enables a high power-factor to be obtained at full load for all speeds.

The drawback of this method is that the brush-shifting arrangements are made more complicated. This is due to the need for shifting the centre-lines of the pairs, in addition to altering their distances apart. The question of whether the gain is worth the extra cost depends partly on the speed range, partly on the size of motor (a large size giving greater saving by p.f. improvement), and partly on the skill of the operator.

23. Transformer Connexions for Speed Control

When the speed of an A.C. series or repulsion motor is to be varied by altering the applied p.d. (see Arts. 2, 4, and 6) all that is necessary is the provision of a number of tapping points on the secondary of the transformer or autotransformer, with switches to connect to these as required.

In passing from one tapping point to the next it is necessary to avoid short circuiting the part of the secondary winding between the two taps. The simplest way, as used on the Hamburg-Altona Railway and on the original L.B. and S.C. Railway equipment is to open one switch before closing the next. This has the drawback of breaking the full current each time; so more usually one or more impedances (called **preventive coils**) are employed to prevent a short circuit.

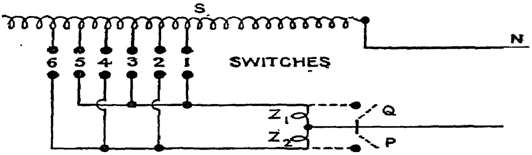

Fig. 8.36.—PREVENTIVE COIL CONTROL.

One method of using two impedances is shown in Fig. 8.36. Here S is the secondary winding of the transformer, M and N the leads to the motors, and Z_1 Z_2 the impedances. The order of switching is:—close 1, close 2, open 1, close 3, open 2, etc. One of the impedances is always in circuit, and if suitably chosen there will be little sparking on opening switch 1 after switch 2 has been closed, and so on.

To avoid leaving Z_2 in circuit after the full voltage has been applied an additional switch P (shown dotted) may be fitted. By using another switch for short circuiting Z_1 (viz. Q in Fig. 8.36) the motor can be run from any tapping point without impedance in series. This arrangement makes intermediate control positions possible as follows:—close 1, close Q, close 2, open 1 and Q, close P, close 3, open 2 and P, close Q, etc.

This requires higher impedances to subdue sparking since only one of them at a time is in the circuit between adjacent tapping points. This may be avoided by opening Q at the instant of closing 2, opening P at the instant of closing 3, etc. But as this requires exact adjustment of the movements of the switches the former method is preferable.

If the two impedance coils are wound on a common core, *i.e.* they consist of a choking coil with a centre tap, then when two neighbouring switches are closed this arrangement acts as a balancing transformer. It causes the motor current to divide itself almost equally between the two closed switches; the difference being equal to twice the magnetising current needed to generate the voltage between two successive tapping points. The order of switching can then be made nearly the same as the original, viz.:—close 1 and 2, open 1 and immediately afterwards close 3, open 2 and immediately afterwards close 4, etc. But the final position is with the last two switches (5 and 6 in Fig. 8.35) both closed. This has the advantage that each switch carries only half the motor current except for any momentary rise which may occur between opening 1 and closing 3.

When a similar arrangement is used for regulating the voltage of a supply network the switches P and Q are included. With these the supply voltage can be made that of any tapping point, by closing the corresponding switch and either P or Q; or it can be made half-way between the voltages of two tapping points, by closing both the corresponding switches and leaving P and Q open.

The sparking can be reduced further by connecting a " preventive " resistance in parallel with the preventive choking coil, with their middle points connected together. This reduces the choking effect of the half-coil when only one switch is closed, and so allows the current through this switch to rise more rapidly in the interval before the next switch is closed. The current in the arc produced by opening a switch can therefore die out more quickly. Some waste of power is caused by this resistance even when two switches are closed, and if the resistances are too low the waste becomes excessive, or the two portions of the current depart too far from equality.

24. Control by Induction Regulator

Other methods of control employ an induction regulator (see Chapter IX., Art. 13), *i.e.* a transformer whose voltage ratio can be adjusted. The simplest method is to make the maximum voltage of the regulator equal to *half* the range of voltage required; *e.g.* with a regulator giving a maximum of 100 volts, and a tapping point on the autotransformer giving 250 volts, any p.d. between 150 volts and 350 volts can be applied to the motors by varying the position of the regulator.

The advantages of this method are that only a single switch is required, that the change in the voltage is smooth instead of by steps, and that the only sparking that occurs is on opening the switch when cutting off power from the motors. The drawbacks are that the induction regulator is heavy, that the movement of it and its gearing absorbs considerable energy, and that it lowers the power-factor of the load by its magnetic leakage.

In Lydall's method an induction regulator is combined with a transformer with a number of tapping points as shown in Fig. 8.37. The primary, P, of the regulator can be rotated, and is connected across the full voltage of the secondary winding, S, of the autotransformer. The secondary winding (R) of the regulator is connected like a preventive coil (cf. Fig. 8.36).

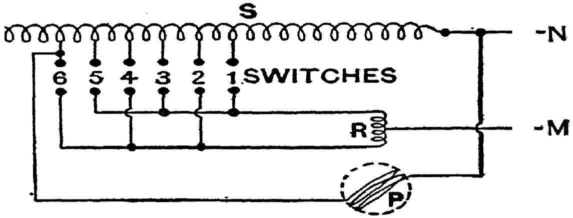

Fig. 8.37.—LYDALL'S METHOD OF CONTROL.

Switch 1 is closed when the regulator is in the position in which its secondary e.m.f. opposes the transformer e.m.f. The regulator is then turned through 180° (electrical) raising the p.d. applied to the motors by double the voltage induced in the half of R in circuit. Switch 2 is then closed and switch 1 opened. As the current from 2 flows through R in the opposite direction to that from 1, the induction regulator e.m.f. is opposed to the transformer e.m.f. as at the start. The regulator is turned again through 180° to raise the motor p.d., and then switch 3 is closed and switch 2 is opened. This action is repeated for each step.

This gives a nearly smooth increase of voltage, and the size of the induction regulator is much smaller than when it has to produce the whole change of voltage. By gearing the switch controller barrel to the induction regulator all the movements can be effected by a single handle.

In an improved modification a two-phase induction regulator is used. Each phase in turn is used as a preventive coil after the regulator has been turned through 90° (electrical). The two centre points are joined together through another preventive coil with the motor lead taken from its centre.* This further reduces the current which has to be interrupted when a switch is opened.

QUESTIONS ON CHAPTER VIII

1. The single-phase series motor may be said to be an adaptation of the direct series motor to the conditions imposed by working it with alternating current. Explain carefully the various points in the adaptation. [C. & G., II.

2. Describe some form of single-phase commutator motor and state its advantages and disadvantages.

3. Make a sketch of the magnetic circuit of a single-phase series commutator motor and explain carefully how the various electric circuits are located. Indicate the essential differences in the design of this motor as compared with the direct current series type. [C. & G., Final.

4. Describe the method of action of a repulsion motor.

5. Describe a single-phase compensated repulsion motor. Under what conditions has such a motor unity power-factor ?

6. Draw the vector diagram of a single-phase series motor with neutralized armature reaction. What are the advantages of this type of motor for traction work as compared with a motor of the repulsion type ?

7. Explain the method of operation of some form of single-phase motor capable of working at a high power-factor. Draw curves connecting torque, speed, and load. How does the efficiency of such a machine compare with that of a direct current motor of the same output ?

8. Give an account of a single-phase commutator motor, and of the method of determining its circle diagram. In what respects has this motor advantages compared with other single-phase motors ?

9. Give a list, without explanation, of the relative advantages of direct current motors, and single-phase and three-phase commutator motors.

10. Give an account of the most economical method of obtaining speed variations of large induction motors.

11. Draw up a list of the successive positions of the various switches in Fig. 8.36 when using balancing transformer control. Start with switches Q and No. 1 closed, and end with Q and No. 3 closed; distinguish between transition positions and running positions of the set of switches.

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* See *Electric Traction* (Wilson and Lydall), vol. II., chap. VI.

CHAPTER IX

ROTARY CONVERTORS

1. Rotary Convertors

When direct current is required from an A.C. supply, *e.g.* for propelling trams, it can be obtained by means of a motor-generator set, comprising an A.C. motor and a D.C. generator. The motor may be either of the synchronous or of the induction type.

An alternative method when a synchronous motor is used would be to make the magnets the stationary portion and to use the same magnets for the D.C. generator. This would effect some saving of material, but the expense and complication of the double-wound armature usually outweigh this advantage.*

A much better method is to employ a **rotary convertor.** In this the *same* armature *winding* is used both for the synchronous motor and for the D.C. generator. It is built up in the usual manner for D.C. armatures, and has in addition, at the end of the armature away from the commutator, a number of slip-rings. These are connected to suitable points of the armature winding, and the slip-ring brushes are connected to the A.C. supply.

In a two-pole machine, or one with a simple wave-winding, each slip-ring is connected to one point of the armature winding. In a multipolar simple lap winding each slip-ring is connected to as many points as there are pole-pairs. The tapping points for any one slip-ring are twice the pole-pitch apart, thus the slip-rings act as equalising rings (see Volume I., Chapter VIII., Art. 17). The types of rotary employed are:—

	No. of Slip-rings	Electrical Angle between Tapping Points
Single-phase	2	180°
Three-phase	3	120°
Two-phase	4	90°
Six-phase	6	60°
Twelve-phase	12	30°

Instead of using the above names the rotaries may be called according to their number of rings, three-ring instead of three-phase, four-ring instead of two-phase, etc. Nine-ring rotaries have been suggested but have not come into general use.

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* See further "The Binary Converter," by F. Creedy, *The Electrician*, vol. XCVI., p. 570.

The six-phase and the twelve-phase types can be supplied from a three-phase high-pressure circuit (see Arts. 9 and 10). If either is to be supplied from two-phase high pressure it can be done by combining the Scott (or other) method of transformation from two-phase to three-phase (Chapter XII., Art. 23) with the methods for supply from three-phase, or by other special arrangements of the transformers.

2. Voltage Ratios

In order that the alternating e.m.f. may be sinusoidal it is necessary for the field distribution to be sinusoidal, and this will be assumed to be the case in what follows.

In a single-phase rotary the alternating e.m.f. is maximum when the tapping points are at the neutral points of the field. If the commutator brushes have no lead they are connected to the conductors which are at any instant at these neutral points. Hence, if the effect of resistance is neglected:—

$$\text{Maximum A.C. voltage} = \text{D.C. voltage,}$$

$$\text{R.M.S. value of A.C. volts} = \left(\frac{1}{\sqrt{2}}=\right) 0{\cdot}707 \times \text{D.C. volts.}$$

In a polyphase rotary the p.d. between tapping points is less than for monophase tappings under the same conditions because there are fewer coils in series between the tapping points in the former case. But the reduction of voltage is less than the reduction in the number of coils in series. The reason for this is that the maximum difference in phase between the various coils in series is reduced when their number is reduced.

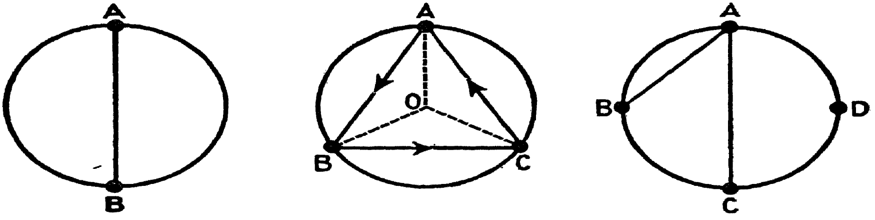
The magnitude of voltage of each coil is the same. The phase difference between the voltages of any coil and of the next one is constant, being the constant distance between two neighbouring coils expressed as an electrical angle. Consequently the vector diagram for the whole armature is a regular polygon.

Unless the number of coils is unusually small the polygon may be replaced by a circle (cf. Chapter IV., Art. 5). The tapping points are represented by points on this circle, and the electrical angles between the tapping points are given by the angles between the radii drawn to these points (see Fig. 9.01).

Thus for a monophase rotary the tapping points are at the ends of a diameter of the circle. This diameter [AB, Fig. 9.01 (*a*)] is the sum of the vectors representing the p.d.s of the separate coils, and therefore represents the p.d. between the tapping points. If it represents the maximum value of the p.d. the diameter must equal the D.C. voltage, and if it represents the R.M.S. value it equals 0·707 × D.C. volts, as obtained previously.

In a three-phase rotary the tapping points will be represented by three points A, B, C [Fig. 9.01 (*b*)] such that each of the angles AOB, BOC, COA is 120°, where O is the centre of the circle. The straight line AB is the sum of the vectors, represented approximately by the arc AB, which give the p.d.s of the coils between these tapping points. Therefore AB represents the p.d. between these points. Similarly the straight lines BC, CA represent the p.d.s between the corresponding tapping points. And AB, BC, CA represent three equal voltages with phase differences of 120°, *i.e.* three-phase voltages.

The magnitude of each of these lines is $\sqrt{3}/2$, *i.e.* 0·866 times the diameter of the circle. Therefore the R.M.S. value of the three-phase voltages is $\sqrt{3}/(2\sqrt{2})$, *i.e.* 0·612 times the D.C. volts.



(*a*) MONOPHASE. (*b*) THREE-PHASE. (*c*) TWO-PHASE.

Fig. 9.01.—VECTOR DIAGRAMS OF ROTARY CONVERTOR.

Similarly in a two-phase rotary the tapping points are represented by four points A, B, C, D [Fig. 9.01 (*c*)] 90° apart. The p.d. between two opposite tapping points is the same as that of a monophase rotary, viz. 0·707 × (D.C. volts). The p.d. between two neighbouring tapping points, *e.g.* A, B, is given by the length of AB, and:—

$$AB = AC/\sqrt{2} = 0{\cdot}500 \times (\text{D.C. volts}).$$

The cases of six-phase and twelve-phase rotaries are dealt with in Art. 6.

3. Current Ratios

If the losses are neglected the output is equal to the input. Hence in a monophase rotary:—

$$V_cI_c = VI\cos\phi,$$

where V_c = D.C. terminal volts, and I_c = D.C. amperes.

If the power-factor is unity, $V_c I_c = VI$;

$$\therefore\ I/I_c = V_c/V = \sqrt{2} = 1{\cdot}414.$$

For other power-factors the alternating current is greater in inverse proportion to the power-factor.

Similarly in a three-phase rotary at unity power-factor:—

$$V_c I_c = \sqrt{3}\ VI,$$

where V = R.M.S. volts between slip-rings,

I = amperes in leads to slip-rings;

$$\therefore\ I/I_c = V_c/\sqrt{3}V = 2\sqrt{2}/(\sqrt{3}.\sqrt{3}) = \tfrac{2}{3}\sqrt{2} = 0{\cdot}943.$$

And in a two-phase rotary the current in each of the four leads is $\frac{1}{2}\sqrt{2} = 0{\cdot}707$ times the D.C. amperes.

The alternating current in the armature of a three-phase rotary, which is necessarily Δ-connected, is $1/\sqrt{3}$ times that in the lines, and therefore has an R.M.S. value equal to ($2\sqrt{2}/3\sqrt{3}$, *i.e.*) 0·544 times the D.C. amperes. Its maximum value is $\sqrt{2}$ times as great, *i.e.* ($4/3\sqrt{3}$ =) 0·770 times the D.C. amperes.

In a two-phase rotary the alternating currents in two neighbouring portions of the armature [such as DA, AB in Fig. 9.01 (*c*)] are 90° out of phase. The current in the line leading to A is the vector difference of these two and therefore is $\sqrt{2}$ times as great. Thus the R.M.S. value of the current in the armature is 0·500 times the D.C. amperes, and its maximum value is 0·707 times the D.C. amperes.

In all these cases if the power-factor is not unity, the alternating currents are increased in inverse proportion to the power-factor.

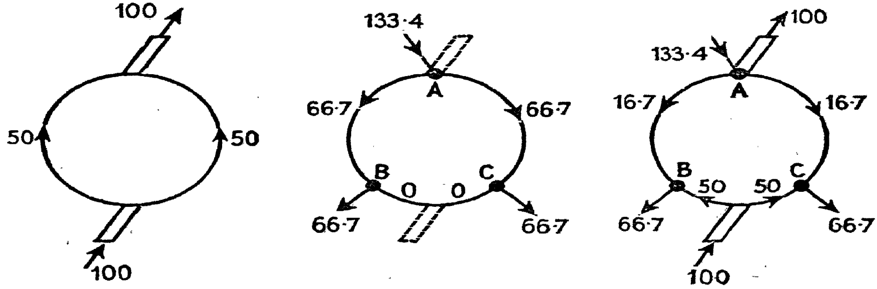
4. Outputs of Rotary Convertors

The main advantage of using rotaries is that the cost of such machines is much less than that of motor-generators of the same output. This reduction in cost is increased by the fact that (except in the case of a monophase rotary) the size of the rotary is less than that of a D.C. generator of the same output. It is an exception to the rule about the impossibility of doing two things well at the same time.

The reason for this reduction in size is that the alternating current, being that of a motor, flows against the e.m.f.; whereas the direct current, being that of a generator, flows with the e.m.f. The two currents therefore cancel out to a considerable extent and the copper losses are reduced thereby.

The amount of this reduction can be found by comparing the losses in a rotary with those in the same armature used only as a D.C. generator giving the same output. If R denotes the resistance of the armature between the D.C. brushes (apart from that of the brushes themselves), and the direct current is taken as 100, the I^2R loss in the D.C. generator is 10 000R. The resistance of each of the two parallel paths is 2R: and each carries a current of 50 [see Fig. 9.02 (*a*)] giving a total loss of $2 \times 50^2 = 2R = 10\,000R$ as stated above.

Compare with this a three-phase rotary in the position in which one tapping point is connected to one D.C. brush [Fig. 9.02 (*b*)]. If the power-factor is unity the A.C. in BC is zero: and the currents in AB and CA are each 0·866 of their maximum value, that in CA being negative. Taking the maximum value obtained in Art. 3, this gives a current flowing from A to B equal to $0{\cdot}866 \times 0{\cdot}770 \times 100 = 66{\cdot}7$; and an equal current flowing *to* C from A. The current in the A.C. lead to A is, therefore, 133·4 (its maximum) and in the other two leads 66·7 each.



(*a*) D.C. GENERATOR. (*b*) A.C. MOTOR. (*c*) ROTARY.

Fig. 9.02.—CURRENTS IN THREE-PHASE ROTARY.

The rotary armature carries the algebraic sums of the separate currents as shown in Fig. 9.02 (*c*) These can be checked by the currents in the A.C. and D.C. leads; *e.g.* at A, lead currents = $133{\cdot}4 - 100 = 33{\cdot}4$ inward; armature currents = $16{\cdot}7 + 16{\cdot}7 = 33{\cdot}4$ outward.

Denoting the lower D.C. brush by D, resistance of DB = resistance of DC = $\frac{1}{3}$ of $2R = \frac{2}{3}R$. And resistance of AB = resistance of AC = $\frac{2}{3}$ of $2R = \frac{4}{3}R$. Therefore total I^2R loss in rotary armature

$$= (16{\cdot}7)^2 \times (\tfrac{4}{3}R + \tfrac{4}{3}R) + 50^2 \times (\tfrac{2}{3}R + \tfrac{2}{3}R) = 4\,070R.$$

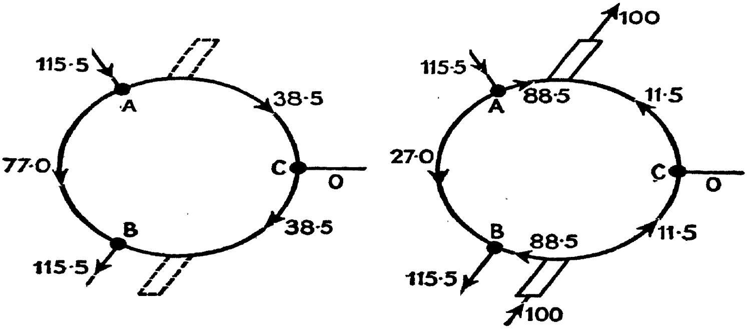
Repeating the above process for the position $\frac{1}{12}$th period later [Fig. 9.03 (*a*)] the current in AB is now at its maximum value 77·0 (cf. Art. 3) and those in BC and CA at half their negative maxima. Combining these with the D.C. generator currents, which have the same values as before [Fig. 9.02 (*a*)], the rotary currents are as shown in Fig. 9.03 (*b*).

Denoting the upper D.C. brush by E:—

Resisistance of AE = resistance of DB = $\frac{1}{6}$ of 2R = $\frac{1}{3}$R,
,, ,, AB = $\frac{2}{3}$ of 2R = $\frac{4}{3}$R,
,, ,, DCE = 2R.

Therefore total I^2R loss in rotary armature

$$= (88\cdot5)^2 \times (\tfrac{1}{3}R + \tfrac{1}{3}R) + 27^2 \times \tfrac{4}{3}R + (11\cdot5)^2 \times 2R = 6\,450R.$$



(*a*) A.C. MOTOR. (*b*) ROTARY.

Fig. 9.03.—CURRENTS $\frac{1}{12}$TH PERIOD LATER.

Another $\frac{1}{12}$th period later tapping point B is connected to brush D. The rotary currents are, therefore, similar to those in Fig. 9.02 (*c*), reversed (see Fig. 9.04). The total I^2R loss in the rotary armature therefore is again equal to 4 070R.

The loss thus changes every $\frac{1}{12}$ period between the above two values. The average of the two is 5 260R. But since the change in the value of the loss is more rapid at the lower value than at the higher a more accurate mean is obtained by taking ($\frac{2}{3}$ of 6 450R + $\frac{1}{3}$ of 4 070R), *i.e.* 5 660R.

If the output of the rotary is increased its copper loss increases as the square of the current. Therefore to make its average loss equal to that of the D.C. generator the rotary current must be increased to $100 \times \sqrt{\{10\,000/5\,660\}}$, *i.e.* to 133.

Therefore a three-phase rotary can give 33 per cent. more output than the same machine used as a D.C. generator with the same rate of heating (see also Art. 5).

This increased output of polyphase rotaries results in the commutator (which has to handle the whole output) being larger in comparison with the armature than in the case of a D.C. generator.

5. Variation of Heating

An inspection of Fig. 9.02 (*c*) shows that when tapping point A is a short distance from the D.C. brush the coils between these two carry a current of nearly 116·7. A coil [such as the one in contact with E in Fig. 9.03 (*b*)], one-quarter of the way from one tapping point to the next, never carries more than 88·5, and a coil midway between two tapping points never more than 50.

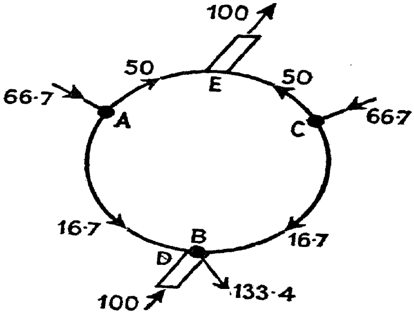


Fig. 9.04.—Rotary Currents $\frac{1}{6}$th Period from Start.

Thus the heating in a rotary is not evenly distributed, but decreases from the tapping points to the coils midway between them. Though conduction makes the temperature rise more even than the heating, the irregularity causes the safe output to be somewhat lower than that calculated as in Art. 4 from the total heat produced.

To find the extent of this variation of heating consider the current-time graph for various conductors. In the case of a three-phase rotary the alternating current is the same in all the conductors of one phase, *e.g.* BC in Figs. 9.02, 9.03, and 9.04. It is a sinusoidal wave of maximum value 0·770 times the D.C. amperes, as shown by OPQRS in Fig. 9.05. Its reversal occurs when the tapping points are at equal distances from, and on opposite sides of one of the D.C. brushes [Fig. 9.02 (*c*)].

The D.C. in any particular conductor remains constant while the armature rotates through one pole-pitch. It then changes to an equal current in the reverse direction and remains at the new value during rotation through another pole-pitch. It then returns to its original value and repeats the cycle.

In the case of the conductor midway between the tapping points the reversal of the D.C. occurs at the same instant as that of the A.C. Consequently the D.C. is as shown by TVWXS in Fig. 9.05, neglecting the time of commutation.

The rotary current in this conductor is the algebraic sum of the D.C. and the A.C. It is therefore given by TYVWZX (Fig. 9.05), viz. a sine wave with the positive half displaced downwards and the negative half displaced upwards by the same amount. The maximum positive value in the first half (*i.e.* at Y) is 27 per cent. of the D.C. amperes.

The equation of TYV is $i = 77 \sin \theta - 50$. Therefore the mean value of i^2 is the mean value of $(77 \sin \theta - 50)^2$, *i.e.* of $(5\,930 \sin^2 \theta - 7\,700 \sin \theta + 2\,500)$ taken over the half period from $\theta = 0$, to $\theta = \pi$. The mean value of $\sin^2 \theta$ is $\frac{1}{2}$ (see Volume I., Chapter V., Art. 4), and that of $\sin \theta$ is $2/\pi$, *i.e.* 0·637. Thus the mean value of i^2 is $(5\,930 \times \frac{1}{2} - 7\,700 \times 2/\pi + 2\,500)$, *i.e.* $(2\,965 - 4\,905 + 2\,500)$, *i.e.* 560.

The D.C. always has the value 50 and so its mean square is 2 500. Therefore the ratio (heating of rotary)/(heating of D.C. generator) is equal to 560/2 500, *i.e.* 0·224, *for the centre conductor.*

Moving in either direction from the centre the A.C. does not change, but the instant of reversal of the D.C. becomes either sooner or later than that of the A.C. This results in a less complete cancelling out of the two currents and consequently the heating is increased. This increase becomes greater the nearer the conductor is to a tapping point.

For a conductor next to a tapping point the difference between the instants of reversal of the two currents is 60° (electrical) in a three-phase rotary. Consequently the currents are as shown in

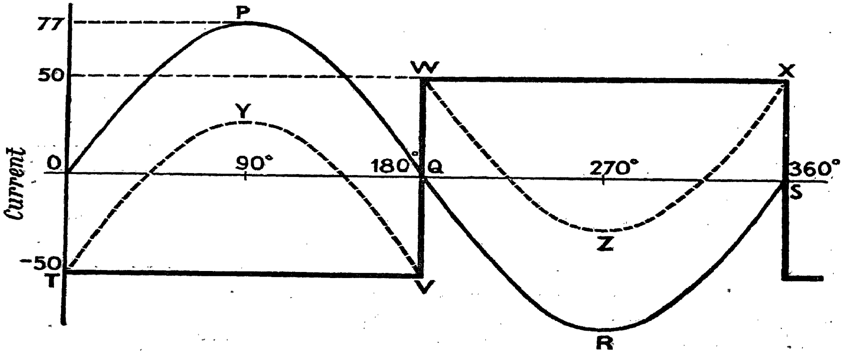


Fig. 9.05.—CURRENT IN CENTRAL CONDUCTOR OF A PHASE.

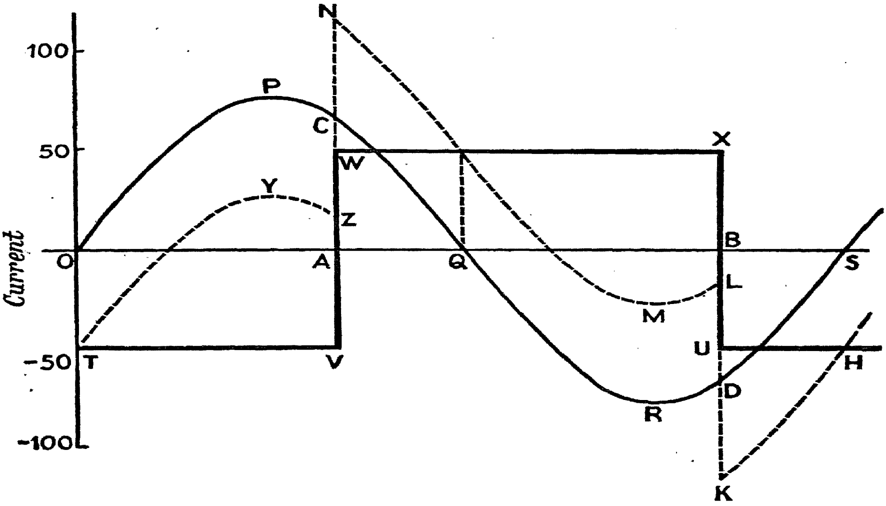


Fig. 9.06.—CURRENT IN CONDUCTOR NEXT TO TAPPING POINT.

Fig. 9.06 where the A.C. is OPQRS, the same as before, but the D.C. is TVWXU, reversing 60° sooner. The combined rotary current has the wave form shown by the broken line. TYZNMLKH, being the same as before for 120°, and then becoming 50 above the A.C. instead of 50 below it.

The equation of the rotary current is now $i = 77 \sin \theta - 50$ from 0° to $\frac{2\pi}{3}$ (120°), and again beyond $\frac{5\pi}{3}$ (300°),

$$\text{and } i = 77 \sin \theta + 50 \text{ from } \frac{2\pi}{3} \text{ (120°) to } \frac{5\pi}{3} \text{ (300°).}$$

Therefore the mean value of i^2 over half a period is equal to

$$\frac{1}{\pi}\left[\int_0^{\frac{2\pi}{3}} (77 \sin \theta - 50)^2 d\theta + \int_{\frac{2\pi}{3}}^{\pi} (77 \sin \theta + 50)^2 d\theta\right]$$

$$= \frac{1}{\pi}\int_0^{\pi} (5930 \sin^2 \theta + 2\,500)\, d\theta - \frac{1}{\pi}\int_0^{\frac{2\pi}{3}} 7\,700 \sin \theta d\theta$$

$$+ \frac{1}{\pi}\int_{\frac{2\pi}{3}}^{\pi} 7\,700 \sin \theta d\theta,$$

$$= 5\,930 \times \tfrac{1}{2} + 2\,500 - (7\,700/\pi)\left\{\Big[-\cos\theta\Big]_0^{\frac{2\pi}{3}} - \Big[-\cos\theta\Big]_{\frac{2\pi}{3}}^{\pi}\right\}$$

$$= 2\,965 + 2\,500 - (7\,700/\pi)\left\{\left(1 - \cos\frac{2\pi}{3}\right) - \left(1 + \cos\frac{2\pi}{3}\right.\right.$$

$$= 5\,465 + 4\,905\cos\frac{2\pi}{3},$$

$$= 5\,465 - 4\,905\cos\frac{\pi}{3},$$

$$= 5\,465 - 4\,905 \times \tfrac{1}{2} = 3\,010.$$

If intermediate conductors are considered it will be found that

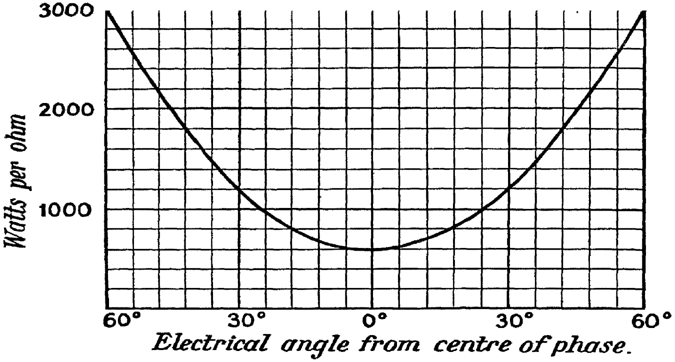


FIG. 9.07.—VARIATION OF HEATING IN THREE-PHASE ROTARY.

the mean rate of heating in them is given by $(5\,465 - 4\,905\cos\alpha)$ where α = electrical angle between the conductor and the centre of the winding between two tapping points. Hence the heating in different conductors varies as shown in Fig. 9.07.

The mean rate of heating is given approximately by ($\tfrac{1}{3}$ of 3 010 + $\tfrac{2}{3}$ of 560), *i.e.* 1 380. The more exact value of the mean is

$$\int_{-\pi/3}^{\pi/3} (5\,465 - 4\,905\cos\alpha)\,d\alpha/(2\pi/3),$$

$$\textit{i.e.}\ 5\,465 - (4\,905 \times 3/2\pi)\Big[\sin\alpha\Big]_{-\pi/3}^{\pi/3}$$

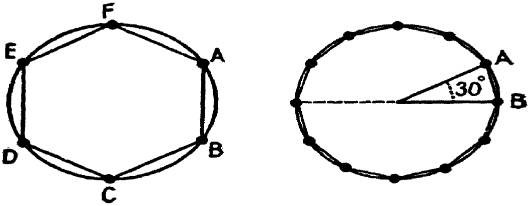
i.e. $5\,465 - 2\,340 \times 2\sin\pi/3$, *i.e.* $5\,465 - 4\,053$, *i.e.* 1 410.

This corresponds with a steady current of $\sqrt{1\,410}$, *i.e.* 37·6. But the current for the same output, if the machine is used as a D.C. generator, is 50 in each conductor. Therefore for equal heating the rotary output can be increased in the ratio $\sqrt{\{2\,500/1\,410\}}$ *i.e.* 50/37·6, *i.e.* 1·33; which is the same result as that found in Art. 4.

6. Outputs of Six- and Twelve-Phase Rotaries

If six equidistant tappings are used then the maximum voltage per phase is, as given by the straight lines AB, BC, etc., in Fig. 9.08 (*a*), half the D.C. volts. Therefore the R.M.S. volts per phase are $\frac{1}{2}/\sqrt{2}$, *i.e.* 35·4 per cent. of the D.C. volts. Consequently, for unity power-factor and negligible losses the R.M.S. current, I_A in the armature, is given by $6 \times \cdot 354 \times I_A = I_C$, whence $I_A = 0{\cdot}471\, I_C$, and the maximum alternating current in the armature is $\sqrt{2}$ times I_A, *i.e.* $0{\cdot}667\, I_C$ (compared with $0{\cdot}770\, I_C$ for 3-phase).

If the mean losses are worked out by the methods of Art. 4 or of Art. 5, it will be found that the heating loss in the rotary is only 26·8 per cent. of that for the same machine giving the same output as a D.C. generator. Hence, for equal total heating the output as a rotary may be $\sqrt{1/0{\cdot}268}$, *i.e.* 1·93 times the output as a D.C. generator.



(*a*) SIX-PHASE. (*b*) TWELVE-PHASE.

Fig. 9.08.—VOLTAGES OF SIX-PHASE AND TWELVE-PHASE ROTARIES.

Similarly, for a twelve-phase rotary [see Fig. 9.08 (*b*)] the maximum A.C. voltage per phase is equal to sin 15° (*i.e.* 0·259) times the D.C. volts, and the R.M.S. value is 18·3 per cent. of the D.C. volts. So under the same conditions as above, $12 \times \cdot 183 \times I_A = I_C$; whence $I_A = 0{\cdot}455\, I_C$, and the maximum value is $0{\cdot}644\, I_C$.

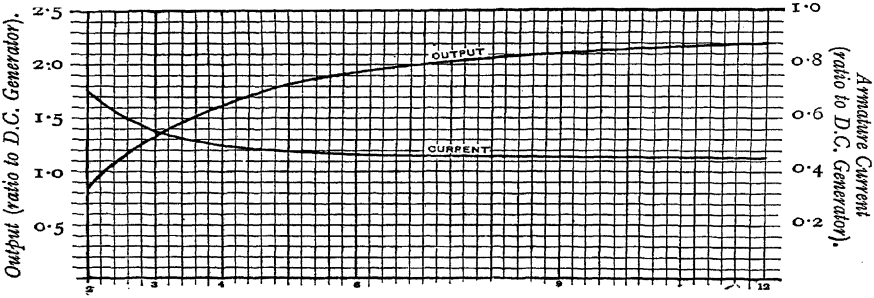
The heating loss as a rotary is thereby further reduced to 20·7 per cent. of the loss as a D.C. generator. The output of a 12-phase rotary is, therefore, $\sqrt{1/0{\cdot}207}$, *i.e.* 2·20 times the output of the same machine used as a D.C. generator.

The preceding results and some others are summarised in the following table:—

No. of Slip Rings	Voltage Ratio	Current Ratio		Output Ratio
		Armature	Leads	
2	·707	0·707	1·414	0·85
3	·612	·544	·943	1·33
4	·500	·500	·707	1·62
6	·354	·471	·471	1·93
9	·242	·459	·314	2·11
12	·183	·455	·236	2·20

N.B.—Current ratio for leads × number of leads = 2·83 for all cases.

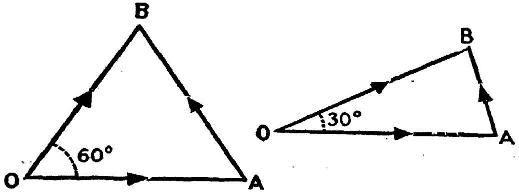
The voltage and current ratios are (A.C., R.M.S.)/D.C. at unity power-factor, the A.C. volts being between neighbouring tapping points. Some results not worked out have been added for comparison. The third and fifth columns are plotted in Fig. 9.09.



Number of Slip-rings.

Fig. 9.09.—VARIATION OF OUTPUT AND ARMATURE CURRENT WITH NUMBER OF SLIP-RINGS.

The current in each of the leads of a 6-phase rotary is the vector *difference* between two armature currents 60° out of phase, and is, therefore, equal to the alternating current in the armature, see Fig. 9.10 (*a*).



(*a*) SIX-PHASE. (*b*) TWELVE-PHASE.

Fig. 9.10.—ARMATURE AND LEAD CURRENTS.

Similarly, in a 12-phase rotary the current in each lead is equal to armature

A.C. × 2 sin 15°, *i.e.* to (·455 × 2 × ·259 times: =) 0·236 times the D.C. amperes [see Fig. 9.10 (*b*)].

7. Advantages of Large Number of Phases

In addition to the output of a given size of machine being increased by using more tapping points, the variation in the heating is diminished at the same time. This is due mainly to the diminution in the electrical angle between the centre point of a phase and the tapping points. As the heating increases with the distance from the centre, the smaller the angle the more uniform the heating.

This is illustrated in Fig. 9.11 (*a*) and (*b*). In the former, the horizontal scale gives the electrical angle from the centre of a phase, and so shows how the heating in the same conductors is diminished by increasing the number of phases. In (*b*) the horizontal scale is in fractions of the phase for each case, and so shows more clearly the diminution in the variation of the heating over the phase. The same facts are illustrated in the following table:—

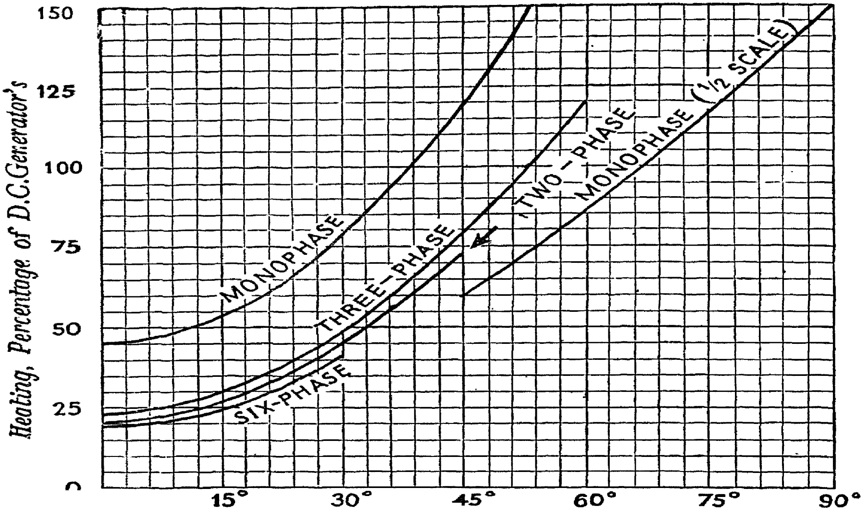
NO. OF SLIP RINGS	PERCENTAGE HEATING		
	MAXIMUM	MINIMUM	MEAN
2	300	45·4	138
3	120·5	22·4	56·4
4	72·7	20·0	37·9
6	41·9	19·1	26·8
12	24·5	19·0	20·7
∞	18·93	18·93	18·93

The heating is expressed as a percentage of that in the same armature used as a D.C. generator giving the same output.

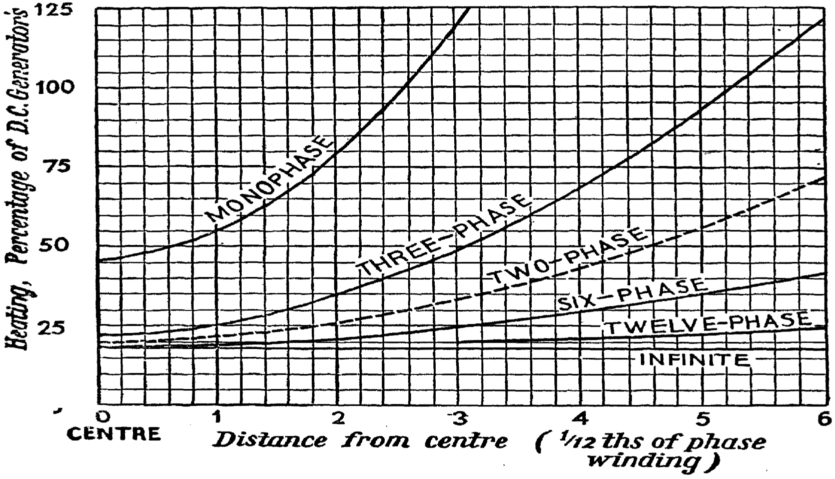
The last line gives the values for the ideal case of an infinite number of phases. The output for this case could be increased to $1/\sqrt{\cdot 189}$, *i.e.* 2·30 times the output of the same machine used as a D.C. generator with the same total heating loss (cf. Art. 6).

8. Effects of Power-Factor

When the power-factor differs from unity the alternating current is increased. This has the effect of increasing the minimum heating. At the same time the points at which the minimum heating occurs cease to be at the centres of the phases. The electrical angle by which a point of minimum heating is behind (or in front of) the centre of the phase winding is equal to the angle of lag (or lead) of the current. This follows from the fact that in a conductor in



(*a*) Variation with Angle.



(*b*) Variation with Position in Phase.

Fig. 9.11.—Variation of Heating in Rotaries.

such a position the alternating and direct currents reverse simultaneously, and therefore cancel out to the maximum extent possible.

As an example, consider the case of a 3-phase rotary, as in Art. 5, but with the current lagging 30° ($\cos \phi = 0{\cdot}866$). The maximum value of the alternating current is increased to ·770/·866, *i.e.* 0·889 of the direct current. The equation of the combined A.C. and D.C. is therefore:—$i = 88{\cdot}9 \sin \theta - 50$ for the conductors in which they reverse simultaneously. Hence, as in Art. 5, the mean square of the combined current is $(88{\cdot}9^2 \times \frac{1}{2} - 8\,890 \times \frac{2}{\pi} + 2\,500)$,

$$\text{i.e. } (3\,950 - 5\,660 + 2\,500), \text{ i.e. } 790.$$

Thus for conductors in this position the ratio

(heating of rotary)/(heating of D.C. generator)

for this case is 790/2 500, *i.e.* 0·316; compared with 0·224 for the centre conductor with unity power-factor.

The second effect of lowering the power-factor is due to the above-mentioned shifting of the point of minimum heating. The distance between this point (M) and one of the tapping points (T) is thereby increased, becoming 90° in the example in place of the original 60°. This causes an increase in the heating of the neighbourhood of this tapping point (see Fig. 9.12) relatively to the minimum.

Although the heating near the other tapping point (S) is diminished relatively to the minimum, this does not compensate for the increase near the former tapping point. Hence the average heating is increased in a greater ratio than the minimum heating is increased.

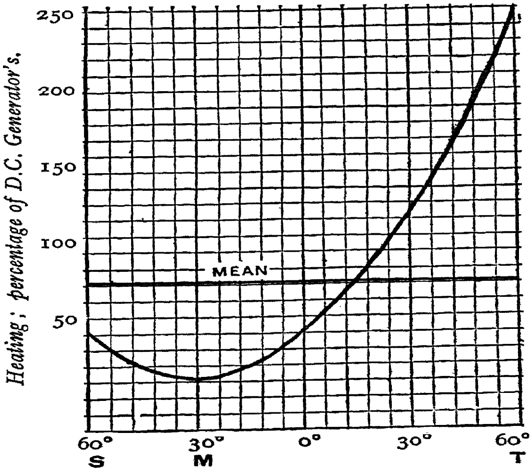


Fig. 9.12.—ELECTRICAL ANGLE FROM CENTRE OF PHASE.

Applying the methods used in Art. 5 to the above example gives:—

Mean rate of heating at T $= 3\,950 + 2\,500 - 5\,660 \cos 9c^\circ$
$= 6\,450$, *i.e.* 2·58 times D.C. heating,

and mean rate of heating at S
$= 3\,950 + 2\,500 - 5\,660 \cos 30^\circ = 6\,450 - 4\,900 = 1\,550$, *i.e.* 0·62 times D.C. heating.

The mean rate of heating averaged over the whole winding is, therefore, approximately

$$\frac{(\frac{2}{3} \times 790 + \frac{1}{3} \times 6\,450) \times 90^\circ + (\frac{2}{3} \times 790 + \frac{1}{3} \times 1\,550) \times 30^\circ}{120^\circ},$$

i.e. $\frac{2}{3} \times 790 + \frac{1}{4} \times 6\,450 + \frac{1}{12} \times 1\,550$, *i.e.* 2 270, *i.e.* 0·91 times the D.C. heating.

By integrating the expression for the heating at various points a more exact value for this average is obtained, viz. $6\,450 - 5\,660 \times (9/4\pi)$ which equals 2 400, *i.e.* 0·96 times D.C. heating. Thus the allowable output is reduced to 2 per cent. more than that of the same machine used as a D.C. generator, as against 33 per cent. more at unity power-factor.

If similar calculations are carried out at various power-factors it will be found that, as the power-factor falls, the heating for a fixed output increases at an increasing rate, *e.g.* the increase of heating due to the fall of power-factor from 0·9 to 0·8 is greater than the increase due to a fall from 1 to 0·9. On the other hand, the percentage increase diminishes.

The general formula for the ratio (rotary mean heating rate)/(D.C. generator heating rate) is $(\text{constant}/\cos^2 \phi) + 1 - 16/\pi^2$. The value of the constant diminishes as the number of rings increases. Its value for a particular number of rings can be found by putting $\cos \phi = 1$, and using the results given in the table in Art. 7. (N.B. $1 - 16/\pi^2 = -0{\cdot}621$.)

The consequent diminution in output for the same total heating becomes *smaller* at lower power-factors, *e.g.* a 3-phase rotary at 0·9 power-factor gives 18 per cent. less output than at unity power-factor, and at 0·8 power-factor the output is 32 per cent. less than at unity power-factor, so that the decrease is less than double.

If the angle of lag (or lead) is greater than half the angle between the tapping points there will be no coils in which the A.C. and D.C. reverse simultaneously (cf. Fig. 9.12). The minimum heating in each phase will then occur close to one tapping point (S), and the maximum close to the other (T) as before. This is not likely to occur except in a rotary with many phases.

The greater the number of phases the worse the effect of a given lowering of the power-factor on the output, not only in actual value, but when expressed as a percentage of the value at unity power-factor. This is because the same actual increase of heating is a greater percentage of its value at unity power-factor when there are many phases.

These points and others are illustrated by the graphs in Fig. 9.13.

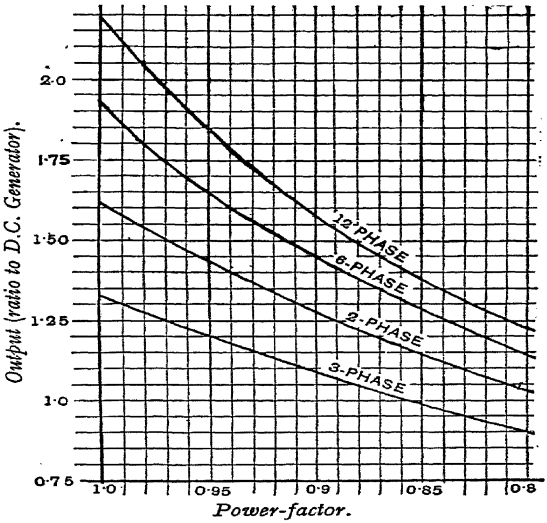


Fig. 9.13.—VARIATION OF OUTPUT WITH POWER-FACTOR.

9. Supply to Six-Phase Rotaries

The advantages of six-phase rotaries over three-phase ones as regards output (and the consequent higher efficiency) would be rather dearly purchased if they required a six-phase high-pressure supply. But any polyphase supply can be transformed into any other. Hence it is only a matter of so arranging the connexions of the transformers (which are necessary for any type of rotary) that they deliver six-phase energy instead of three-phase.

There are three methods in use for providing current to six-phase rotaries from a three-phase high-voltage supply. These are known respectively as (*a*) double-delta, (*b*) diametral, and (*c*) hexagonal or ring.

In the **double-delta** method of connexion the primaries of the transformers may be connected either Y or delta. Each has two secondaries containing the same number of turns of the same section [see Fig. 9.14 (*a*)]. Three of these, one from each phase, are connected together in delta and leads taken from this delta to three of the tapping points, omitting alternate ones. The other set of three windings are also connected in delta but with reversed connexions; *e.g.* I*a′* to II*b′* compared with I*b* to II*a*. They thus produce p.d.s suitable for connexion to the alternate tapping points previously omitted.

The correctness of this method of connexion is shown more clearly in Fig. 9.14 (*b*) in which the armature winding of the rotary is shown as a circle, and the tapping points as numbered dots on this circle (cf. Art. 2). This shows that any two secondaries belonging to the same phase are connected across two pairs of tapping points (*e.g.* 5-1, and 4-2 for phase I) between which the p.d.s are equal in magnitude and of the same phase.

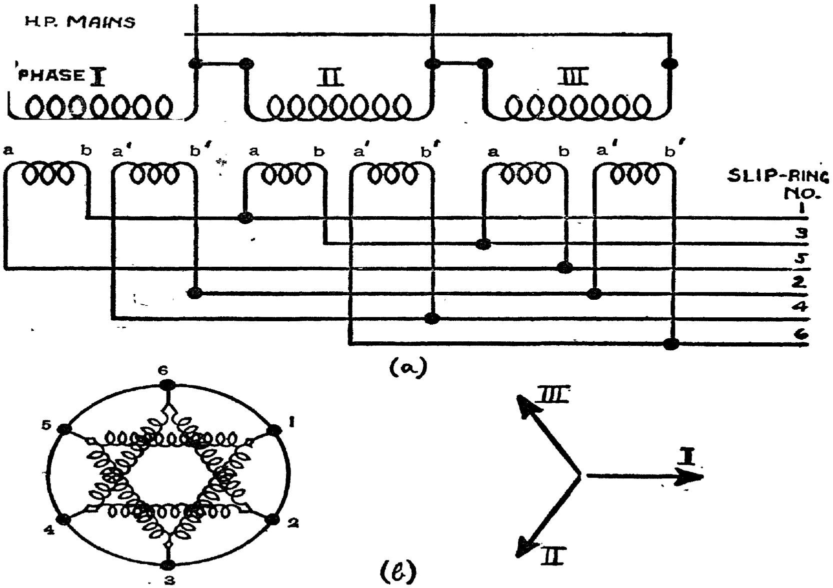


Fig. 9.14.—DOUBLE-△ SUPPLY TO SIX-PHASE ROTARIES.

For the diametral method, only one secondary is needed on each transformer. The ends of each of these are connected to tapping points 180° electrical apart, whence the name. The method is shown in Fig. 9.15 (*a*) and (*b*), the former being a somewhat simpler type of diagram than Fig. 9.14 (*a*) owing to the omission of the primary connexions to the lines.

The centre points of the three secondaries are at the same potential, viz. midway between the positive and negative D.C. mains. If tappings are taken from these points and joined to form a star-point S, the connexions become **double-Y**. One Y is formed by S 1 3 5, and the other by S 4 6 2. If desired the two Y's can be kept separate in the transformer windings.

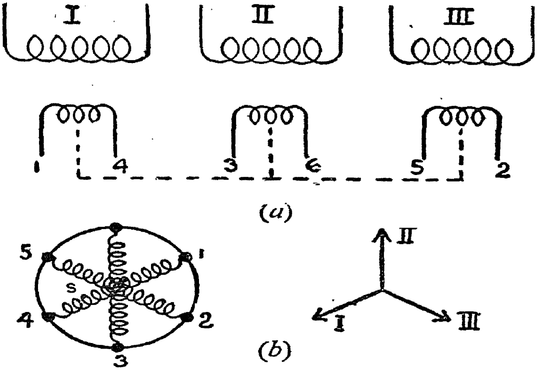


Fig. 9.15.—DIAMETRAL SUPPLY TO SIX-PHASE ROTARY.

For the **hexagonal** (or ring) method two equal secondaries per phase are required as for the double-delta, but their voltages are lower and their currents correspondingly greater (see Example 1). The connexions are shown in Fig. 9.16, corresponding to the (*b*) diagrams for the previous methods. The diagram corresponding with the (*a*) diagrams can be obtained readily from this. The secondaries of phase I are connected across slip-rings 2-3, and 6-5 respectively, and similarly for those of the other two phases. It will be seen that the six secondaries form a closed mesh.

Example 1. *A six-phase rotary is to supply* 200 *A. D.C. at* 500 *volts from three-phase mains at* 3 300 *line volts. If the Δ-connected primary windings of the transformers have each* 800 *turns, find the number of turns and the currents in the secondary windings for each of the three methods of connexion, neglecting all losses.*

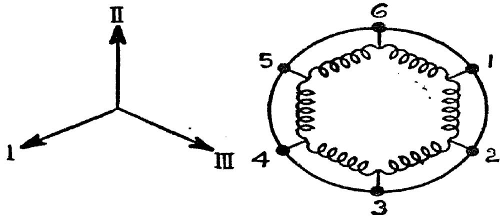


Fig. 9.16.—HEXAGONAL SUPPLY TO SIX-PHASE ROTARY.

(*a*) Double-delta.

P.d. across each secondary $= (0{\cdot}612 \times 500) = 306$ volts (R.M.S.):

just as for a 3-phase rotary (see Arts. 2 and 6);

$\therefore$ No. of turns in each secondary $= 800 \times 306/3\,300$ $= 74$ turns.

Current in each lead (see Art. 6) = ·471 × 200 A. = 94 amperes;
∴ Current in each secondary = $94/\sqrt{3}$ = 54 amperes.

[Note that this is half the current for a 3-phase rotary of the same output with Δ-connected secondaries.]

(*b*) Diametral (or double-Y).

P.d. across each complete secondary = (·707 × 500) = 354 volts (R.M.S.) just as for a two-ring rotary;

∴ No. of turns in each secondary = 800 × 354/3 300 = 86 turns.

For double-Y half of this, viz. 43 turns, form each secondary.

Current in each lead = 94 A. as in (*a*);
∴ current in each secondary = 94 amperes.

[Note that this one-third of the current for a two-ring rotary of the same output.]

(*c*) Hexagonal.

P.d. across each secondary (see Art. 6) = (·354 × 500) = 177 volts (R.M.S.)
∴ No. of turns in each secondary = 800 × 177/3 300 = 43 turns.
Current in each lead = 94 amp. as in (*a*);
∴ Current in each secondary = 94 amperes,

since the current in the lead is the vector *difference* between two equal secondary currents with 60° phase difference [see Fig. 9.10 (*a*)].

[Note that full load secondary ampere-turns per phase are—

(*a*) 2 × 74 × 54 = 8 000
(*b*) 86 × 94 = 8 100
(*c*) 2 × 43 × 94 = 8 100;

i.e. the same in all cases after allowing for the effect of approximations in the calculations. This means that the same amount of copper is required for all methods except for the effect of the greater total amount of insulation required for the larger number of turns in the double-delta.]

10. Supply to Twelve-Phase Rotaries

With a high-voltage 3-phase supply a suitable arrangement of the transformer windings enables 12-phase rotaries to be supplied and their higher output and efficiency to be obtained. In small machines the gains are not worth the extra cost and complication, but this does not apply to larger machines.

One method for this purpose is to combine the double-delta and diametral methods of connexion for six-phase rotaries. Each of these gives a supply to six of the twelve slip-rings, and the two sets of six are connected to alternate tapping points. This requires three secondaries in each phase, and their connexions are shown in Fig. 9.17.

The odd numbered slip-rings are shown connected double-delta; 1-5-9 and 3-7-11 forming the two deltas. The even numbered slip-rings are connected in the diametral way, and if desired the middle points of the three secondaries connected to these (2-8, 6-12, 10-4) may be connected together to form a star point. The secondaries of phase I are connected to 1-5, 11-7, and 12-6 respectively, and similarly for the other phases.

The numbers of turns in the secondaries will be the same as for the corresponding methods of connexion for six-phase rotaries. The currents will be half as great as for the latter, each group of windings supplying half the total power; *e.g.* if a twelve-phase rotary were used for the same transformation as in Example 1 (Art. 9), each phase would require three secondary windings. Two of these, for connecting in double-delta would each have 74 turns and carry 27 A.; the third would have 86 turns and carry 47 A. Thus the total full load ampere-turns per phase would be 8 000 approximately, as before.

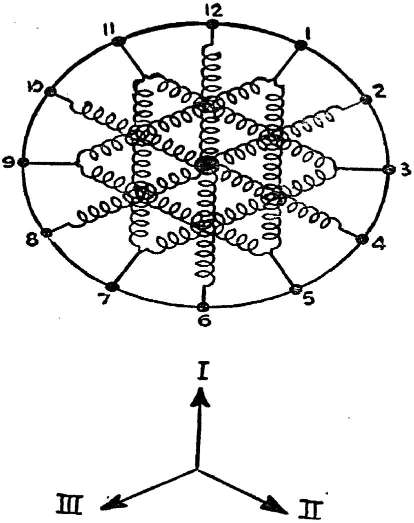


Fig. 9.17.—THREE-PHASE SUPPLY TO TWELVE-PHASE ROTARY.

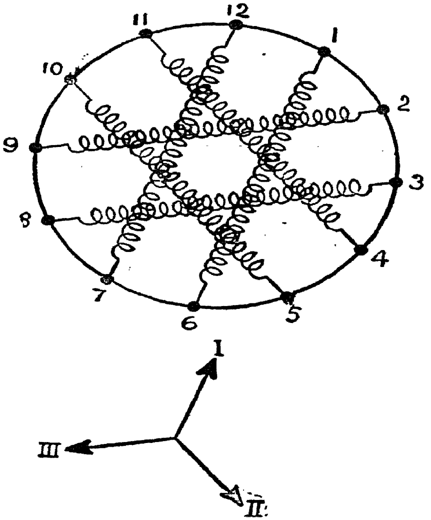


Fig. 9.18.—ALTERNATIVE SUPPLY TO TWELVE-PHASE ROTARY.

Twelve-phase rotaries likewise can be supplied from 3-phase mains by combining the double-delta and hexagonal methods for six-phase rotaries (requiring four secondaries per phase), but not by combining the latter with the diametral method. A method requiring only two secondaries per phase is shown in Fig. 9.18, and is called the " parallel chord " connexion. The two secondaries of phase I are connected across 1-6 and 12-7 respectively, those of phase II across 4-11 and 5-10, and those of phase III across 8-3 and 9-2 (see further Art. 11).

For methods of connexion in which a star point is obtained the transformers can be utilised in place of a balancer if the D.C. supply is on the 3-wire system (Volume I., Chapter XVIII., Art. 5). The middle wire is connected to the star point and thus the p.d.s across the two sides of the three-wire system are kept nearly equal. The difference between the regulations of the transformers due to unequal loading of the two sides causes the p.d. on the heavily loaded side to fall a little below that on the lightly loaded side.

Even when the D.C. supply is from generators and not from rotaries a similar arrangement can be used. One or more of the generators is supplied with slip-rings, usually three in number.

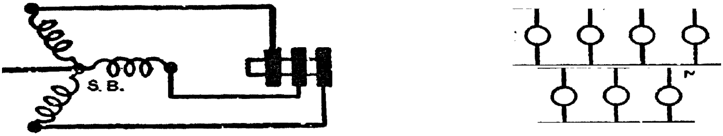


Fig. 9.19.—STATIC BALANCER CONNEXIONS.

These are connected to the terminals of star-connected coils, like a three-phase transformer with only primary windings. The middle wire is connected to the star point (see Fig. 9.19). Such an arrangement is called a **static balancer.** The closeness with which the p.d. of the middle wire is kept half-way between those of the outers depends on the smallness of the resistances and leakage reactances of the static balancer windings.

On the other hand methods which provide closed paths in the secondaries (double-delta and hexagonal) have the advantage of reducing the harmonics liable to be set up with Y-connected primaries.

These are due to the reduced permeability at high flux-density flattening the flux-waves. These produce peaked e.m.f. waves approximating to sinoidal waves plus negative triple harmonics. The latter produce a resultant e.m.f. and current of triple-frequency in the secondary mesh, and this current diminishes the triple harmonics, cf. Chapter XI., Art. 7.

11. Disadvantage of Parallel-Chord Connexion

It can be seen from the vector diagram given in Fig. 9.10 (*b*) that, with equal currents in neighbouring phases of a 12-ring rotary, the corresponding line current is 75° and 105° respectively out of phase with the currents in the two phases. Thus in Fig. 9.20 if the current flowing from 11 to 12 is represented by the straight line joining these points, and the equal current from 12 to 1 by the line 12-1, then the current in the line joined to 12 is given in phase, though not in magnitude, by the line drawn to 12 from the centre (C) of the circle representing the armature winding.

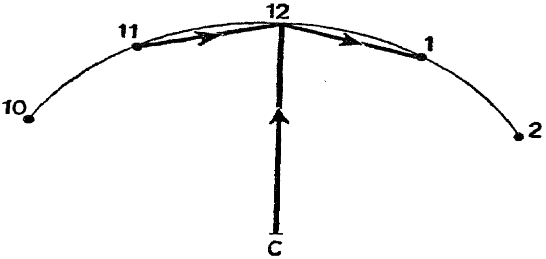


Fig. 9.20.—CURRENTS IN 12-PHASE ROTARY.

In the case of diametral, or of Y-connexion this is in phase with the voltage of the corresponding transformer secondary. And with delta connexion, currents in phase with the secondary voltages give a resultant current of the required phase.

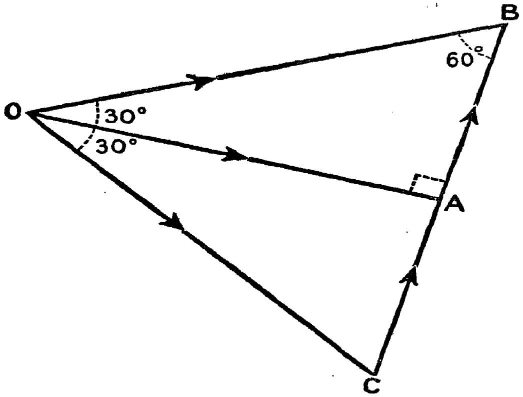


Fig. 9.21.—CURRENTS WITH PARALLEL CHORD CONNEXION.

But with a parallel chord connexion the current flowing in at 12 must be equal but opposite to that flowing in at 7; since they are supplied by the same secondary. They cannot both be in phase with the corresponding radii, and to give unity power-factor that flowing in at 12 must be retarded by 15° from its radius, and that in at 7 advanced by 15°, which brings them exactly opposite in phase.

To enable this to be the case the currents in 11-12 and 12-1 can no longer be equal. The phase differences of the line current (AB in Fig. 9.21) become 60° with current 11-12 (OB), and 90° with current 12-1 (OA). (Compare with Fig. 9.10.) It follows that the current in 12-1 is 0·866 of the current in 11-12.

Similarly the current from 1 to 2 is represented by OC at 30° on the other side of OA, giving a line current CA, in phase with the voltage of the secondary 6-1. This may be continued all round, showing that the phase currents are equal alternately to OB and OA. Since the average power input must remain unchanged it follows that $1{\cdot}866\ OB = 2 \times 0{\cdot}455\ I_c$, and so $OB = 0{\cdot}488\ I_c$, whence $OA = 0{\cdot}422\ I_c$. The corresponding maximum values are $0{\cdot}690\ I_c$ and $0{\cdot}597\ I_c$.

On carrying out two calculations on the lines of Art. 5 it is found that this results in increasing the mean heating of *both*, to 21·5 per cent. and 21·0 per cent., compared with 20·6 per cent. for a 12-ring rotary supplied in the normal way (see Art. 7). Taking the mean of these two gives an output ratio (Art. 6) of 2·17 compared with the normal value of 2·20. The reduction of output therefore is not serious.

Various methods that have been used for supplying 12-ring rotaries from 2-phase mains suffer from a similar drawback but to a greater extent. Two-phase supply is now rare, so that the details are omitted.

12. Voltage Regulation

The fact that in obtaining the voltage ratios of rotary convertors (Art. 2) no statement is made as to the field strength indicates that the voltage cannot be adjusted by merely altering the excitation.

The reason why, with constant A.C. volts, any change of excitation has very little effect on the D.C. volts although the speed is constant, is that the change in the armature reaction opposes the change of field. If the excitation is increased it tends to make the alternating current lead the applied p.d. (see Chapter V., Art. 6). Since a leading current in a synchronous motor weakens the field (see Chapter V., Art. 10) the field remains nearly at its original strength in spite of the change in the excitation.

Similarly if the excitation is reduced the alternating current tends to lag, and consequently to strengthen the field so that the result again is an almost constant field.

As the load on the rotary increases, the resistances of the windings cause an increased fall in the D.C. volts below the no-load value, even with constant alternating pressure. The regulation of the transformers which supply the rotary increases this variation, so that a means of regulating the D.C. voltage is necessary when it is desired to keep this constant. It is still more necessary when the D.C. voltage is to rise with load giving an over-compounding effect (cf. Volume I., Chapter X., Art. 6).

The methods employed depend either on varying the A.C. pressure applied to the rotary, or on adding a D.C. pressure to that developed by the rotary.

13. Induction Regulator

The A.C. pressure applied to the rotary may be varied by tapping the transformer windings at a number of points and varying the connexions to the rotary according to the voltage required. The switches for this purpose have to be arranged in a special way so as to avoid short-circuiting a section of the transformer winding when altering the connexions (see Chapter VIII., Art. 23). A similar result can be obtained by using a separate boosting transformer with tap-changing switches.

Another method is to use an *induction regulator* which is a transformer with a variable ratio of transformation. It is built like an induction motor (see Fig. 9.22), the stator forming the

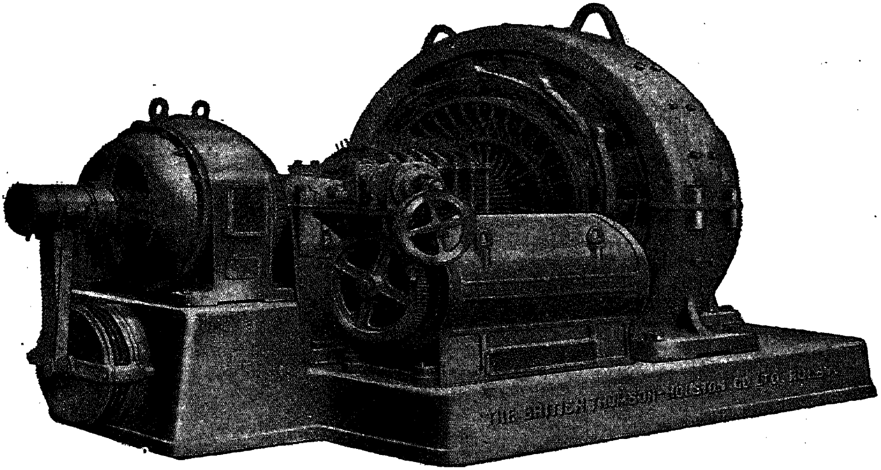


Fig. 9.22.—ROTARY CONVERTOR WITH INDUCTION REGULATOR AND PONY MOTOR.

primary and the wound rotor the secondary. The rotor is not free to rotate but its position can be altered through one pole-pitch by means of a worm and wheel. This may be operated by hand, or as in the illustration, by a small motor.

The connexions of one phase are shown in Fig. 9.23 in which S is one secondary of the main transformers supplying the rotary. The primary (stator) of the induction regulator (P P), is supplied from this. The secondary (rotor) winding R of the induction regulator is connected in series with the main secondary winding to the slip-rings of the rotary convertor.

The p.d. applied to the slip-rings is thus the vector sum of the voltages of the transformer secondary (S) and of the induction regulator secondary (R), these two being necessarily of the same frequency. In one position of R the two voltages will be in phase opposition and so the resultant voltage is equal to their arithmetical difference. If R is turned through a pole-pitch its voltage comes into phase with that of S, and a resultant equal to their arithmetical sum is obtained. Any intermediate phase difference can be obtained by placing R in the corresponding position and so the slip-ring voltage can be given any value between $(V_S - V_R)$ and $(V_S + V_R)$.

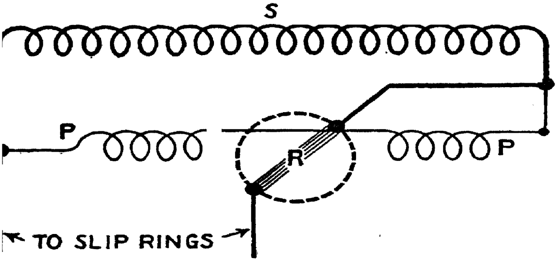


Fig. 9.23.—CONNEXIONS OF INDUCTION REGULATOR.

In a polyphase induction regulator all the primary windings are placed on the same stator just as in a polyphase induction motor. The secondary windings are placed similarly on the one rotor. Consequently any movement of the rotor alters all the voltages applied to the slip-rings simultaneously by the same amount. *Two* connexions per phase are required for the rotor and these may be either through slip-rings or by flexible leads. It is, however, possible to use a three-phase regulator with a six-phase rotary convertor if it is not necessary to maintain the star point at a p.d. exactly half-way between those of the D.C. brushes.

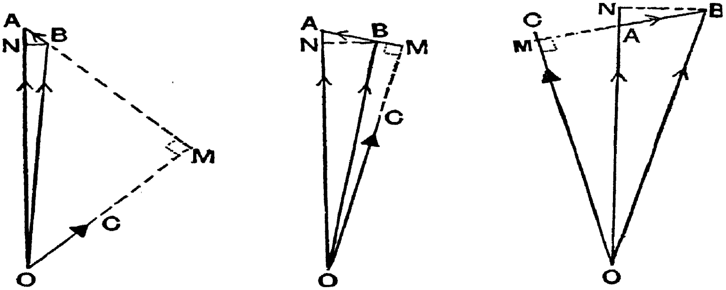
A slight disadvantage of this method is that the magnetic leakage of the regulator increases the regulation of the transformers (but see next Art.).

14. Compound Winding with Reactance in A.C. Leads

A *compound* winding by itself does not produce an increase of the D.C. voltage on load. But when it is desired to make the rotary convertor self-regulating, a compound winding together with reactances in the A.C. leads will cause an increase indirectly.

The method of action is shown by the vector diagrams of Fig. 9.24 which illustrate what occurs in one phase. In these OA represents the p.d. of the transformer secondary, supposed constant. This is made up of the p.d. on the slip-rings (OB), and that across the series reactance (BA). The latter is 90° ahead of the current (OC) if the resistance of the reactance is negligible.

When the load is light the field is weak and so the A.C. lags behind the slip-ring p.d. (see Chapter V., Art. 6). Hence as shown at (*a*) the slip-ring p.d. (OB) is then considerably less than that of the transformer secondary.



(*a*) LIGHT LOAD. (*b*) MEDIUM LOAD. (*c*) HEAVY LOAD.

Fig. 9.24.—REGULATION BY COMPOUND WINDING.

As the load increases the series turns strengthen the field, and so the current comes more nearly into phase with the slip-ring p.d. (see *b*). The reactance p.d. is then approximately perpendicular to OB, and so the value of the latter rises nearly to the full value of the secondary p.d.

At still heavier loads the rotary convertor will take a leading current owing to its stronger field. The angle OAB will therefore increase (see *c*), and so the slip-ring p.d. can reach a value higher than that supplied by the transformer.

The drawback of this method is that except at one particular load the current is out of phase with the slip-ring p.d. Consequently the efficiency and the maximum output of the rotary are reduced (see Art. 8). But if the power-factor is kept above 0·95, except at light loads, the relative phase of the current can vary by over 36° which gives a considerable variation in slip-ring voltage (see further Example 2).

Even without a series reactance a compound winding will produce some of the above effect, owing to the equivalent reactance of the transformers. And the effect may be intensified by increasing their magnetic leakage instead of providing a separate reactance.

Example 2. *A rotary convertor is supplied from a transformer giving a constant secondary p.d. of* 300 *volts, with a coil in series having a reactance of* 0·28 *ohm and negligible resistance. The field is compound wound so that at different loads the rotary convertor takes :—*

(*a*) 100 A. lagging 45° *:* (*b*) 200 A. lagging 15° *:* (*c*) 300 A. leading 15° *: phase differences measured from* secondary *p.d.*

Find for each case the slip-ring p.d., the power supplied to the rotary convertor, and its power-factor.

(*a*) With 100 A. supplied to the rotary convertor.

In the vector diagram, Fig. 9.24 (*a*); OA = 300 volts.

$$BA = 100 \times 0{\cdot}28 = 28 \text{ volts.}$$

Draw BN, perpendicular to OA. Then:—

$$\angle ABN = \angle AOC = 45°;$$

$$\therefore\ AN = AB \sin 45° = 28 \times 0{\cdot}707 = 19{\cdot}8 \text{ volts,}$$

$$BN = AB \cos 45° = 19{\cdot}8 \text{ volts;}$$

$$\therefore\ OB = \sqrt{ON^2 + BN^2} = \sqrt{\{(300 - 19{\cdot}8)^2 + (19{\cdot}8)^2\}} = 281 \text{ volts.}$$

(*b*) With 200 A. supplied to rotary convertor [see Fig. 9.24 (*b*)].

$$BA = 200 \times 0{\cdot}28 = 56 \text{ volts,}$$

$$AN = 56 \sin 15° = 14{\cdot}5 \text{ volts,}$$

$$BN = 56 \cos 15° = 54{\cdot}1 \text{ volts;}$$

$$\therefore\ OB = \sqrt{\{(300 - 14{\cdot}5)^2 + (54{\cdot}1)^2\}} = 291 \text{ volts.}$$

(*c*) With 300 A. supplied to rotary convertor [see Fig. 9.24 (*c*)],

$$BA = 300 \times 0{\cdot}28 = 84 \text{ volts,}$$

$$AN = 84 \sin 15° = 21{\cdot}8 \text{ volts,}$$

$$BN = 84 \cos 15° = 81{\cdot}1 \text{ volts;}$$

$$\therefore\ OB = \sqrt{\{(300 + 21{\cdot}8)^2 + (81{\cdot}1)^2\}} = 332 \text{ volts.}$$

$$\angle AOB = \tan^{-1} (BN/ON).$$

(*a*) $BN/ON = 19{\cdot}8/280{\cdot}2 = {\cdot}071;$

$$\therefore\ \angle AOB = 4°;$$

$$\therefore\ \angle BOC = 45° - 4° = 41°;$$

$$\therefore\ \text{Power-factor} = \cos 41° = 0{\cdot}75 \text{ lagging;}$$

$\therefore$ Power supplied to rotary $= 281 \times 100 \times 0{\cdot}75$ watts $= 21{\cdot}1$ kilowatts.

This may be checked by calculating the power given out by the transformer, which is $(300 \times 100 \times \cos 45°)$ watts $= 21{\cdot}2$ kilowatts.

(*b*) $BN/ON = (54{\cdot}1/285{\cdot}5) = 0{\cdot}1895;$

$$\therefore\ \angle AOB = \tan^{-1} 0{\cdot}1895 = 11°;$$

$$\therefore\ \angle BOC = 15° - 11° = 4°;$$

$$\therefore\ \text{Power-factor} = \cos 4° = 0{\cdot}998 \text{ lagging;}$$

$\therefore$ Power supplied to rotary $= 291 \times 200 \times \cdot 998 = 58 \cdot 1$ kilowatts.

(c) $\quad BN/ON = (81\cdot 1/321\cdot 8) = 0\cdot 252;$

$$\therefore \angle AOB = \tan^{-1} 0\cdot 252 = 14°;$$
$$\therefore \angle AOC = 15° + 14° = 29°;$$
$$\therefore \text{Power-factor} = \cos 29° = \cdot 875 \text{ leading;}$$
$$\therefore \text{Power supplied to rotary} = 332 \times 300 \times \cdot 875 \text{ watts} = 87\cdot 2 \text{ kilowatts.}$$

15. Booster and Split-Pole Rotary Convertors

One obvious method of applying the second way of regulating the D.C. voltage (see Art. 12), viz. by adding a variable amount to this, is to employ a booster (see Volume I., Chapter XVI., Art. 2). This might be shunt-wound, but it is more convenient to make it

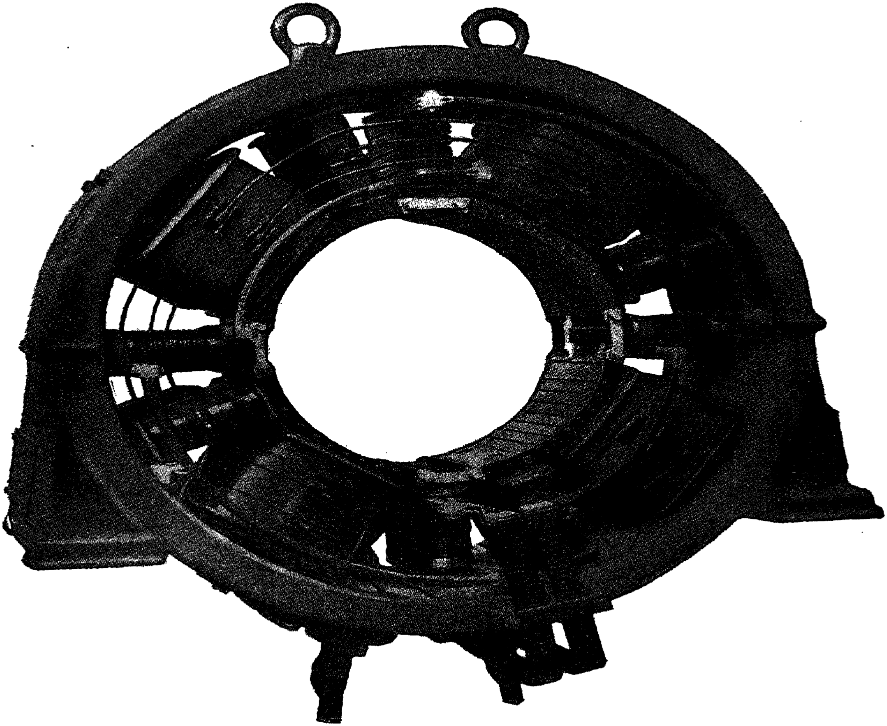


Fig. 9.25.—B.T.H. Co.'s SPLIT-POLE ROTARY CONVERTOR.

series-wound and so obtain automatic regulation. A divertor regulator may be added to its field winding, if hand regulation is desired in addition.

The same effect, however, can be obtained by boosting the A.C. voltage. This is done by means of an alternator with stationary field (excited by D.C. as described above), and rotating in synchronism with the rotary convertor. The advantage of this method is that it avoids the use of a second commutator carrying the full current, as required by a D.C. booster.

The booster armature is mounted on the shaft of the rotary convertor between the slip-rings and the main armature, which drives the booster as well as itself. The drawback of the method is that the booster has a bad effect on the commutation of the rotary convertor if this is fitted with commutating poles.

In the "split-pole" method of regulation each pole is made in two or three parts (see Fig. 9.25), a main pole and one or two regulating poles. In the figure there is one regulating pole to each main pole, the remaining small poles being interpoles (see Volume I., Chapter IX., Art. 25). The regulating poles can have their excitation varied *and reversed*, independently of the main poles.

The action depends on the fact that the D.C. e.m.f. varies directly as the algebraic sum of the main and regulating fluxes (see Volume I., Chapter VIII., Art. 14), *i.e.* as the total flux per group of poles. But the A.C. e.m.f. is the R.M.S. value of the total e.m.f. (cf. Chapter II., Art. 10) due to the separate fluxes. Keeping the R.M.S. value constant the algebraic sum can be varied widely. Thus with constant A.C. voltage the D.C. voltage can be adjusted as desired, a 25 per cent. range being possible in a rotary convertor running on a supply at 25 cycles per sec.

16. Starting of Rotary Convertors

All the methods of starting available for ordinary synchronous motors (see Chapter V., Art. 11) may be used for rotary convertors. But when starting from a D.C. supply this is fed into the commutator of the rotary itself, as there is no separate exciter. The risk of overloading the exciter is, therefore, not present. The synchronising of the A.C. side is best done on the *high* pressure side of the transformers, since owing to the smaller currents on this side the switches can be operated more rapidly.

Starting from the A.C. side is done exactly as for a synchronous motor, *i.e.* by transformer tappings or a pony motor. In the former case an additional precaution is necessary because the rotary convertor is self-excited. If, when synchronous speed is reached, the polarity of the field happens to be reversed by the A.C. starting currents the rotary will work correctly, but will have the wrong polarity for connecting to the D.C. bus-bars. This could be allowed for by making the main D.C. switch of the reversing type, but is more often done by " slipping a pole."

This means allowing the speed to drop slightly below the synchronous value until the armature has fallen one pole-pitch behind the position it would have occupied if synchronous speed had been maintained. The operation is effected by means of a field-reversing switch.

The actuation of this switch demagnetises the field and reduces the D.C. voltage to zero, the armature meanwhile dropping below synchronous speed. The switch is then returned to its original position and the voltage builds up again. If the return of the switch is made when the armature has fallen back by not quite a pole-pitch, the polarity on again coming into synchronism will be the reverse of the former polarity. The full transformer voltage is then applied and the rotary convertor may then be connected to the D.C. bus-bars.

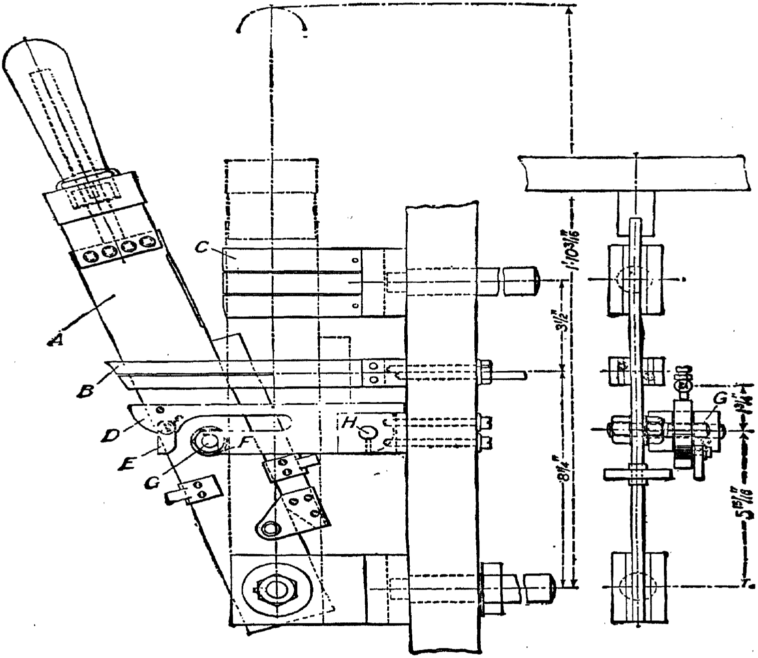
For rotary convertors of over 300 kW. output, this method is liable to cause troublesome sparking at the commutator. This can be avoided by using a D.C. brush-lifting device, which, however, is costly.

When starting with a pony motor with one pole-pair less than the rotary convertor has, synchronising is effected by resistance in the rotor circuit of the motor. An alternative is to use a short-circuited rotor (or one of solid *iron*) and to synchronise by altering the field regulator of the rotary, thus varying the load on the motor.

17. Self-Synchronising Rotary Convertors

These rotary convertors are obtained by connecting the stator windings of the pony motor in series with the A.C. side of the rotary (cf. Chapter V., Art. 12). The action is almost exactly the same as in a synchronous motor connected in this way, but is complicated by the rotary convertor being self-excited.

The synchronising is improved by Messrs. Siemens & Co.'s method of placing a non-inductive resistance in parallel with the pony motor. This brings the current in the rotary convertor armature more nearly into phase with the voltage, and so increases the synchronising torque. With this method the field circuit is kept open until synchronous speed has been reached. This has the advantage of avoiding an excessive voltage across the pony motor, which occurs during closed-field starting at those instants when the rotary e.m.f. assists the supply p.d. instead of opposing it.



SIDE VIEW. END VIEW.

Fig. 9.26.—SWITCH FOR SELF-SYNCHRONISING ROTARY.

The other method described in Chapter V., Art. 12, is unsuitable for rotary convertor starting, since an induction motor's rotor cannot be used to generate D.C.

The General Electric Co.'s self-synchronising switch is used in conjunction with a pony motor with the same number of poles as the rotary and a set of reactance coils. The first operation is to close the switch (or circuit breaker) which connects the pony motor's stator to the transformer secondaries. This starts the rotary convertor, and it excites itself. When the voltage has risen to about 25 per cent. of its normal value on the D.C. side (but slowly alternating due to the slip) the main switch is pushed into its preliminary position.

This is shown for one pole of the switch by the full lines in Fig. 9·26. The result is to connect the transformer secondaries through the reactance coils (which bridge across the B and C contacts) to the hinge contacts and thence to the slip-rings of the rotary. The current taken is limited by the reactances but is sufficient to bring the rotary up to synchronous speed.

The main switch is then pushed home, so short-circuiting the reactances and connecting the rotary directly to the transformer secondaries. This action operates a tripping device which disconnects the pony motor.

A device is attached to the main switch to prevent its being accidentally pushed straight home. The steel pin (G, Fig. 9.26) on the switch blade comes against the upper arm, D, of the slotted piece of steel. Thence it passes to the end of the lower arm, F, being prevented from entering the slot by the hinged piece E. The switch is thus stopped in its preliminary position as shown by the full lines in the figure.

To push the switch home it is first pulled a little forward so that the pin, G, enters the slot between D and F, and allows the switch to reach the running position. The slotted piece is hinged at H in such a way that it cannot rise above the position shown, and can fall below this by only a small angle.

18. Inverted Rotary Convertors

When used to convert D.C. power into A.C. power a rotary convertor is said to be "inverted." The same advantages as to output and efficiency as in the ordinary use are obtained by making the rotary convertor six-phase or twelve-phase when a three-phase supply is required. The connexions are exactly the same as before, but the low-pressure windings of the transformers are now the primaries, and the high-pressure ones the secondaries.

The chief difference is that the speed is no longer fixed. It varies inversely as the field *strength*, just as in an ordinary D.C. motor. There is, therefore, some danger of an excessive rise of speed, particularly if the A.C. load is an inductive one which weakens the field.

The risk of this is greatly diminished by adding a series winding to the field magnets. The number of turns required in this to keep the speed constant is greater the lower the power-factor. The number should, therefore, be sufficient for the lowest power-factor, and a divertor added for use when the power-factor is higher. A further precaution is to use a separate exciter driven by the rotary. Then any rise of speed is checked by the increase in the exciter p.d.

Generally a speed-limiting device is added. This revolves with the shaft, and consists of an arm with weighted end controlled by a spring. When the speed exceeds normal by a predetermined amount the weight moves out under centrifugal force a sufficient distance to release a non-rotating trigger. This closes a tripping circuit which opens the circuit-breaker in the leads from the D.C. supply (see Fig. 9.27).

Messrs. Siemens' method is to supply the main excitation from the D.C. supply, and auxiliary excitation from a special exciter mounted on the shaft of the rotary convertor. This exciter is like a small three-phase rotary convertor, but with no windings on its field magnets. As shown in Fig. 9.28, its slip-rings are connected to a three-phase series transformer with its primary in the A.C. mains supplied by the inverted rotary.

The exciter armature is thus supplied with currents proportional to and in phase with the A.C. supplied by the rotary. If the load is inductive these currents will produce a flux in the exciter magnets by armature reaction. This in turn sets up a p.d. between the D.C. brushes (B, B, Fig. 9.28) of the exciter, which sends a current through the auxiliary field winding of the rotary, and strengthens its field.

The armature reaction of the exciter increases with the A.C. load, and with its angle of lag. Hence the auxiliary excitation varies in the correct way to compensate for the armature reaction in the rotary at all loads and power-factors (cf. Chapter IV., Arts. 11, 12). Should the load be a leading one, the armature reaction of the exciter is reversed, and so the p.d. between B, B is reversed,

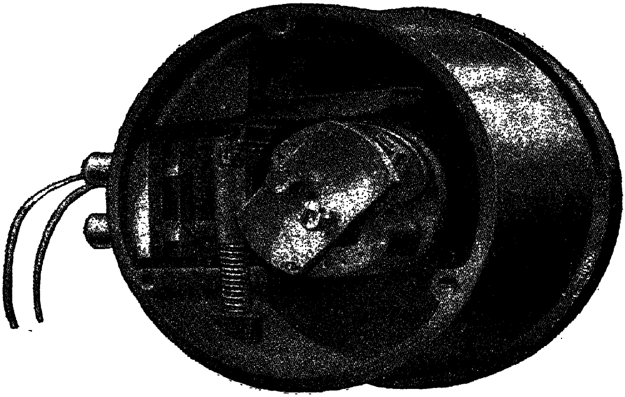


Fig. 9.27.—B.T.H. Co.'s SPEED LIMITING DEVICE.

and the auxiliary excitation weakens the rotary field. This, again, is correct, since the armature reaction of the leading current in the rotary tends to strengthen its field. The speed can thus automatically be maintained nearly constant at all loads, instead of the variations being merely diminished, as in the usual exciter method. Moreover, the exciter p.d. can be low, as it supplies only part of the excitation; and if it fails, the main excitation partially checks the rise of speed.

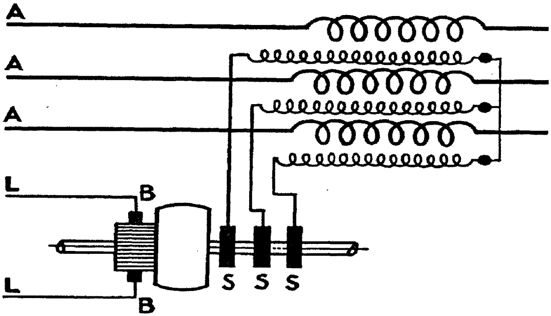


Fig. 9.28.—EXCITER FOR INVERTED ROTARY.
A A A = A.C. mains. B B = D.C. Brushes.
L L = Leads to auxiliary winding.

19. Double-Current Generators

A rotary convertor if driven by a separate motor, or by an engine, can be used to supply D.C. and A.C. simultaneously. It is then called a "double-current generator." In such cases the two currents flow on the whole in the same direction, and so there is very little gain in output by the double use.

The *maximum* heating now occurs midway between the tapping points (cf. Art. 5). On the other hand, the effect of power-factor in diminishing the allowable output is much smaller. This is because the increase in the A.C. for the same output is largely counteracted (instead of being intensified) by the change of phase (cf. Art. 8).

The above points are illustrated by the following table, which is drawn up for equal D.C. and A.C. outputs:—

NO. OF SLIP RINGS	OUTPUT (D.C. GEN. = 1)		OUTPUT (A.C. GEN. = 1)	
	Cos $\phi = 1$	Cos $\phi = 0·9$	Cos $\phi = 1$	Cos $\phi = 0·9$
3	1·025	0·99	1·11	1·19
4	1·05	1·02	1·05	1·16
6	1·07	1·04	1·00	1·11
12	1·08	1·05	0·99	1·07

The last two columns are the ratios of the outputs to the outputs of the same machine used as an A.C. generator. The reason for the decrease in these values with increasing number of slip-rings is that the improved breadth coefficient has a greater effect when the whole output is A.C. than when it is equally divided between A.C. and D.C.

The smallness of the advantage of a double-current generator, and the fact that the A.C. and D.C. voltages are not independent, results in such machines being used very rarely.

20. The Cascade Convertor or Motor-Convertor

This convertor was devised by O. S. Bragstad and J. L. la Cour in 1902. It consists of an induction motor driving, and connected "in cascade" (see Chapter VII., Art. 12) with a rotary convertor (see Fig. 9.29).

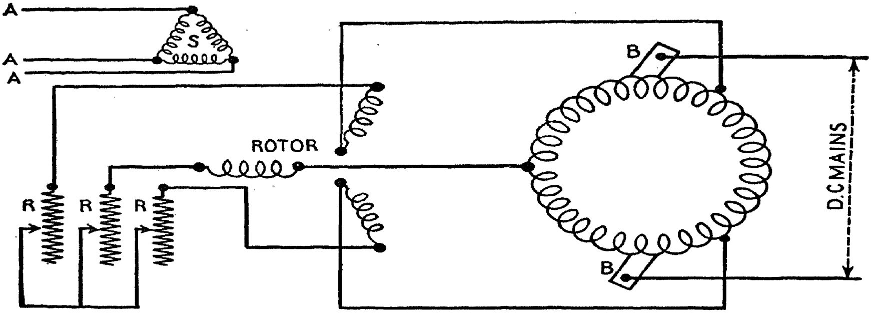


Fig. 9.29.—CONNEXIONS OF CASCADE CONVERTOR.

A A A = A.C. mains. B B = D.C. Brushes. R R R = Starting resistances.
S = Stator of induction motor.

The induction motor has a stator of the ordinary type, and a wound rotor connected to starting resistances (R, R, R) through slip-rings. The other ends of the phases of the rotor winding, instead of being joined together to form a star-point, are connected to tapping points on the armature of the rotary convertor portion. No slip-rings are needed for these connexions, since the rotor and the armature are mounted on the same shaft and revolve together. Hence a large number of phases (9 or 12) is used to obtain high efficiency, not three, as shown in the figure.

When in action the frequency of the rotor currents (see Chapter VI., Art. 8) is equal to (supply frequency) × (fractional slip). Hence the rotary convertor must run at the synchronous speed corresponding to this reduced frequency. But the rotor's speed is the same, and so the speed of the two is the synchronous speed corresponding to the total number of poles (cf. Chapter VII., Art. 12).

The speed of the induction motor being fixed in this way it ceases to have the ordinary characteristics, and behaves instead in two capacities. It acts partly as a synchronous motor and partly as a frequency transformer. The transformed power is transmitted through the connecting leads to the armature of the rotary convertor portion, and there converted into D.C. The power developed by the synchronous motor action is transmitted mechanically to the armature, driving it as a D.C. generator.

The rotary convertor portion, therefore, has a double action too, D.C. generator and rotary convertor. Its output compared to its size is, therefore, less than if it were acting purely as a rotary convertor, and its efficiency is lowered correspondingly.

The relative amounts of power transmitted by the transformer and motor actions of the induction motor part, neglecting rotor resistance, are as slip (s) is to speed ($1 - s$), where s = fractional slip. For at standstill the whole power is transformed. And at other speeds the voltage generated in the rotor is reduced to $s \times$ standstill value, and so the power transformed becomes $s \times$ total power.

Let p_1 = number of pole-pairs of induction motor portion;

p_2 = number of pole-pairs of rotary convertor portion;

f = supply frequency;

m = speed in revolutions per *sec.*

Then frequency in rotor $= sf =$ frequency of supply to rotary convertor;

$$\therefore\ m = sf/p_2; \qquad \therefore\ s = mp_2/f.$$

Again:—Synchronous speed for induction motor $= f/p_1$;

$$\therefore \text{ actual speed of induction motor} = (1 - s)\,f/p_1 = m;$$

$$\frac{\text{Power transmitted electrically from rotor}}{\text{Power transmitted mechanically from rotor}}$$

$$= \frac{s}{1 - s} = \frac{mp_2/f}{mp_2/f} = \frac{p_2}{p_1},$$

e.g. for equal numbers of poles half the total power is transmitted in each way. And for unequal numbers of poles, the greater the number of poles in the rotary convertor portion the greater the fraction of the total power converted by transformer and rotary convertor action, and the less converted by synchronous motor and D.C. generator action.

The above expressions for the speed show further that

$$m = \frac{sf}{p_2} = \frac{(1 - s)f}{p_1} = \frac{sf + (1 - s)f}{p_2 + p_1} = \frac{f}{p_1 + p_2},$$

i.e. the speed of the cascade convertor is, as already stated, the synchronous speed of a motor with a number of poles equal to the sum of the numbers of poles of the induction motor and rotary convertor portions.

21. Relative Advantages

The two great advantages of the polyphase rotary convertor are, firstly, that it is small, and therefore cheap, for its output; and secondly, that in spite of this its efficiency is high. These are partially but by no means completely counteracted by the necessity of providing transformers. The extent of these advantages depends on the output required and on the high-pressure voltage.

For 500 kW. and medium voltages (up to 2 kV.) the combined cost of rotary convertor and transformers will be about 5 per cent. less than that of a synchronous motor-generator and about equal to the cost of an induction motor-generator. But the combined efficiency at full load will be about 93 per cent. for rotary and transformer, against about 87 per cent. for the motor-generators.

At higher voltages the advantages of the rotary combination are greater because it is then advisable to use transformers with motor-generators. Even if transformers are not used the cost of the motor increases with voltage faster than the cost of transformers. But even at 1 000 V. the mercury arc rectifier (see Chapter XI., Art. 1) has a higher efficiency than even the rotary.

A cascade convertor is intermediate both in cost and in efficiency, the full-load efficiency of a 500 kW. one being about 91 per cent. As regards transformers it is in the same position as an induction type motor-generator.

This last is most convenient for starting, as it requires no synchronising nor a starting motor. The cascade convertor is started up in the same way, although when running its speed is fixed. The rotary convertor requires synchronising and is on a level with the synchronous motor-generator in this respect.

As regards power-factor the induction motor-generator is worst, always requiring a lagging current. All the other three can be worked at unity power-factor, and reach their best efficiency if this is done. They can be used with leading power-factors to correct the lag due to other loads on the A.C. supply. But whereas a synchronous motor-generator can be used in this way with little loss of output and lowering of efficiency (see Chapter V., Art. 13), a rotary convertor suffers greatly in these ways if the power-factor is not very nearly unity (see Art. 8). The cascade convertor, partaking of the characteristics of both, is intermediate in this respect too.

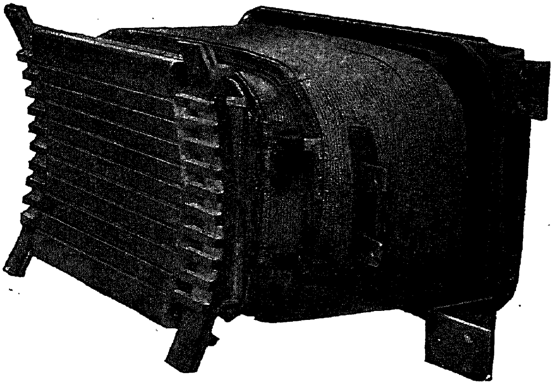
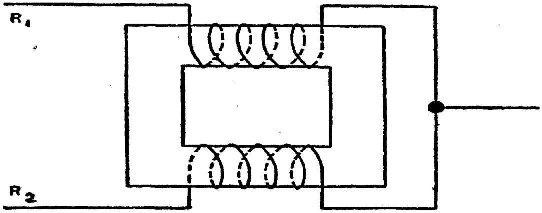


Fig. 9.30.—B.T.H. Co.'s AMORTISSEUR FOR ROTARY CONVERTOR.

Rotary convertors require damping coils or amortisseurs to prevent phase-swinging (see Chapter IV., Arts. 22, 23), to which they are rather more liable than are synchronous motors. And when running in parallel the former are very liable to divide the load unequally. Consequently it is standard practice to have separate transformers when two or more rotaries are used in parallel. When they are supplied direct from an alternator without intermediate transformers this may have separate windings for each rotary. An alternative is to use a balancing transformer so as to keep the supply currents close to equality.

The connexions of one phase of this are shown in Fig. 9.31. The two windings have the same number of turns and so long as the currents supplied to the two rotaries are equal and in phase no flux is produced. But any difference between the currents sets up a flux, the e.m.f. due to which tends (by Lenz' Law) to correct the inequality.



A. Leads from alternator. R_1, R_2. Leads to rotaries.
Fig. 9.31.—BALANCING TRANSFORMER FOR ROTARIES.

Cascade (motor-) convertors are seldom used for frequencies of 25 or less. But above this value rotaries are liable to commutation difficulties, and to instability under sudden loads. The cascade convertor overcomes this trouble and has an efficiency much closer to that of the rotary than a motor generator has. But modern improvements in rotary design have overcome the difficulty almost completely. In small sizes, however, the cascade convertor can be wound for a large number of phases at little extra cost, whereas the additional slip-rings necessary in a rotary convertor make the cost too high for the reduction in armature size to be worth while in such cases.

QUESTIONS ON CHAPTER IX

1. Find the A.C. amperes and volts per phase in a three-phase rotary convertor running at unity power-factor and delivering 100 amp. D.C. at 600 volts. Neglect the losses.

Draw diagrams showing the currents in the various parts of the armature windings of this rotary at two instants $\frac{1}{12}$th period apart. Calculate the rate of heating in the rotary at each of these instants if the armature resistance between D.C. brushes is 0·36 ohm. Estimate the time average of the heating rate. Hence find the output of the machine if used as a D.C. generator with equal losses.

2. Draw graphs of the currents in a three-phase rotary delivering 100 A. (*a*) for a conductor connected directly to a slip-ring; (*b*) for one midway between two slip-rings.

Find the R.M.S. values of each of these currents.

3. Draw graphs showing the wave-form of the current in the armature of a six-phase rotary convertor giving 100 amperes D.C., (*a*) for a conductor midway between two tapping points; (*b*) for a conductor next to a tapping point.

Find the R.M.S. of each of these currents, and hence estimate the maximum output of the rotary compared with that of the same armature used as a D.C. generator. (Take unity power-factor.)

4. A ten-pole rotary convertor delivers 1 500 amperes direct current at 550 volts. If the ratio of the alternating to the direct line currents is 0·94, find the ratio of the coil heating at a point midway between slip-rings to that at one adjoining a slip-ring, the power-factor being unity in the windings, and the supply three-phase.

5. Find the ratios of alternating to direct voltages and currents in three-phase and six-phase rotary convertors. How are these ratios modified by the power-factor of the machine and by the wave-form of the supply electromotive force? Under what conditions of load is an inverted rotary likely to race? How may this be prevented?

6. What are the advantages of using a large number of phases in a rotary convertor? Show that in a twelve-phase rotary convertor working at unity power-factor, the heating in coils close to the tapping point is 30 per cent. more than that in coils mid-way between two tapping points.

7. Examine the distribution of current in a three-phase rotary convertor and prove that the armature reaction of such a machine has no demagnetising component. How is the power-factor controlled?

8. Draw diagrams showing three methods by which a six-phase rotary convertor can be supplied from high-pressure three-phase mains. If the D.C. output is 100 A. find the current in each transformer secondary with double-Δ connexions.

9. If a six-phase rotary is supplied from three-phase high-pressure mains using double-Δ connexions, find the currents in the transformer secondaries and in the leads connecting them to the rotary when the D.C. output is 200 A. at 500 volts. Take unity power-factor and neglect the losses in the rotary.

10. Show how from high pressure three-phase mains a supply can be obtained:

(*a*) for a six-phase rotary convertor;
(*b*) the same, by an alternative method;
(*c*) for a twelve-phase rotary convertor.

In each of the three cases find the p.d.s across the secondary windings of the transformers if the D.C. p.d. is 500 volts.

11. A rotary convertor with six slip-rings is fed from the secondaries of a three-phase transformer with star-connected primaries. Each primary coil has ten times as many turns as each secondary. A load of 200 amperes at 500 volts is taken from the rotary. Draw carefully a diagram of the connexions, and also a vector diagram showing the magnitude and phase relationship of the voltages of the line, of the transformer coils, and of the slip-rings. Calculate approximately the voltages on the mains, and the currents in the primary coils. Assume the power-factor to be unity and the efficiency to be 100 per cent.

12. Give a diagram of connexions for a substation with transformers and rotary convertors for supplying power at 500 volts direct current, the primary supply being at 10 000 volts, three-phase, 33 cyc. Calculate the ratio of turns in the transformers, the current in the windings, and in the connexions to the slip-rings, when working at full load, the power-factor being 0·95.

13. Give a diagram of connexions for a twelve-phase rotary convertor supplied from three-phase mains.

If the primary volts are 2 000 (R.M.S.) and the D.C. volts are to be 230, and there are 800 turns in each of the Δ-connected primaries, calculate the number of turns for all the secondary windings.

14. Make a list of the numbers of the conductors connected to each ring of a six-phase rotary convertor when the armature is lap wound with six poles, 162 slots, and two conductors in each slot.

15. Describe one method of adjusting the D.C. voltage of a rotary convertor through the range from 440 to 550 volts, the high pressure p.d. on the transformer being constant.

16. Give an account of the construction and operation of an induction regulator. If 600 kVA. are required at a pressure varying from 5 000 volts to 7 000 volts from a supply at a constant p.d. of 6 000 volts, what must be the capacity in kVA. of the induction regulator used ?

17. Describe two methods by which rotary convertors may be automatically overcompounded, and discuss their relative merits. How is a voltage variation obtained in the case of the split-pole convertor ?

18. Describe two methods of starting and synchronising a rotary convertor.

19. Explain the switchboard connexions that must be made in a substation containing self-synchronising rotary convertors, and compare the

advantages of this type of convertor with that in which the rotary is started up without this special device. Give a diagram of connexions of the self-synchronising rotary, and explain the method of starting up.
[C. & G., Final, B.

20. If a six-phase rotary convertor has a full-load efficiency of 94 per cent. at unity power-factor, calculate its efficiency at a power-factor of 0·9:

(*a*) with the same load;

(*b*) with its new maximum continuous load.

Assume the copper losses to be two-thirds of the total losses originally.

21. Draw a diagram of connexions for a static balancer: (*a*) monophase; (*b*) six-phase.

22. A dynamometer voltmeter and a permanent magnet moving coil voltmeter were used simultaneously to measure the drop in the winding of the commutating poles of a rotary convertor. A considerable difference was observed between the readings on the two instruments although they read alike when connected together to a cell. Explain the probable reason for the difference between the readings, and say which instrument should give the higher reading, and why.
[C. & G., Final.

CHAPTER X

THERMIONIC VALVES AND RECTIFIERS

1. Neon Lamps

When an electrical discharge is produced in a tube containing gas at low pressure, called a Geissler tube, the gas itself glows. The successful application of this phenomenon to commercial lighting was made by D. McFarlane Moore, about 1905. Alternating current is almost essential since a high pressure is required, which can be obtained simply from an A.C. supply by a step-up transformer.

The diameter of the tube used was about 45 mm., and lengths up to 60 metres were used. Owing to this great length the tube had to be jointed on the site, and evacuated after erection. The pressure was 12 000 volts for the longest length, and the current about 0·3 A.

The chief difficulty of this type of lamp is that the gas pressure *diminishes* with use. A similar phenomenon occurs with X-ray tubes and other electrical discharge apparatus. This has been overcome in the Moore lamp by the use of an automatic valve (see Fig. 10.01). A glass tube, T, connected to the main tube, is expanded at its outer end into a bulb, B, open at the top end, A. The lower end of the bulb contains a cone-shaped carbon plug, C, normally covered with mercury. A tube-shaped glass plunger, P, dips into the mercury, and is attached to a core of iron wires, I. The solenoid, S, is connected in series with the primary of the lamp transformer.

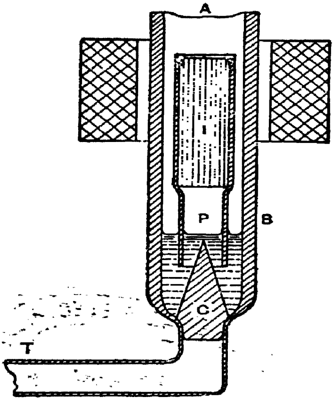


Fig. 10.01.—Valve of Moore Lamp.

When the gas pressure falls the lamp current increases, and the primary current increases correspondingly. The solenoid then raises the core, I, and the plunger, P, with it, causing the level of the mercury to fall and so uncover the tip of the carbon plug, C. This allows gas to ooze into the tube through C, until the rise of gas pressure and consequent diminution of current allow P to fall and re-seal the tube. In a long tube the feeding in of gas occurs about once a minute. The pressure is about $\frac{1}{10}$ of a mm. of mercury, and the valve keeps this within a 10 per cent. variation.

Atmospheric air is the gas simplest to use, but nitrogen gives a higher efficiency (about $1\frac{3}{4}$ watts per C.P.). In this case air is led over phosphorus to absorb the oxygen before the gas comes to the valve. The drawback of nitrogen is that the light is too pink. Carbon dioxide gives a white light but is only about one-quarter as efficient as nitrogen.

Mr. Moore developed a new type of CO_2 lamp, with self-contained gas generators, suitable for connecting to Edison type screw sockets such as are used for large incandescent lamps. The gas column is about 30 cm. long and 22 mm. diameter. The electrodes are contained in enlarged portions at each end, and behind each is a bulb, about 25 mm. diameter, containing calcium carbonate. Wires of high resistance are embedded in the carbonate and are connected in series with resistors and in parallel with the gas column. This is of a lower pressure than in the previous type, and consequently a reduction in gas pressure *increases* the gas resistance. The current in the resistance wires is thereby increased, and CO_2 gas is driven off by decomposition of the carbonate. This is claimed to keep the gas pressure within ·001 mm. of mercury of its normal value, and so to produce a very steady light. The white colour makes it useful for the correct matching of coloured materials.

The main advantage of the Moore lamp is that it has a large light source of low intrinsic brilliancy. It therefore gives very uniform light distribution so that when such is required, its actual efficiency is much higher than the figures indicate.

The rare gas neon has been used for Moore lamps. It gives an efficiency nearly twice as good as that of nitrogen but the light is of a strong orange pink. About 1920 this method was applied to lamps of the same size as ordinary incandescent lamps. By placing the electrodes close together these lamps can, owing to the properties of neon, be made for voltages down to 110 volts A.C., or 150 volts D.C. Thus no transforming apparatus is required on many supply systems.

No provision is made for adjusting the vacuum during use so that the lamp is as simple as an ordinary incandescent lamp. By adjusting the initial vacuum to a pressure of about 10 mm. of mercury, which is somewhat above that requiring minimum voltage for discharge, the effect of the change of vacuum is minimised. With constant p.d. the current will increase slowly with use, and afterwards diminish slowly. Consequently over a long period the alteration of current (and of candle-power) is small.

For alternating current the electrodes are of equal size. They may be either two thick wire spirals, interlaced but not touching (see Fig. 10.02), or two flat plates. On D.C. the gas glows only near one electrode, viz. that connected to the negative main. It is therefore preferable to make this larger (*e.g.* of two connected flat plates) and the other smaller (*e.g.* of a straight round rod) than in lamps for A.C. A series resistance of about 2 000 ohms is mounted in the cap portion of the lamp for the purpose of diminishing the fluctuations of the current when the p.d. varies.

The efficiency is low, about 10 watts per C.P., but the lamp absorbs only 5 watts. It is therefore very useful when only a small light is required, *e.g.* as a night light. When used as an indicator of the supply being on with electric kettles, cookers, and similar non-luminous consuming devices, a pattern absorbing only ½ watt is employed.

For advertising purposes long tubes and high voltages are used, as for the Moore lamp but without the complication of a gas pressure valve. By varying the composition of the glass, and by adding other substances to the neon gas, a wide range of colours can be obtained.

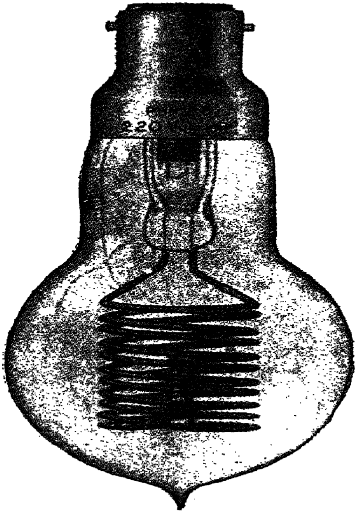


Fig. 10.02.—NEON LAMP.

2. The Thermionic Valve

If a filament lamp is furnished with an additional electrode, a current can flow under certain conditions between the filament and this electrode through the "vacuum," *i.e.* through the low pressure residual gas. The necessary conditions are (*a*) the filament must be incandescent, (*b*) the pressure applied to the vacuum must tend to send a current from the electrode to the negative end of the filament.

Under these conditions a single cell (C, Fig. 10.03) will send a considerable current, measurable on a milliammeter (A), across the vacuum. If this cell (C) has its connexions reversed the current is stopped almost completely. The explanation of this is that the incandescent filament, F, emits **electrons,** *i.e* atoms of *negative*

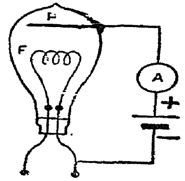


Fig. 10.03.—Edison Effect.

electricity. These are attracted by the electrode, P, which is maintained at a potential higher than the negative end of the filament by the cell, C. This passage of electrons from filament to electrode constitutes an electric current *in the opposite* direction, viz. from the electrode P, to the filament F.

If additional cells are connected in series with C the current is increased, because the electrode P is thus raised to a potential higher than a greater fraction of the filament than before (see further Art. 3).

The action is similar to that of the neon lamp, the main differences being (*a*) the heated filament produces free electrons and so a very low voltage can produce a current; (*b*) no glow is produced by the current through the gas.

This piece of apparatus is called a thermionic valve, but the name covers other more complicated valves as well. This type is sometimes called a **diode** because it has two electrodes, viz. the filament and the cold electrode. It is also named the **Fleming Valve,** because, though Edison first observed the phenomenon, Dr. Fleming was the first to make experiments on it and discover most of the properties of the discharge.

If the voltage applied between the filament and the anode (sometimes termed the **plate**) is varied a curve of the type shown in Fig. 10.04 is obtained. The scale of this depends on the particular valve used and on the filament voltage. The current rises with increase of voltage, slowly at first, then more rapidly at a nearly constant rate between A and B, and then again more slowly.

The slower increase of the current towards C is due to "saturation." When the potential applied is sufficient to attract to the plate all the electrons given off by the filament no further increase of current can be produced by raising the potential. The current, therefore, reaches a constant value known as its "saturation value." No further increase occurs until corona starts (see Chapter XII., Art. 3).

In "hard" valves, viz. those in which the vacuum is high, so that there is very little residual gas ($1/10^7$ atmosphere, or less), this

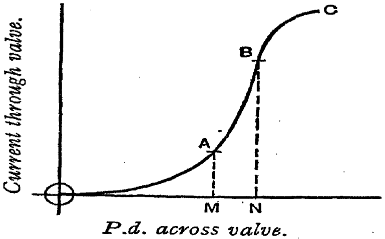


Fig. 10.04.—Current Curve of Diode.

phenomenon is very clearly marked. But in "soft" valves, although the increase of current with voltage becomes much less rapid, the increase continues. This is due to ionisation of the residual gas, *i.e.* some of its molecules become negatively charged, and so can act as carriers of electricity. The current is thereby increased above the value which it can reach when it consists only of the movement of free electrons. The same is true of "dull-emitter" valves (see Art. 4).

3. The Three-Electrode Valve

The thermionic valve, which is used largely in radio-telegraphy and radio-telephony, has three electrodes, and may, therefore, be called a **triode**. It was devised independently by Lee de Forest, who called it an **audion**; and by R. von Lieben, E. Reisz, and S. Strauss as a development of the Wehnelt cathode. This cathode consists of a platinum wire coated with a thin layer of an oxide of calcium, barium, or strontium. This was discovered by Dr. A. Wehnelt (1904) to emit an abnormally large number of electrons.

The third electrode is now called the **grid**. It consists of a metallic network placed between the filament and the plate. It has an independent terminal, and its object is to alter the movements of the electrons in the valve, and so change the relation between the electron current and the pressure producing it.

The original "bright emitter" valves with filaments run at 2 000° to 2 500° C. have been superseded by "dull emitter" (D.E.) valves which give a much larger rate of electron emission at 1 400° C. or less. The power consumption of the filament is reduced very much thereby. Thus the ordinary radio receiving D.E. valve takes about 0·1 A. at 2 V., compared with 0·7 A. at 4 V. taken by the bright emitter.

The filaments for dull-emitters fall into two main classes, (*a*) coated filaments, (*b*) thoriated filaments.

In the former a central metallic core is covered with a coating made of a mixture of the oxides of barium and strontium, sometimes with the addition of calcium oxide. Originally the core was platinum, but other metals, *e.g.* nickel, are now used.

The thoriated filament is made of tungsten, with a small percentage of thorium oxide. By suitable heat treatment a layer of thorium of molecular thickness is formed on the surface of the filament. This gives it the "dull-emitter" property of producing the same emission as the tungsten filament at a reduced temperature. The wearing away of the thorium layer in use is made good by diffusion from inside the filament. But it is destroyed if the

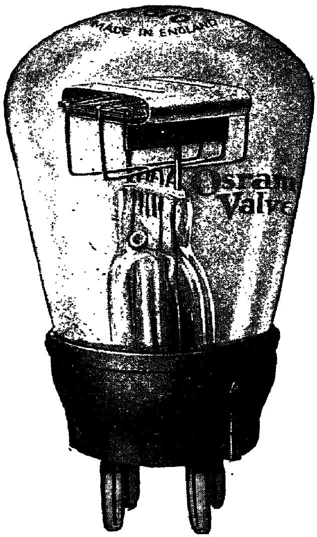


Fig. 10.05.—DULL-EMITTER VALVE.

filament is run at a temperature above 2 250° C. The presence of free gas likewise destroys this layer, consequently the pressure is reduced below the very low value of 0·00001 mm. of mercury (about one hundred-millionth of atmospheric pressure).

A valve of the latter type is shown in Fig. 10.05. The filament is mounted in a V-shape owing to its length. The anode is therefore made as a flattened cylinder, and the grid wires are arranged in a similar shape between the two.

4. Characteristics of a Triode

The behaviour of a triode valve depends on the following quantities:—

(*a*) The filament voltage,

i.e. the p.d. of B above A in Fig. 10.06. This can be adjusted by a series resistance, R.

(*b*) The anode voltage,

i.e. the p.d. of P above the *negative* end (A) of the filament. This can be adjusted by altering the number of cells in the anode battery Q.

(*c*) The grid voltage (usually called grid bias),

i.e. the p.d. of G above or below the negative end of the filament. This can be adjusted by a "potentiometer" resistance, H, which, if connected as shown, allows the grid potential to be made above or below that of A. Alternatively the adjustment may be made by altering the number of cells connected.

Note that the anode and grid voltages are reckoned from the negative end of the filament.

The effect of the grid can be seen by observing on the milli-ammeter, M, the current from the anode to the filament for various

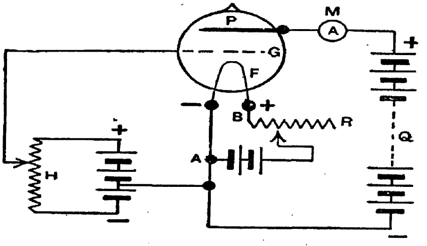


Fig. 10.06.
CONNEXIONS FOR OBTAINING TRIODE CHARACTERISTICS.

values of the grid potential, keeping the other voltages fixed. The result is a curve of the type shown in Fig. 10.07. This is one of the "static characteristic curves" of the triode. The adjective "static" is often omitted, but is inserted when needed to indicate that the relations are taken with steady currents and voltages. Under working conditions the relations may be altered. Usually the e.m.f. in the anode circuit is constant, but the anode voltage drops as the current increases. This drop is due to the resistance in the external part of the anode circuit. The curve connecting anode current with grid voltage under these conditions is called the "dynamic characteristic." It is less steep than the static characteristic, and depends on the external circuit as well as on the valve itself. "Working characteristic" is a more descriptive name.

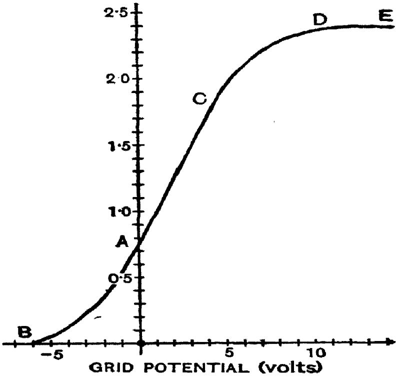


Fig. 10.07.—EFFECT OF GRID POTENTIAL.

The shape of this static characteristic curve can be explained as follows. When the grid is negative it repels the electrons and so prevents many of them from passing from the filament to the anode. The greater the negative potential of the grid the fewer the electrons which pass. This accounts for the fall in the anode current in passing from A to B, at which latter point the grid's potential is sufficient to stop all the electrons.

On the other hand, when the grid potential is positive it attracts the electrons. The higher potential of the anode causes them to continue their motion to this, and so produce an anode current. As the positive potential of the grid is raised the anode current increases as shown by AC. Beyond a certain value, however, the rate of increase diminishes (CD), and finally ceases more or less completely (DE) according to the "hardness" of the valve. This corresponds with the saturation current in a diode or Fleming valve (Art. 2).

If the filament p.d. is raised the filament becomes hotter, and gives off electrons more rapidly. The result is that the saturation current is increased. Moreover, as shown by the curves of

Fig. 10.08, for a given grid potential an increase of filament p.d. increases the anode current, and increases the change in anode current due to a given change of grid voltage.

A third way of examining the behaviour of the triode is to vary the anode potential. The results can be shown by drawing a number of curves similar to that of Fig. 10.07, each for a different constant anode potential, keeping the filament p.d. the same for all the curves. From these static characteristics (see Fig. 10.09) the following deductions can be made:—

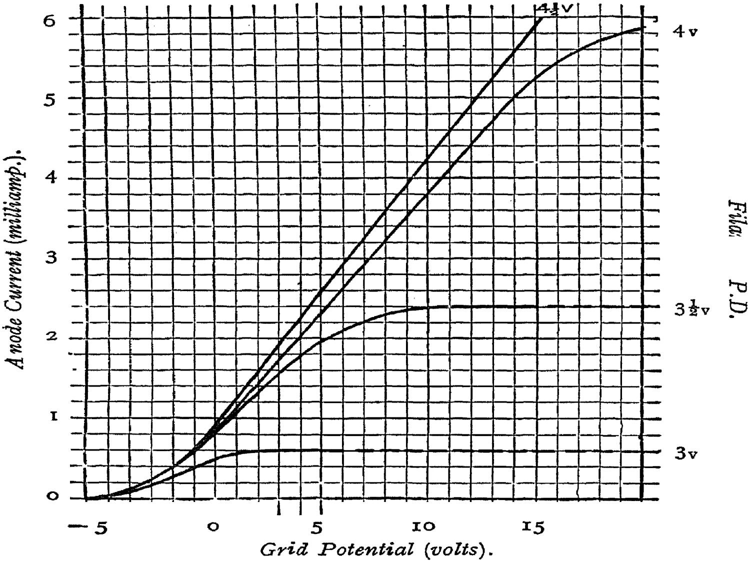


Fig. 10.08.—EFFECT OF FILAMENT P.D.

An increase of anode potential:—

(*a*) Increases the saturation current, but beyond a certain point (about 40 volts in the example) the increase is small;

(*b*) Increases the change of anode current per volt change of grid potential (as shown by the slope of the nearly straight portion of the curves, and called the mutual A.C. conductance of the valve) up to a certain point (about 50 volts in the example), beyond which the ratio of change ceases to increase;

(*c*) Reduces the grid potential which gives the greatest ratio of change of anode current to change of grid voltage, *i.e.* makes this occur at a lower positive or larger negative value.

The same applies to the values of grid potential to give the saturation current (top of curve), and to prevent any current at all (bottom of curve), *i.e.* these are both made less positive or more negative by an increase of anode potential.

Still another way of testing the triode's action is to find the change of anode potential to keep the anode current constant when the grid voltage is varied. A convenient method of keeping the anode current constant is to include a saturated diode in the anode circuit.* The resulting curves are shown in Fig. 10.10.

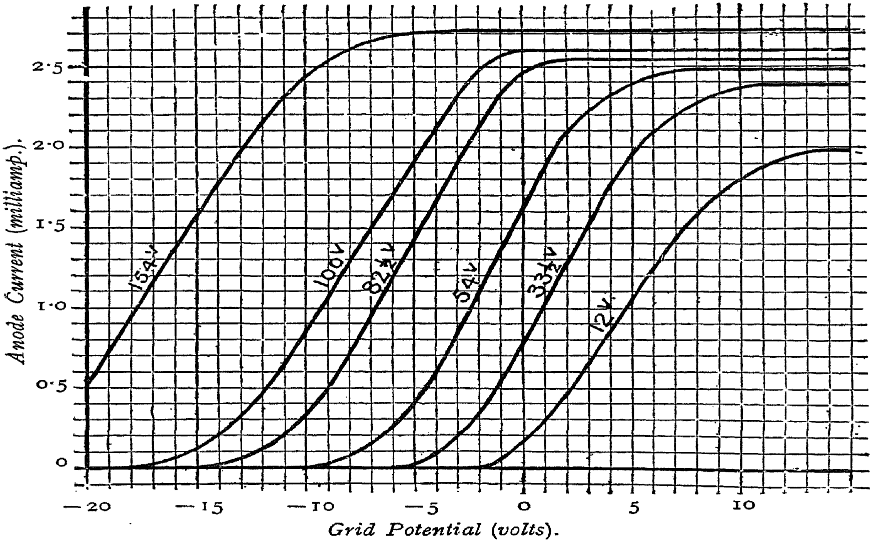


Fig. 10.09.—EFFECTS OF ANODE POTENTIAL.

It will be seen that until the grid voltage becomes too positive all the curves are straight lines of the same slope. The ratio (change of anode voltage)/(change of grid voltage) is called the "amplification factor." These experiments show that, over a wide range, the amplification factor of a triode has a constant value.

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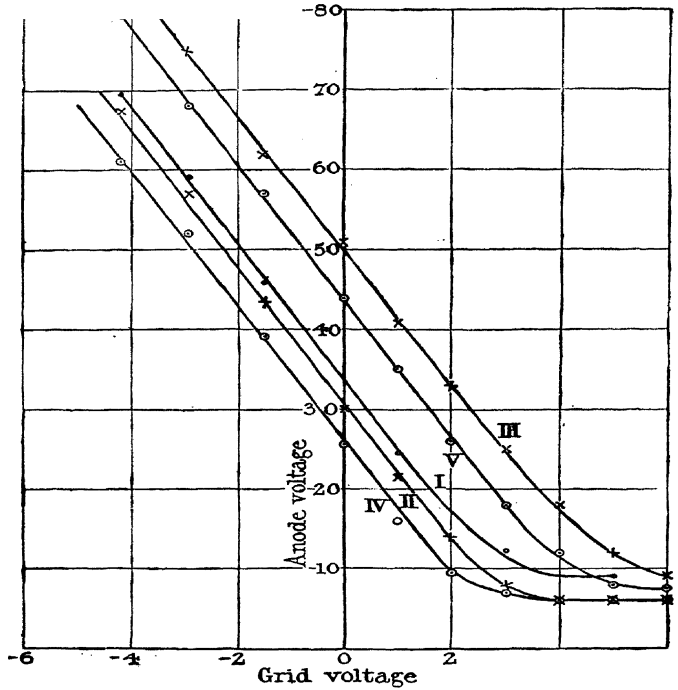
* See "Some Thermionic Tube Circuits for Relaying and Measuring" (Eccles and Leyshon), *J.I.E.E.*, vol. 59, p. 433.

In the curves, given the amplification factor is 8·8 over the ranges of constancy.



Reproduced from *Journ. I.E.E.*, Vol. 59, p. 433.

Fig. 10.10.—EFFECTS OF GRID POTENTIAL.

CURVE	FILAMENT VOLTAGE	ANODE MILLIAMP.
I.	2·5	·052
II.	3·0	·070
III.	3·0	·410
IV.	4·0	·070
V.	4·0	·410

The characteristics of dull-emitter valves are similar in character. The main differences are that the usual filament voltage for D.C.

valves is 2 V.; and that the saturation values are less definite. In modern valves the saturation valves cannot be reached without using such high voltages that there is danger of damaging the filament. The D.E. anode currents are often much larger.

5. Parameters of Triodes

Usually a valve is run at a definite filament voltage, but even then the anode current depends on both the anode voltage and the grid voltage. The behaviour of a given valve can be followed most simply by means of certain relationships which are called the parameters of the valve.

One of these, the amplification factor (μ), has been defined in Art. 4. The definition put into symbols becomes:—

$$\mu = \frac{\delta v_a}{\delta v_g} \; (i_a \text{ constant}).$$

As pointed out already, the value of μ is constant over a considerable range.

If the anode current for a fixed value of the grid voltage is plotted against anode potential, curves are obtained similar in shape to those of Fig. 10.09. The slope of these curves is called the "anode A.C. resistance" (r_a) or the "slope resistance." Sometimes it is called the valve impedance, but the use of this term is deprecated. The definition of this parameter in symbols is:—

$$r_a = \frac{\delta v_a}{\delta i_a} \; (v_g \text{ constant}).$$

It too is nearly constant over a considerable range.

The third parameter is called the "mutual A.C. conductance" (g_m) of the valve. It is the ratio of (change in anode current) to (change in grid voltage) at a fixed anode voltage, or in symbols:—

$$g_m = \frac{\delta i_a}{\delta v_g} \; (v_a \text{ constant});$$

and it is constant over the same range as the other two. It is a conductance since it is a current divided by a voltage (usually expressed in milliamperes per volt); "mutual" because the current and voltage are in different, though interlinked circuits; and "A.C." because it is the ratio of the changes and not of the quantities themselves.

It follows from the definitions that at any operating point

$$\mu/r_a = \frac{\delta v_a}{\delta v_g} \div \frac{\delta v_a}{\delta i_a} = \frac{\delta i_a}{\delta v_g} = g.$$

This relation is true at any point, whether this is within the range of constancy or not; but its chief use is for points within the range. This range is that covered by the nearly straight parts of the characteristics, provided there is no grid current: the latter is realised if the grid bias is always negative.

6. Equivalent Circuit of Triode

The general expression for any small change of anode current (δi_a) is $\frac{\delta i_a}{\delta v_a}\,\delta v_a + \frac{\delta i_a}{\delta v_g}\,\delta v_g$, which $= \frac{1}{r_a}\,\delta v_a + g_m \delta v_g = (\delta v_a + \mu \delta v_g)/r_a$.

Provided the changes are kept within the nearly straight parts of the characteristics, so that μ, r_a, and g_m are constant, the relation holds for large changes as well as for small ones. Thus under such conditions $i_a = (v_a + \mu v_g)/r_a$, where i_a, v_a, and v_g now mean the *changes* of these quantities.

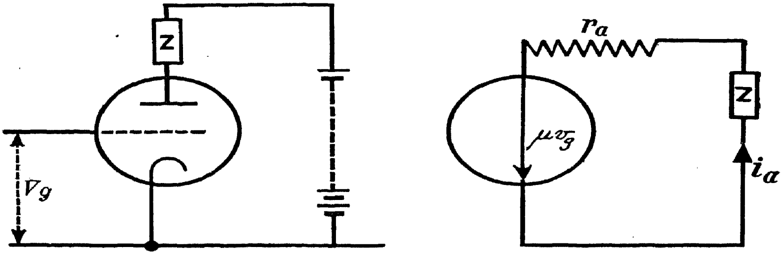


Fig. 10.11.—EQUIVALENT CIRCUIT OF TRIODE.

It follows that a triode may be replaced by the equivalent circuit shown in Fig. 10.11, where $\dot{Z}$ is the anode circuit vector impedance. For if v_3 is the increase in the p.d. across $\dot{Z}$, $v_a = -v_3$, since the total voltage in the anode circuit is constant.

Therefore from the preceding equation,

$$\mu v_g = i_a r_a - v_a = i_a r_a + v_3,$$

which latter evidently is true in the equivalent circuit.

Thus the triode can be replaced by a source of e.m.f. of value μv_g in series with a resistance of value r_a, as far as the change in the current in $\dot{Z}$ is concerned.

For exact work the inter-electrode capacitances have to be considered in addition in certain cases (see Arts. 9 and 14).

7. Uses of Triode in Radio Reception

In this application valves are used for the following three purposes:—

(*a*) To amplify the signal received by the aerial at a high ("radio") frequency; these radio frequencies are equal to 3×10^8/(wave-length in metres), since the waves travel at 3×10^8 metres/sec., *i.e.* the velocity of light;

(*b*) To convert the signal, either before or after amplification, to a lower ("audio") frequency which can affect a telephone;

(*c*) To amplify the signal at the audio frequency.

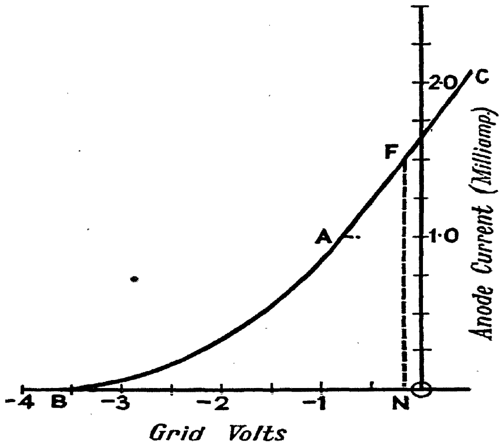


Fig. 10.12.—AMPLIFICATION BY TRIODE.

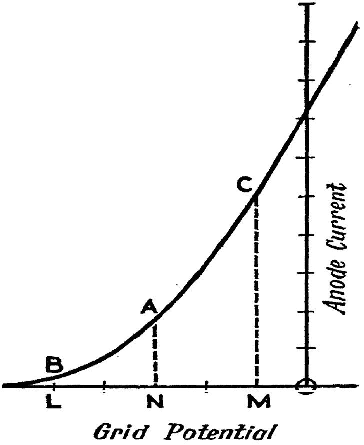


Fig. 10.13.—"DETECTION" BY TRIODE.

For amplification the straight and steepest part of the anode current-grid potential characteristic (Fig. 10.12) is the best to use. Therefore the grid potential when no signal is being received should be adjusted to the value (ON) corresponding with the middle point (F) of the nearly straight part (AC) of the characteristic. A given change in the grid potential, due to a signal, will then produce a proportional change in the anode current.

Fairly high values of the filament p.d. and the anode potential are advantageous too. But provided the variations of the grid potential do not bring the conditions outside the nearly straight part (AC) there is little advantage in pushing up the potentials excessively. In most arrangements it is, however, advisable to have a *negative* value, not only of the grid bias (ON), but of the grid potential for all signals received. Otherwise grid current flows and reduces the amplification.

A common way of using the valve for (*b*), generally called "detection," is to work on the lower part of the characteristic. For instance, if the grid potential is ON (Fig. 10.13) with no signals, the anode current is then AN. If an alternating voltage is applied to the grid by a received signal the grid potential will vary between OL and OM, where L and M are at equal distances from N. The anode current will therefore vary between BL and CM. Owing to the upward curvature of the characteristic the mean value of this varying current is higher than the previous steady value, AN; consequently a sound will be produced in a telephone included in the anode circuit. This is called anode bend detection.

When detecting a telephony signal the "carrier wave" (the radio frequency wave sent out by the transmitting station) increases the anode current when the receiver is tuned to its wavelength. When a sound is to be transmitted the carrier wave is "modulated," *i.e.* alternately increased and decreased in amplitude at the audio frequency. The anode current of the detector valve is thus increased or decreased at audio frequency, though always greater than when the receiver is untuned, or when the carrier wave is not being radiated. Hence in this case the valve should be worked near the centre of the curved portion.

Another method, which is used very often for detection, is to apply the radio frequency voltage (whether amplified or not) to the grid through a small condenser of about 300 $\mu\mu$F. A high resistance (about 2 megohms), called the grid leak, is connected either across the condenser or between the grid and the filament. The condenser being of low impedance at radio frequency allows the grid to follow the voltage variations almost exactly. If these are of constant amplitude the *mean* grid current is constant, but an increase of amplitude raises the mean grid current in the same way as the anode current is increased in the previous method. This *lowers* the grid potential, since grid current flows through the resistance from filament to grid, and hence the anode current is reduced. So these changes in anode current depend on the comparatively slow (audio frequency) alterations in amplitude of the signal, and not on its (radio frequency) changes.

For these actions to occur the grid must become positive during at least part of the cycle of grid potential changes, for no grid current flows when the grid voltage is zero or negative.

8. Triode Connexions

(*a*) A simple method of connecting a valve for high frequency amplification is shown in Fig. 10.14. In this (A) is the aerial, connected in series with the primary (B) of a transformer, and a variable condenser (C) to earth (E). This condenser is often of fixed value, and is sometimes omitted: when made variable it is placed between E and B as a rule. When a radio signal is received electrical oscillations are set up in the aerial circuit (AE).

These produce an alternating voltage in the secondary (D) of the transformer, which is " tuned " by the variable condenser (H) so as to be in resonance with the desired signal. This voltage is superposed on the steady p.d. applied between the grid (G) and the filament (F) by the grid battery (K), if used. As explained in Art. 7 these variations in grid voltage result in a much larger variation in the current between the anode (P) and the filament. The increased energy is supplied by the anode battery (M).

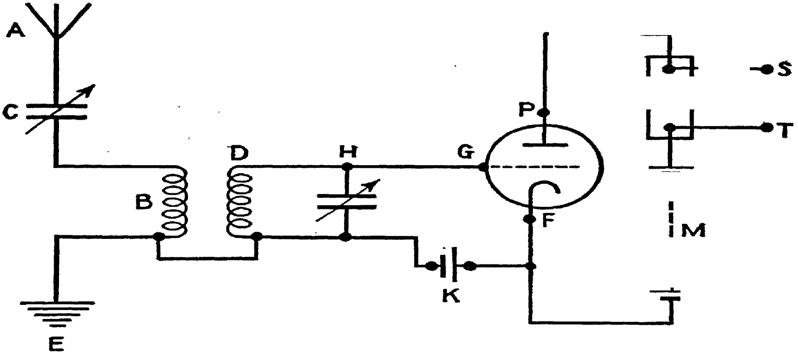


Fig. 10.14.—CONNEXIONS FOR HIGH FREQUENCY AMPLIFICATION.

The impedance, Z, in the anode circuit, provides the amplified voltage which is applied to the grid of the next valve by means of the terminals S, T. A number of different methods are used for " coupling " the valves, *i.e.* for applying the voltage amplified by the first valve to the grid of the next valve (see Art. 9).

(*b*) For detection by the first method described in Art. 7 the connexions shown in Fig. 10.15 are suitable. The aerial (A) is connected to earth (E), through an inductance (B) and a variable condenser (C) for tuning purposes. The terminals of the inductance

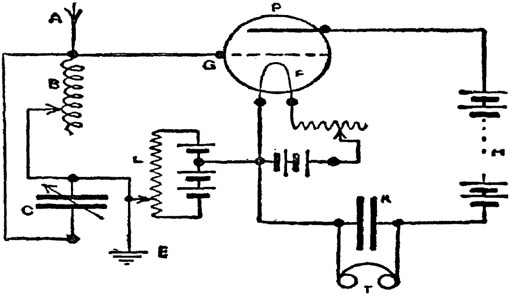


Fig. 10.15.—Connexions for Detection.

are connected to the grid (G) and the grid potential slide (L). Thus the voltage due to the received signals varies the grid voltage. Instead of connecting directly to the aerial circuit, a transformer may be interposed as shown in Fig. 10.14.

The high frequency variation of the grid voltage causes a change in the mean value of the anode current (see Art. 7) if the steady value of the grid potential is adjusted to suit the voltage of the anode battery (H) in use. This causes a sound in the telephones (T). The condenser (K) in parallel with the telephones is to permit the passage of the high frequency anode current. Otherwise the high inductance of the telephone coils would make the anode current variations exceedingly small so that any sound produced might be imperceptible (see further Art. 9).

(*c*) When low frequency ("note") amplification is used in addition, the connexions of the detecting valve (Fig. 10.15) can be modified as follows. The telephones (T) are replaced by an inductive coil or by the primary of a transformer. The terminals of this coil (or of the secondary when a transformer is used) are connected in the grid circuit of the L.F. amplifying valve (see Fig. 10.16) in the same way as coil B in that of the detecting valve (see Fig. 10.15). S is the secondary of the transformer, and the grid bias battery is not shown, though it is desirable to use one.

The anode current passes through the primary (M) of a second transformer. The secondary (N) of this transformer induces in the telephones or loudspeaker (T) a current corresponding to the *changes* in the plate current. This is preferable to connecting

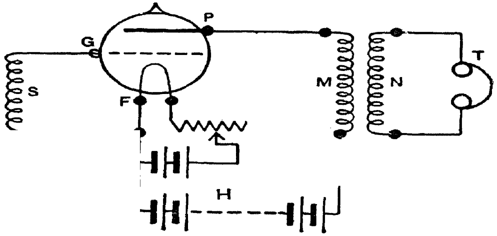


Fig. 10.16.—Connexions for L.F. Amplification.

the telephones directly in the anode circuit, particularly when working high up on the valve characteristic (cf. Art. 7) because the steady anode current may damage the telephones, and in any case reduces their sensitiveness.

This transformer may have an iron core since it is working at "audio" frequency, say 100 cyc. to 9 000 cyc. Those used for high frequency amplification often have non-metallic cores to avoid the losses due to eddy currents at high frequencies. But "dust" cores are used sometimes for this purpose. These consist of fine particles of iron embedded in an insulating material. Their presence reduces greatly the core reluctance, without causing serious eddy current losses even at radio frequencies.

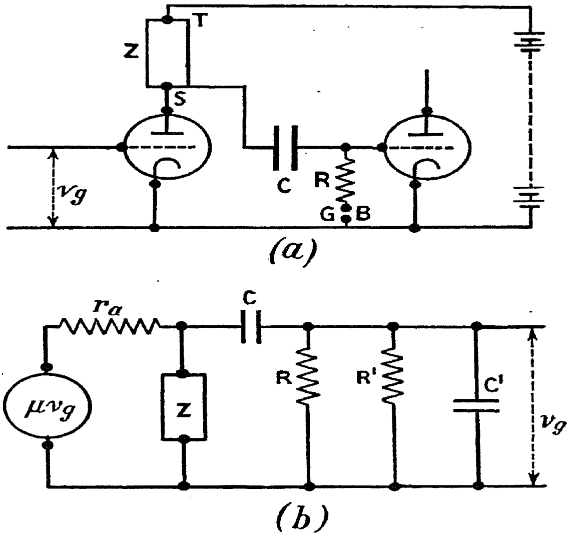


Fig. 10.17.—COUPLING OF TRIODES.
(*a*) CURRENT CONNEXIONS.
(*b*) EQUIVALENT CURRENT.

9. Coupling of Triodes

It may be noted that the telephones in Fig. 10.15 are connected on the filament side of the anode battery, whereas the transformer primary (M) in Fig. 10.16 is on the anode side of this battery. The latter is preferable to avoid making the mean potential to earth of the battery vary with the output voltage across M.

But if the detector valve is coupled to the L.F. amplifier by an inductive coil as described in Art. 8 (*c*), this coil must not be placed on the anode side of battery H. If this were done the voltage of the battery would be applied to the grid of the L.F. valve.

If a transformer is used for coupling these two valves its primary should be inserted on the anode side of battery H, for the reason given above.

For coupling a H.F. amplifier to a detector a transformer of a suitable kind may be used too. That is to say that the impedance Z in Fig. 10.14 may be a transformer with its primary connected in the anode circuit of the H.F. valve; and with its secondary connected between the grid and the filament of the detector, *i.e.* in the position occupied by coil B in Fig. 10.15. The aerial is no longer connected as shown, but the retention of the variable condenser C will increase the selectivity of the arrangement.

For other methods of coupling it is necessary to connect a condenser between S in Fig. 10.14 and the grid of the next valve [see Fig. 10.17 (*a*)]. This prevents the steady potential of the anode battery from reaching the grid, while permitting the rapid variations to do so. A high resistance, R (called the grid leak), is connected as shown to maintain the steady potential of the grid. The filament of the second valve is connected to the negative of the anode battery of the first valve and not to T.

If the valve is replaced by its equivalent circuit (see Fig. 10.11, Art. 6), and all steady sources of potential omitted, the complete equivalent circuit becomes as shown in Fig. 10.17 (*b*). In this R′ and C′ are the input resistance and capacitance of the second valve, and v_g' is the variable voltage applied to its grid.

10. Amplification of Coupled Valves

The capacitance C in Fig. 10.17 is usually about 300 $\mu\mu$F., which at a frequency of 1 megacycle per sec. gives an impedance of about 500 ohms, and less at higher frequencies. This is negligible in comparison with the other impedances, so that Z_a, R, R′ and C′ are effectively in parallel. If $\overline{Z}$ denotes their combined impedance $v'_g/\mu v_g = \overline{Z}/(\overline{Z} + r_a)$, the addition being vectorial. Thus the voltage amplification of the arrangement $= v'_g/v_g = \mu\overline{Z}/(\overline{Z} + r_a)$. This cannot exceed μ, and approaches that value more closely as $\overline{Z}$ is increased.

r_a is 10 000 ohms or more, therefore $\overline{Z}$ should be made 50 000 ohms, or more if possible, to keep the amplification high. R is usually one or two megohms, so it has no appreciable effect on the value of $\overline{Z}$. R′ depends on the anode load of the second valve, but will usually be several hundred thousand ohms so that its effect is small too. But C′ is about 50 $\mu\mu$F. so that its impedance at 1 megacycle per sec. is only about 3 000 ohms.

If Z_a is a pure resistance (resistance-capacitance coupling) $\overline{Z}$ must be less than 3 000 ohms, so that the amplification is only about $\mu/5$. Therefore this method is seldom used except for long (low radio frequency) waves, or for audio frequency amplification.

By making Z_a inductive, $\overline{Z}$ can be made to have a much higher value, and so the amplification can be increased. The highest impedance of a choke and a condenser in parallel occurs at their resonant frequency, but this would make the circuit unstable. Consequently the choke is designed to resonate at a frequency slightly below the lowest radio frequency which is to be amplified.

A further alternative is to make Z_a consist of an inductance in parallel with a variable condenser. This enables the value of $\overline{Z}$ to be adjusted to a suitable value for different frequencies, and so gives greater selectivity though at the cost of an extra adjustment. This method is known as "tuned anode."

The difficulties introduced into radio frequency amplification by the inter-electrode capacitances of valves have been overcome by various complicated circuits. But the introduction of screen-grid valves (see Art. 12) has reduced these capacitances so much that the complication has become unnecessary.

11. Oscillations Generated by Valve

Both for producing radio signals and for detection by beats, high frequency oscillations are required. A triode can be used for this purpose by connecting it as shown in Fig. 10.18. Between the grid (G) and the negative terminal of the filament (F) a variable inductance (A) and a variable capacitance (C) are connected in parallel. These form an oscillatory circuit whose frequency can be adjusted by altering A or C.

In line with A is a coil B, called the **reaction** coil. This can be rotated so as to vary the mutual inductance of the two coils. When this is made large the coils are said to be tightly (or closely) coupled, and when it is small they are called loosely coupled. B is connected in the anode circuit, and so a variation in the anode current tends to produce a current in A, *i.e.* in the grid oscillatory circuit, by the inductive action of B on A.

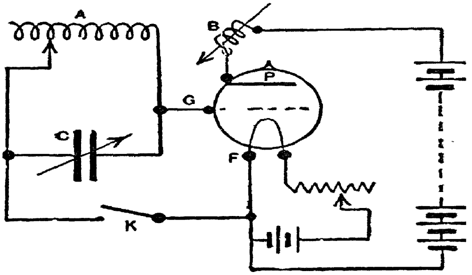


Fig. 10.18.
Generation of Oscillations.

Any small oscillation in the grid circuit produces an oscillation of the same frequency in the anode current. This tends, through the action of B on A, to strengthen the grid oscillation. According to the tightness of the coupling the energy given by B to A is greater or less than the energy lost in the grid circuit. In the former case the oscillations increase until the losses become equal to the energy supplied. The oscillations then continue as long as the key (K) is kept closed and the filament and anode p.d.s and the coupling are unaltered.

For beat reception, coil A and a coil carrying the signals (whether amplified or not) together produce currents which are used to operate further valves. This method is known as **heterodyning,** because the oscillations for producing the beats are generated by an independent source of power.

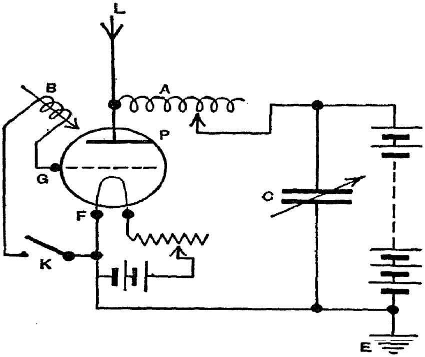


Fig. 10.19.—Triode Transmitter.

The combination of the two frequencies produces a variation whose frequency is equal to the difference between those of the two components (cf. Chapter IV., Art. 17). The advantage of this is that by adjusting the heterodyning frequency the resulting frequency can be given any desired value, and so it becomes easy to obtain high selectivity.

The "reaction" principle is employed in many receiving circuits, apart from heterodyning. By keeping the coupling loose the signals can be strengthened without causing the generation of oscillations when only the carrier wave is being received.

For producing radio signals the connexions are modified (see Fig. 10.19). B is placed in the grid circuit, and A and C (connected in series instead of in parallel) in the anode circuit, with the battery across C only. One end of the oscillatory circuit formed by A and C is connected to earth (E), and the other to the aerial (L). Thus A and C become the aerial tuning inductance and condenser. The method of action is as before.

12. Screen-Grid Valve

When high frequency amplification is used there is considerable liability to generate oscillations causing "howling" in the receiver.

This is due to coupling between the anode and the grid circuits, as in Fig. 10.19, but effected through the capacitances between the anode and grid, and between the grid and the filament.

To avoid this effect an additional grid (called the screen grid) is placed in the valve between the ordinary grid and the anode. In the earliest pattern the valve was cylindrical, with a cap at each end. One of these carried three pins, two for the filament and one for the control (ordinary) grid. The other cap had two pins, one connected to the anode and the other to the screen grid. In later patterns an ordinary 4-pin cap is used, but with the usual anode pin joined to the screen grid. The anode is connected to a terminal mounted on top of the valve [see Fig. 10.20 (*a*)].

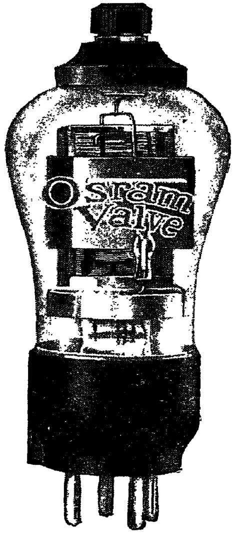


Fig. 10.20 (*a*).
SCREEN-GRID VALVE.

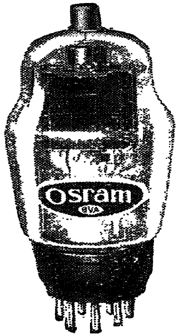


Fig. 10.20 (*b*).
INDIRECTLY HEATED SCREEN-GRID VALVE.

When an indirectly heated cathode is used (see Art. 14) the valve has a 7-pin base [see Fig. 10.20 (*b*)]. Three of these pins are connected to the heater and to a centre tap on the heater. The cathode, anode, and screen grid are each connected to a pin, and the seventh is a dummy. The top cap is connected to the ordinary control grid. In all patterns the screen grid is connected to a tapping on the H.T. battery (or other source) giving from half to two-thirds of the voltage applied to the anode. When this screen grid voltage is adjusted correctly the result is that coupling between the anode and grid circuits is eliminated almost completely. Consequently very high amplification can be obtained without risk of "oscillation." A large capacitance is connected between the screen grid and earth to prevent R.F. oscillations of its potential.

13. Pentode Valves

The screen-grid type of valve in a modified form has been applied to audio frequency amplification. The modification consists in adding a third grid, placed between the screen grid and the anode.

This outermost grid is connected inside the valve to the negative end of the filament. The valve is called a pentode because it contains five electrodes, viz. the filament, three grids, and the anode.

The innermost grid is the ordinary (control) grid, and is connected to a base pin in the usual way. The screen grid, between the other two, is connected either to a terminal on the side of the base or (see Fig. 10.21) to a fifth pin mounted at the centre of the base.

In use the screen grid is supplied at a voltage equal, or nearly equal, to the anode voltage. For instance, in the valve shown the two voltages may be equal up to 200 volts. The anode voltage may be raised to 250 V., but the screen grid voltage must then be kept down to 200 V., preferably by a series rheostat of about 7 000 ohms resistance.

The result of this design is to give a valve which has a very high amplification factor (80), and yet can deal with a large variation of grid voltage, *i.e.* with a signal which is strong, or which has been made so by high (radio) frequency amplification. In the ordinary triode valve for audio frequency amplification the factor cannot be made so high (limit about 30), and as the signal strength increases valves with lower factors (down to 8 or less) must be used if distortion is to be kept small. Thus in many cases a single pentode can replace two triodes.

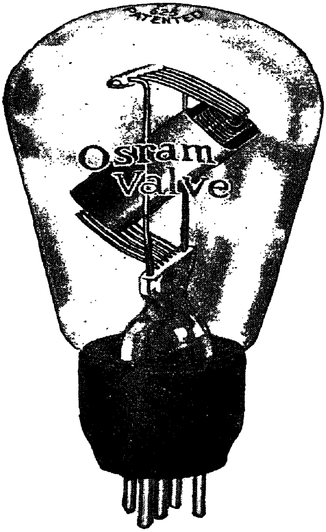


Fig. 10.21.—PENTODE VALVE.

14. Cathode Heating from A.C. Mains

Where an electric supply is available, the use of batteries in radio receiving sets may be partly or completely avoided by obtaining the anode current or the filament current, or both, from the mains. With a D.C. supply either of these can be done by means of resistances. A smoothing device, consisting of series inductances and parallel condensers, is required to prevent the slight fluctuations in the voltage due to commutation or armature teeth effects from producing a hum in the receiver.

When the supply is A.C., transformers will give the desired voltages, but a rectifier is required to make the current unidirectional. More smoothing is required owing to the larger variations of voltage. For filament supply alone the rectifier and smoothing device may be omitted by modifying the ordinary arrangements as follows.

The filament is made short and thick, so that its heat capacity is large compared with its rate of radiation. This reduces its temperature variation due to the alternating current, and so greatly cuts down the hum introduced. Suitable filaments work at 0·8 V., and take 1·6 A. in the case of detector valves, and 0·8 A. for those used as amplifiers (H.F. or L.F.).

Secondly, the return lead of the grid circuit, instead of being connected to one end of the filament, should be joined to the middle point of a "potentiometer" resistance connected between the two ends of the filament. This arrangement keeps constant the *average* voltage between the filament and the grid, apart from signal changes.

The hum can be more nearly completely eliminated by heating the cathode indirectly. This is done by enclosing an insulated heating element in the cathode. The two together replace the filament, which in the ordinary type of valve acts both as heater and as cathode.

The ends of the heater are joined to the "filament" pins. The separate cathode is connected either to a terminal mounted at the side of the base, or to a fifth pin projecting from the centre of the base (as in Fig. 10.21).

For more complicated valves either a 7-pin base [see Fig 10.20 (*b*)] or an "octal base" may be used (see Fig. 10.22). This has a projecting stalk of insulating material with a key on it to fix the position of the valve in its holder. There may be 8 pins at equal distances apart, but in many cases one or more are omitted, *e.g.* the figure shows a 7-pin.

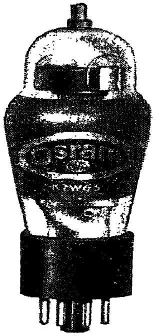


Fig. 10.22. OCTAL BASE VALVE.

Both directly-heated and indirectly-heated types can be made to give results almost identical with those given by the usual type of valve with D.C. heating.

15. Valve Rectifiers

Since the anode current can flow only from this to the filament inside the valve, because the anode does not emit electrons, a valve can be used as a rectifier. This means that if supplied from an alternating voltage it will deliver a current flowing in one direction only. If a single valve is used the current will flow during alternate half-cycles only. By using two or more valves in suitable circuits " full-wave rectification " can be obtained, *i.e.* the output current can flow during both half-cycles of a period. Even so it varies between maximum and zero, so that if a fairly steady current is required an arrangement of " smoothing " chokes and condensers must be added.

If two anodes and one cathode are mounted in one bulb (Fig. 10.23) the valve costs less than two ordinary valves; and the smoothing circuits for full-wave rectification can be used, which are simpler and cheaper than for " half-wave."

16. Other Forms of Rectifiers

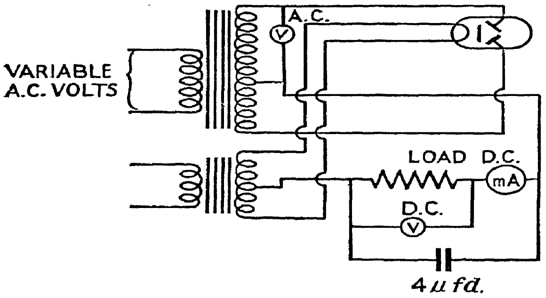


Fig. 10.23.—Circuit for Full-Wave Rectification Using a Two-Plate Valve.

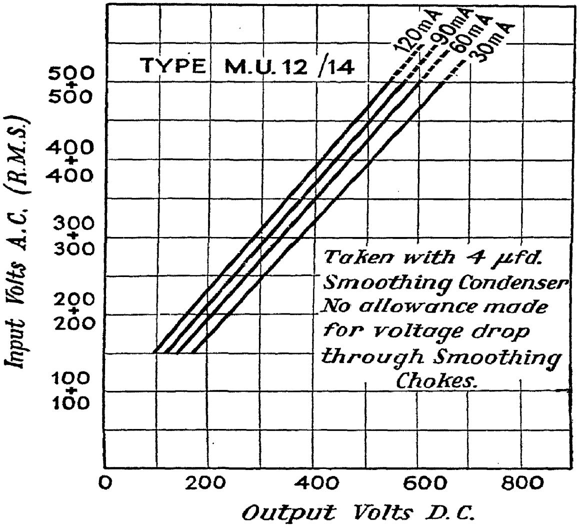
An alternative form of rectifier is the gas-discharge valve. This depends on the glow discharge in an inert gas, such as nitrogen, at low pressure. By suitable shaping of the electrodes, one being pointed and the other flat, the " striking voltage " (viz. that at which the discharge starts) is made higher than the maximum voltage for one direction of discharge, and lower than this for the opposite direction. Since no discharge occurs until the striking voltage is reached, the resulting unidirectional current fluctuates greatly. Even with full-wave rectification there is a considerable portion of each half-period when no current flows.

Another alternative is the electrolytic rectifier, usually consisting of one plate of aluminium and one of another metal in a solution of a salt of aluminium. The action depends on the rapid formation of a highly resistant thin layer on the aluminium when current passes in one direction, and the rapid decomposition of this layer when the current is reversed.

The drawbacks of this type are that the " reverse " current (*i.e.* in the unwanted direction in the load) is considerable, that their life is rather short, that the output wave is distorted greatly, and that the presence of liquid is undesirable, especially as regards maintenance.

The last drawback is done away with by a number of " dry-plate " rectifiers, one type of which utilises magnesium and copper sulphide, but most of these possess the remainder of the above disadvantages.

One of the best for small currents is the copper oxide type. This depends on the fact that the resistance for current passing from copper to an oxide coating on it is about 1 000 times as great as the resistance to current passing from the oxide coating to the copper. Units of such a rectifier are built up of copper washers with one side oxidised and in contact with a lead washer. The object of the latter is to distribute the current evenly over the oxidised surface. A number of such units are connected in parallel, series, or series-parallel, according to the voltage and current to be rectified.



Taken with 4 μfd. Smoothing Condenser No allowance made for voltage drop through Smoothing Chokes.

Fig. 10.24.—PERFORMANCE OF COPPER OXIDE RECTIFIER.

The advantage of all these over the thermionic rectifier is that the filament current is eliminated, simplifying the circuits and increasing the efficiency.

It is important for a rectifier to have a fairly low resistance in the forward direction. This increases the efficiency, and also

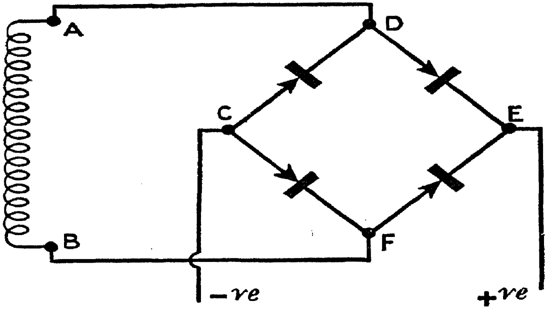


Fig. 10.25.—BRIDGE-CIRCUIT IN FULL-WAVE RECTIFICATION.

improves the regulation. The latter point is of greater importance than the former for small currents at high voltages, such as radio supplies. An example is a copper oxide rectifier, using the voltage-doubler connexion (see Art. 17) with a "smoothing circuit" containing a 40-henry choke, and gives 280 V., D.C. with 4 mA. This drops to 140 V. at 50 mA., about 20 V. of the fall being due to the choke. A full-wave valve rectifier, without choke but with smoothing condenser, gives 500 V., D.C., at 30 mA., dropping to 410 V. at 120 mA. (see Fig. 10.24).

Arc-type rectifiers are dealt with in Chapter XI.

17. Rectifier Connexions

The simplest method for full-wave rectification is the "centre-tap" method. This is shown for a full-wave valve rectifier in Fig. 10.23, but may be used with two rectifier units of any type. Its main drawback is that the secondary voltage of the transformer is double that applied to the rectifier.

The "bridge-circuit" is shown in Fig. 10.25. AB is the secondary of the transformer, and four rectifier units are connected in Wheatstone Bridge formation. When A is positive current flows through DE to the external load, and back through CF to B. When the e.m.f. of AB reverses current flows through FE to the external circuit, and back through CD to A. The current in the load therefore is always in the same

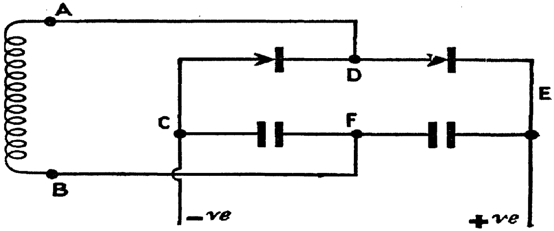


Fig. 10.26.—VOLTAGE-DOUBLER CIRCUIT

direction. The drawbacks of this circuit are the need for four rectifier units, and the fact that drop occurs in two units in series.

Another alternative is the " voltage-doubler " circuit, which is shown in Fig. 10.26. This is similar to the bridge circuit, with two of the rectifiers replaced by condensers. When A is positive current flows through DE, part goes through the load and the rest increases the charge of condenser EF. The current returning from the load reduces the charge of condenser FC. The currents flowing to F from the two condensers return to B.

When A is negative current flows to it through CD. This current is made up of the current returning from the load and the current flowing out from condenser FC at C, due to its being charged up from B. The load current is supplied by the partial discharge of condenser EF. The resulting current flowing into it at F brings the current from B up to equality with that flowing into the secondary at A.

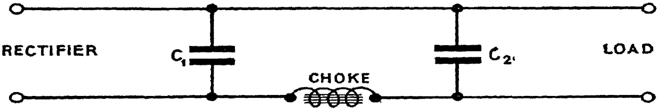


Fig. 10.27.—SMOOTHING CIRCUIT.

The name of this circuit arises from the fact that the voltage applied to the load (neglecting drop in rectifier with forward current) is greater than the voltage of AB, and approaches double that given it by straightforward half-wave rectification.

In this circuit the condensers are essential, but in all rectifier circuits for radio work " smoothing circuits " are required to reduce variation in the output current. A simple but satisfactory circuit for this purpose consists of two condensers and a choke, connected as shown in Fig. 10.27. For radio supplies a choke of 40 henries and 400 ohms is suitable, though $\frac{3}{4}$ of this value may be used. The condensers, C_1 and C_2 should each be of at least 4 μF., and C_2 may be double this capacitance. In the voltage-doubler circuit the two condensers shown in Fig. 10.26 take the place of C_1 in Fig. 10.27.

CHAPTER XI

MERCURY ARC RECTIFIERS

1. Mercury Vapour Lamps and Rectifiers

A low-pressure D.C. mercury vapour lamp cannot be used on an A.C. supply because the arc goes out each time the current reaches zero, and so must be re-struck. In this respect the mercury arc differs from the carbon arc, which persists long enough for the arc to re-establish itself after the voltage has reversed.

To overcome this difficulty two anodes (A_1, A_2) are used, connected to an auto-transformer with a centre point tapping, as shown in Fig. 11.01. By this means A_1 and A_2 are each in turn positive to the mercury cathode, K. By the use of suitable reactances (R, R_1, R_2), the current due to one half-wave is kept flowing until the voltage of the next half-wave has risen sufficiently to maintain the arc. The middle electrode E is used for starting the arc by tilting the tube (cf. Volume I., Chapter XV., Art. 14).

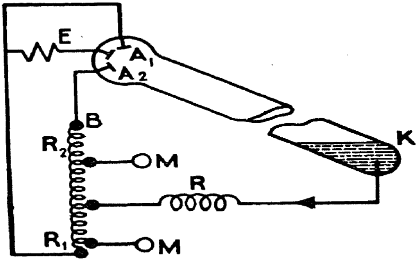


Fig. 11.01.—Mercury Vapour Lamp for A.C.
M, M = A.C. Supply.

This drawback of the A.C. mercury vapour lamp has been put to good use by employing it as a rectifier. The current in R (Fig. 11.01) is always in the same direction. Hence, by placing in this part of the circuit a consuming device which requires unidirectional current the required rectification is effected. The current will not be steady but will fluctuate. The extent of these fluctuations is diminished by increasing the reactance of R. The anodes must be placed further apart the greater the current to be rectified. The arrangement for moderate sizes is shown in Fig. 11.02. The cooling chamber condenses the mercury vaporised by the current. The glass-bulb pattern is made in sizes up to 500 A. at 500 V., and this size can carry 25 per cent. overload for 2 hours. It is made also for 1 200 V. up to 300 A.; and for 3 000 V. up to 200 A.

The larger sizes of mercury arc rectifiers are made with steel containers. The chief difficulties are then the maintenance of the vacuum and the starting of the arc, since tilting is inconvenient.

One method is to use an auxiliary electrode which is dipped into the mercury and then withdrawn. The movement is effected by coils acting magnetically on a rod, which enters the casing through a gland. Many varieties of packing have been devised for making air-tight the glands by which the conductors and the auxiliary electrode enter. But in the larger sizes the usual practice is to employ a vacuum pump to remove any air which leaks in.

In glass-bulb rectifiers a method used in place of tilting is to mount this electrode on a spring. When the main switch is closed a magnet outside the bulb draws the electrode into the mercury: the circuits are arranged so that when current flows through the dipping electrode the pull of the magnet is weakened and the electrode springs out of the mercury. This strikes an arc which transfers itself to the auxiliary (or excitation) electrodes. These two small auxiliary electrodes are contained in the lower part of the tube. This maintains the arc when the current is reduced to a small value. If current is cut off altogether the auxiliary electrodes are kept going by a loading resistance taking a small current.

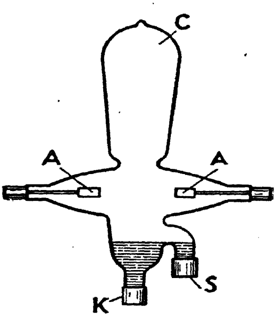


Fig. 11.02.—Mercury Rectifier.

A, A = Anodes. K = Cathode.
S = Starting electrode.
C = Condensing chamber.

For heavy current work mercury rectifiers usually have two or more sets of anodes connected in parallel. For example, a large rectifier which can deal with currents up to 5 000 amperes, and powers up to 2 500 kilowatts, has eighteen anodes (three sets of 6 in parallel, supplied 6-phase). The anodes are water-jacketed, the outer casing of the water-jackets being provided with flanges to assist in removing the heat produced. The cylindrical arc-chamber likewise is water-jacketed, and is mounted on insulators fixed to a foundation ring. The flange for connecting to the vacuum pump is at the top. When a pair, or a number of pairs, of such rectifiers are used the transformer secondaries may be 12-phase, *i.e.* two sets of 6-phase windings, each supplying half of the rectifiers. In some cases 12-anode electrodes have a 12-phase supply (see Art. 11).

The efficiency of the mercury rectifier is high, the chief loss being the drop of about 15 to 30 volts in the arc. Strictly, only part of this is true arc drop, from 10 V. to 15 V. being dropped at the anode and cathode combined. Thus the higher the voltage the higher the efficiency. Moreover the efficiency remains nearly constant down to 25 per cent. of full load, the reduction being due mainly to the greater percentage loss in the auxiliary circuits.

On three-phase circuits with a single rectifier the connexions are modified as shown in Fig. 11.03. There are three anodes (A_1, A_2, A_3), in three horizontal projections of the containing vessel at 120° apart, and a single cathode K. An auxiliary starting electrode for the tilting method is shown at I. Since current is always flowing from at least one anode to the cathode the steadying reactance can be less than for a single-phase rectifier.

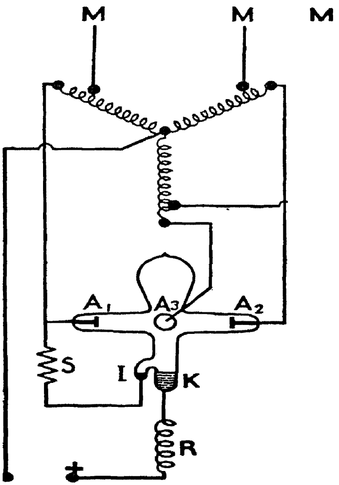


Fig. 11.03.—THREE-PHASE MERCURY RECTIFIER.

$A_1A_2A_3$ = Main anodes.
I = Starting electrode.
K = Mercury cathode.
M, M, M = Three-phase mains.
R = Steadying reactance.
S = Starting resistance.

2. Connexions of Ignition and Excitation Circuits

One method of connecting these circuits is shown in Fig. 11.04. A and B are the secondary terminals of the excitation transformer, with a centre tapping at C. The D.C. relay is closed by gravity, and being polarized is not affected by A.C.; but on the passage of sufficient D.C. it opens the two sets of contacts shown.

On closing the main switch the half-winding CB sends a current through the resistance R, and the magnet M. The latter pulls the ignition electrode IE down into the mercury pool. This closes an alternative circuit from D to F and reduces the current in M, thus permitting the ignition electrode to spring out of the mercury and strike an arc. These actions alternate until the arc is transferred to one of the two excitation electrodes, EA and EA′.

When current flows from either excitation electrode to the mercury pool it passes through the D.C. relay to C. The relay then opens the circuits of the ignition electrode and of the magnet. These are kept open as long as the excitation anodes carry current; but if this current stops the circuits are re-closed, and the above actions are repeated.

The excitation choke with divided windings maintains the current when the voltage applied to one excitation anode (*e.g.* the voltage of CA applied to EA) approaches zero until the voltage (that of CA′) applied to the other excitation anode (EA′) has risen sufficiently. The centre-tapped resistance G assists in this action.

3. Advantages of Large Number of Anodes

If the smoothing effect of the inductance of the transformer windings, and of other windings in the circuits, are neglected, the voltage applied to the D.C. load at any instant is equal to the alternating applied voltage less the drop in the arc and in the internal resistances. Unless the arrangements of the circuits are such as to cause " overlap " (see Arts. 4 and 6) the arc always passes to the mercury pool from the one electrode which is at the instant at the highest positive potential to the pool.

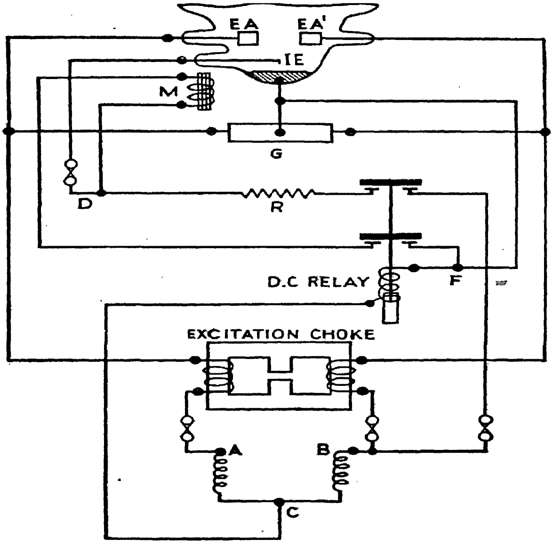


Fig. 11.04.—CONNEXIONS OF IGNITION AND EXCITATION CIRCUITS.

Consequently in a straightforward 3-phase rectifier each anode carries current for ⅓rd of a period, and the D.C voltage varies as shown in Fig. 11.05. Neglecting arc and resistance drops, this voltage varies between the maximum alternating voltage and half this value.

If six anodes are used with a 6-phase supply each anode is in operation for only $\frac{1}{6}$th of a period, and the D.C. voltage is as shown in Fig. 11.06 (*a*). And with 12 anodes the time of operation of each is halved again, and the variation of D.C. volts reduced further [Fig. 11.06 (*b*)]. Thus (neglecting internal drop as before) the D.C. voltage varies from 100 per cent. to 86·6 per cent. of the alternating voltage for 6-phase, and down to 96·6 per cent. for 12-phase.

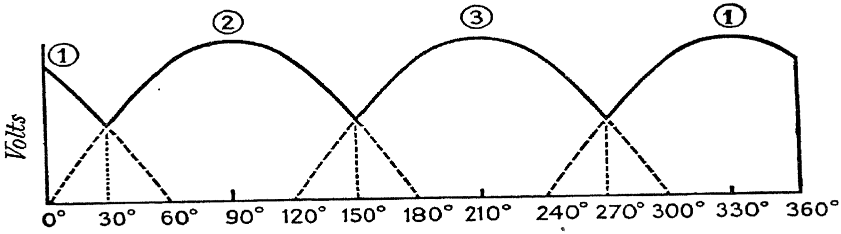
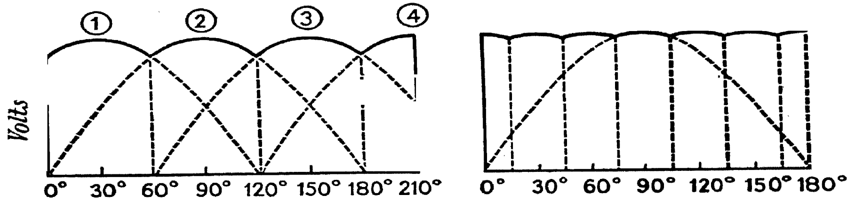


Fig. 11.05.—D.C. VOLTAGE VARIATIONS WITH THREE-PHASE RECTIFIER.



(*a*) WITH SIX ANODES AND A 6-PHASE SUPPLY.

(*b*) WITH TWELVE ANODES AND 12-PHASE SUPPLY.

Fig. 11.06.—D.C. VOLTAGE VARIATIONS.

Thus without smoothing circuits (see Chapter X., Art. 17) the larger the number of phases the less the variation of D.C. voltage. And when smoothing circuits are used in order to produce a given minimum variation, they can be of less size, and so cheaper, the greater the number of phases.

If V_m is the maximum applied alternating voltage the mean D.C. voltage (neglecting internal drop) of a 3-anode rectifier is equal to

$$\int_{\pi}^{\frac{5\pi}{6}} V_m \sin \theta . d\theta \div (2\pi/3),$$

i.e. $V_m \cos (\pi/6)/(\pi/3)$, *i.e.* $(3\sqrt{3}/2\pi)\, V_m$, *i.e.* $0{\cdot}827\, V_m$. And the maximum D.C. voltage is nearly 21 per cent. greater than the mean, and the minimum nearly 40 per cent. less.

In a plain 6-phase rectifier, the mean D.C. voltage

$$= \int_{\pi} V_m \sin \theta d\theta \div (\pi/3)$$

$= 2V_m \cos (\pi/3)/(\pi/3) = (3/\pi)\, V_m$, *i.e.* $0{\cdot}955 V_m$; so that the maximum is about 5 per cent. more than the mean, and the minimum about 9 per cent. less.

In a plain 12-phase rectifier, the mean D.C. voltage

$$= \int V_m \sin \theta d\theta \div (\pi/6) = 2V_m \cos (5\pi/12)/(\pi/6),$$

i.e. $0{\cdot}989 V_m$; the maximum is a little over 1 per cent. higher, and the minimum less than $2\frac{1}{2}$ per cent. lower.

The advantage of 6-phase over 3-phase is therefore very great, and the advantage of 12-phase over 6-phase is considerable but much smaller (cf. the case of rotary convertors, Chapter IX., Arts. 6 and 7).

4. Voltage Ratios with Overlap

With some supply arrangements two or more anodes are in operation simultaneously. This is effected by reducing the voltage applied to one and increasing the voltage applied to the other so as to make them equal. Since this is done by the equivalent of a transformer with 1/1 ratio the resulting voltage is the mean of the voltages otherwise applied to the anodes.

The "angle of overlap" is the name given to the phase angle by which the operation of a given anode is prolonged beyond the instant at which its normal operation ceases. This is the instant at which its normal voltage falls to the normal voltage of the next anode in order of operation.

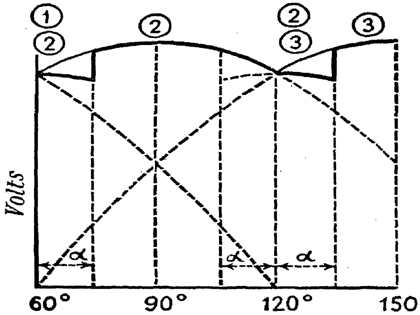


Fig. 11.07.—EFFECT OF 'OVERLAP' ON VOLTAGE.

One effect of overlap is to reduce the mean D.C. voltage, but not necessarily to increase the variation of this correspondingly. To illustrate this the case of the 6-anode rectifier will be considered in detail, the same method being applicable to other cases.

If the operation of anode No. 1 (see Fig. 11·06) is prolonged by the angle α beyond 60°, the voltage during this interval is midway between the normal voltages of Nos. 1 and 2, as shown by the thick line curve in Fig. 11·07. The same applies to Nos. 2 and 3 during the interval from 120° to (120° + α). The D.C. voltage will thus be as shown by the thick line.

If, however, the circuit arrangements are such that anode No. 3 is brought into operation at (120° — α) the D.C. voltage from then to 120° is as shown by the thick broken line. Since this is the usual arrangement, this case will be taken for calculation. The D.C. voltage from 60° to 120° is then symmetrical, so it is sufficient to consider the interval 60° to 90°, giving the following results.

Mean D.C. voltage

$$= \left[\int_{\frac{\pi}{3}}^{\frac{\pi}{3}+\alpha} (V_m/2)\{\sin\theta + \sin(\theta+\pi/3)\}d\theta + \int_{\frac{\pi}{3}+\alpha}^{\frac{\pi}{2}} V_m \sin\theta \, . \, d\theta\right] \div (\pi/6),$$

which reduces to $(3/\pi)\cos\alpha\, V_m$. Thus the mean D.C. voltage is reduced in the ratio $\cos\alpha$ to 1 compared with the value without overlap. If the value of α is less than $\pi/6$ the maximum D.C. voltage remains equal to V_m, and the minimum is reduced to $V_m\left\{\sin\left(\frac{\pi}{3}+\alpha\right)+\sin\left(\frac{2\pi}{3}+\alpha\right)\right\}\Big/2$, *i.e.* $0{\cdot}866 \cos\alpha\, V_m$. The percentage by which the minimum is below the mean is therefore unaltered, but the percentage by which the maximum is above the mean is increased; the total variation is increased but in a much smaller ratio. For example, if α is 15° the mean value is reduced to 0·966 of its former amount, *i.e.* to $0{\cdot}923\, V_m$; and the minimum to $0{\cdot}837\, V_m$, which is again about 9 per cent. less. But the maximum remains at V_m, which is about 8 per cent. above the new mean (compared with 5 per cent., see Art. 3). The total variation is increased to nearly 18 per cent. as against its former value of 14 per cent.

If, however, α is equal to $\pi/6$ giving complete overlap, viz. two anodes always operating simultaneously, the following relations ensue. Mean D.C. voltage $= (3\sqrt{3}/2\pi).V_m = 0{\cdot}827\ V_m$, *i.e.* the same as for 3-phase without overlap, as must evidently be the case. But the D.C. voltage at 90° is now $0{\cdot}75\ V_m$ and is the minimum value, and the maximum is at 60° and is $0{\cdot}866\ V_m$; so that maximum is about 5 per cent. above mean, and minimum about 9 per cent. below. The two variations are thus very nearly the same percentages as without overlap (see Art. 3); and if the present variations are expressed as percentages of the mean voltage without overlap they become about 4 per cent. and 8 per cent. respectively.

If overlap occurs in only one of these ways (thick line of Fig. 11.07) the reduction of voltage is halved, and so the factor $\cos \alpha$ becomes $(1 + \cos \alpha)/2$, which is equal to $\cos^2(\alpha/2)$. This effect is produced by reactance in the transformer secondaries or in associated circuits, but it is usually small.

5. Transformer Ratings

The D.C. load is connected between the mercury pool (which is positive) and the star-point of the transformer secondary windings. These carry not A.C. but a unidirectional current lasting for a fraction of a period. In the case of 3-phase this fraction is ⅓rd, and so the R.M.S. secondary current is $\sqrt{I_D^2/3}$, *i.e.* $0{\cdot}577\ I_D$, where I_D is the D.C. amperes.

As shown in Art. 3, the D.C. voltage $(V_D) = 0{\cdot}827\ V_m$. Hence R.M.S. phase voltage $= V_m/\sqrt{2} = V_D/(0{\cdot}827 \times \sqrt{2}) = 0{\cdot}855\ V_D$. So the total rating of the transformer secondaries is

$$3 \times 0{\cdot}855\ V_D \times 0{\cdot}577\ I_D,\ \textit{i.e.}\ 1{\cdot}48\ V_D I_D.$$

The primary must be delta-connected so that each primary can carry current alone during the ⅓rd period that its secondary is carrying current. Neglecting the no load currents and losses the primary rating will be $1{\cdot}48\ V_D I_D$ too. This increased rating may be expressed alternatively by saying that the transformer utilization factor, viz. output/rating, is $1/1{\cdot}48$, *i.e.* $0{\cdot}676$.

With 6-phase and plain star-connected secondaries the R.M.S. current is $\sqrt{I_D^2/6}$, *i.e.* $0{\cdot}408\ I_D$; and the R.M.S. phase voltage $V_D/(0{\cdot}955 \times \sqrt{2})$, *i.e.* $0{\cdot}741\ V_D$. Total secondary rating $= 6 \times 0{\cdot}741\ V_D \times 0{\cdot}408\ I_D = 1{\cdot}81\ V_D I_D$. But in this case the primary rating is different, since each primary supplies two secondaries

180° apart in phase. So each primary carries current for $\frac{1}{3}$rd of a period and its R.M.S. current is $0{\cdot}577\ I_D$ divided by the turns ratio (cf. the 3-phase case). Hence the total primary kVA. is $3 \times 0{\cdot}741\ V_D \times 0{\cdot}577\ I_D$, *i.e.* $1{\cdot}28\ V_DI_D$. So the transformer utilization factor is $2/(1{\cdot}81 + 1{\cdot}28)$, *i.e.* $0{\cdot}645$, which is slightly less than for 3-phase. The primaries must be delta-connected for the same reason as for 3-phase.

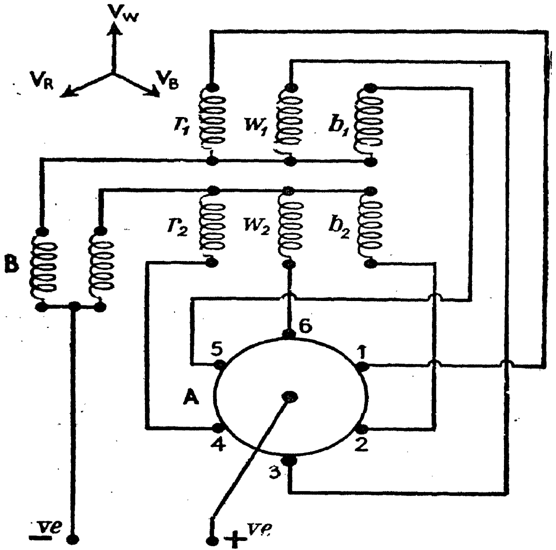


Fig. 11.08.—ACTION OF INTERPHASE TRANSFORMER.

6. Interphase Transformer

Owing to the poor utilization factor of the transformer with the 6-phase-star connexion, it is preferable with 6-anode rectifiers to use a method which gives complete overlap. This consists in using a double-star connexion for the transformer secondaries, with the two star-points connected through an "interphase transformer" (or "reactor"), as shown in Fig. 11.08. The latter is a transformer with 1/1 ratio in which each winding acts as primary and secondary alternately. Its action is as follows.

When the voltage applied to anode No. 1 by secondary r_1 falls below that applied to anode No. 2 by secondary b_2 the interphase transformer (B in Fig. 11.08) adds to the former and subtracts from the latter voltages which are equal, apart from the regulation of B. This maintains the current in anode No. 1 for 30°, at which point the voltage applied to anode No. 3 by secondary w_1 becomes equal to that on anode No. 1 (see Fig. 11.09), and the arc transfers itself from No. 1 to No. 3. A similar action equalises the voltages on Nos. 2 and 3 for 30° until the voltage of w_1 has become equal to that of b_2.

The interphase transformer action then reverses, the previous primary becoming the secondary, and vice versa. This equalises the voltages on anodes Nos. 2 and 3 for a further 30°, when the voltage applied to anode No. 4 by winding r_2 has become equal to that of winding b_2, and the arc is transferred from No. 2 to No. 4.

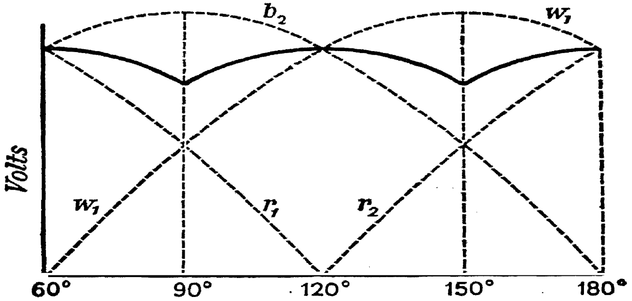


Fig. 11.09.—D.C. VOLTAGE WITH INTERPHASE TRANSFORMER.

These actions proceed to all the anodes in turn, and it will be seen that the result is that there are always two anodes in operation simultaneously. The D.C. voltage is shown by the full line in Fig. 11.09, and its mean value is 0·827 V_m, as proved in Art. 4.

7. Rating of Interphase Transformer

With the connexions of Fig. 11.08 each secondary winding carries half the D.C. amperes for $\frac{1}{3}$rd of a period. This gives a R.M.S. current of $\sqrt{(I_D/2)^2/3}$, *i.e.* 0·289 I_D. Hence the secondary rating is $6 \times V_D/(0{\cdot}827 \times \sqrt{2}) \times 0{\cdot}289\ I_D$, *i.e.* 1·48 V_DI_D. This is much lower than for the 6-phase star, and is equal to the secondary rating for 3-phase star.

Moreover, since each primary supplies two secondary windings it carries $I_D/2m$ for *two*-thirds of each period, where m is the turns ratio. Hence its R.M.S. current is $\sqrt{(I_D/2m)^2 \cdot (2/3)}$, *i.e.* $0{\cdot}408\ I_D/m$, and its rating is $3 \times mV_D/(0{\cdot}827 \times \sqrt{2}) \times 0{\cdot}408\ I_D/m$, *i.e.* $1{\cdot}047\ V_D I_D$. And so the transformer utilization factor is $2/(1{\cdot}48 + 1{\cdot}047)$, *i.e.* $0{\cdot}791$, which is much better than for three-phase star.

The primaries may be Y-connected since two are carrying opposite currents at any instant, but delta connexion is preferable since this forms a closed circuit in which third harmonic currents can circulate. These third harmonic currents are due to the variation in permeability of the transformer core (cf. Chapter IX., Art. 10). With Y-connexion this causes a triple frequency distortion of the A.C. voltage, and corresponding ripples in the D.C. voltage, unless a tertiary winding is added. The latter is a closed delta-connected winding to permit the circulation of triple frequency currents, and this reduces the voltage distortion to a very small amount.

The voltage on each half of the interphase transformer is (see Fig. 11.09) $V_m \left\{\sin\theta - \sin\left(\theta + \frac{\pi}{3}\right)\right\}/2$, from $\frac{\pi}{3}$ to $\frac{\pi}{2}$. This expression is equal to $V_m \cos\left(\theta + \frac{\pi}{6}\right)\sin\frac{\pi}{6}$, *i.e.* $\frac{1}{2}V_m \cos\left(\theta + \frac{\pi}{6}\right)$, the R.M.S. value of which over the range $\pi/3$ to $\pi/2$ is $0{\cdot}147\ V_m$, which equals $0{\cdot}178\ V_D$. (Its maximum value is $0{\cdot}25\ V_m$.) The current is $\frac{1}{2} I_D$ at all times, but in opposite directions at different times, hence this is its R.M.S. value. So the rating of the interphase transformer is $0{\cdot}178\ V_D \times 0{\cdot}5\ I_D$, *i.e.* $\cdot 089\ V_D I_D$.

If this is added to the average rating of the main transformer, the combined transformer utilization factor becomes $0{\cdot}739$. But this calculation is too favourable to this method, since the small interphase transformer will cost more per kVA. than the main transformer.

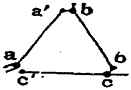
8. Regulation with Interphase Transformer

This transformer requires a magnetising and an iron loss current, like any other transformer. When there is no load on the rectifier these are not being supplied. Hence the connexion becomes practically the same as a plain six-phase star, and so (see Art. 3) $V_D = 0{\cdot}955\ V_m = 1{\cdot}35\ V$ (where V = R.M.S. secondary voltage). When the load becomes sufficient to excite the interphase transformer the operation becomes "complete overlap" or "double three-phase," and V_D drops to (see Art. 4) $0{\cdot}827\ V_m = 1{\cdot}17\ V$. This change-over occurs at a very small load, of about 1 per cent. of full load.

Thus the no-load voltage is more than 15 per cent. above the voltage at light load, and still higher above the full load voltage. This large difference may be troublesome when paralleling the rectifier with others, or with supply apparatus of a different type. It can be avoided by either of the following methods.

A reference to Figs. 11.08 and 11.09 will show that the interphase transformer currents and voltages are of triple the supply frequency. Hence if a supply of triple frequency is given to it the overlap operation will be produced even at no load. This can be done by means of 3-phase transformers connected as shown in Fig. 11.10. The cores are worked at a high maximum flux-density so that the saturation effect gives flat-topped flux waves.

This results in peaked e.m.f. waves in the three secondaries. The fundamentals of these give zero resultant, but the triple harmonics add up and give a resultant voltage of three times the main frequency, as desired. This method is used as well for supplying the lighting circuits on trains driven by A.C. motors, where the low frequency suitable for the motors would cause objectionable flicker.



aa′, bb′, cc′ = Secondary windings.
a and c′ = Terminals connected to load.

Fig. 11.10.
CONNEXIONS FOR TRIPLE FREQUENCY.

This arrangement is expensive for its kVA. output, since this is less than that of *one* of the transformers. The high flux-density makes the losses high too, but this is not of quite so much importance since the output is small, approximately that of the interphase transformer in the case of the rectifier (see Art. 7).

An alternative is to use a resistance capable of carrying about 1 per cent. of the full load of the rectifier. If this is combined with a cut-out which will disconnect it when the actual load rises above this value, and re-connect it when the load falls below again, the overlap operation is maintained throughout.

In many cases the no-load rise of voltage has no serious disadvantages, and then the extra cost and complication of these arrangements become unnecessary.

9. Line Cable Ratings

When the primary windings are Y-connected the line currents are the same as the primary currents. But in the usual case of delta-connected primaries the line current is not usually $\sqrt{3}$ times the phase currents, as for sinusoidal waves (cf. Chapter II., Art. 14).

For example, with 3-phase star connexions (Art. 3) each line will carry the maximum current of one phase for $\frac{2}{3}$rds of each period,

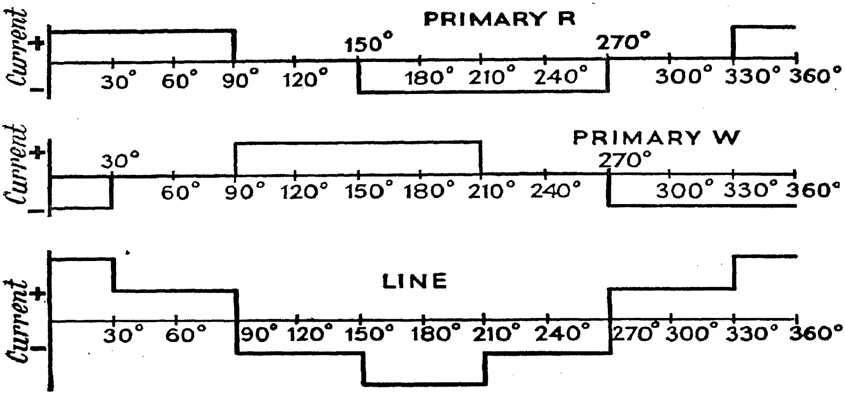


Fig. 11.11.—CURRENTS IN DOUBLE-STAR.

⅓rd period in each direction. Hence (R.M.S. line current)/(R.M.S. phase current) = $\sqrt{(2/3)/(1/3)}$ = 1·414, instead of 1·732. Thus while the primary kVA. is 1·48 times D.C. kW. (Art. 5) the cable carrying capacity corresponds with 1·48 × (1·414/1·732), *i.e.* 1·21 times D.C. kW., plus the losses.

In a 6-phase star similarly, the line current flows during ⅔rds of each period (four ⅙ths of a period with currents in alternate directions) against ⅓rd for the primary current, giving the same ratio of 1·414. But since the primary rating in this case is only 1·28 times D.C. kW., the cable size corresponds with 1·28 × (1·414/1·732), *i.e.* 0·975 times D.C. kW. This result is due to the peak factor of the line current having a value less than $\sqrt{2}$, the peak factor of a sinusoidal wave.

In the double-star 6-phase (Fig. 11.08) each primary carries current for ⅔rds of each period; the wave-form of two of the phases is shown in Fig. 11.11 for the phase voltages shown in Fig. 11.09. Since one line carries the difference between these two currents its wave form is as shown also in Fig. 11.11, with a maximum value double the maximum of the phase currents. The ratio (R.M.S. line current) to (R.M.S. phase current) is therefore $\sqrt{(1+4+1)/3}$ to $\sqrt{2/3}$; which is equal to $\sqrt{3}/1$. So in this case the usual ratio holds, and the line cable size (neglecting losses) corresponds with 1·047 times D.C. kW., the same as the primary rating. In this case the peak factor of the line current is $\sqrt{2}$, the same as for a sinusoidal wave.

10. Six-Phase Branched-Star Connexion

The arrangement of the secondary windings for this alternative method of supply to a six-anode rectifier is shown in Fig. 11.12. Sometimes it is called " triple-star " connexion. Each phase has three equal secondaries, which when connected as shown give the required six-phase voltages. There is no overlap and so the six end windings carry a current of R.M.S. value $\sqrt{\text{I}^2/6}$, *i.e.* 0·408 I_D as in the plain 6-phase star.

But the three inner windings carry current for ⅓rd of each period, having the R.M.S. value of $\sqrt{\text{I}_\text{D}^2/3}$, *i.e.* 0·577 I_D. The R.M.S. voltage of each secondary is $\text{V}_m/(\sqrt{3} \times \sqrt{2})$, *i.e.* 0·408 V_m, *i.e.* 0·408 V_D/0·955, *i.e.* 0·427 V_D. So the total secondary rating is 0·427 V_D (6 × ·408 I_D + 3 × 0·577 I_D), *i.e.* 1·79 $\text{V}_\text{D}\text{I}_\text{D}$. Each primary carries current for ⅔rds of each period, *e.g.* that of phase W carries current while anodes Nos. 1, 3, 4, and 6 are in operation (Fig. 11.12). So the R.M.S. primary current is ($\sqrt{2/3} =$) 0·817 times its maximum value. Consequently the primary rating is

$$3 \times 0{\cdot}427m\text{V}_\text{D} \times 0{\cdot}817\ (\text{I}_\text{D}/m),\ \textit{i.e.}\ 1{\cdot}047\ \text{V}_\text{D}\text{I}_\text{D}\ \text{(cf. Art. 7).}$$

The transformer utilisation factor is therefore 2/(1·79 + 1·047), *i.e.* 0·705. This is about 10 per cent. better than the plain six-phase star, and about 5 per cent. worse than the double star with interphase transformer. But, as pointed out in Art. 7, the figure for the latter is unduly favourable, and so the difference is unimportant. The interphase transformer connexion is generally preferred for moderate rectifier voltages, and the branched star for high voltages.

The primary windings may be either Y-connected or delta-connected; and in the latter case the current wave-forms and the ratio (line current)/(phase current) are the same as for the interphase transformer double-star connexion.

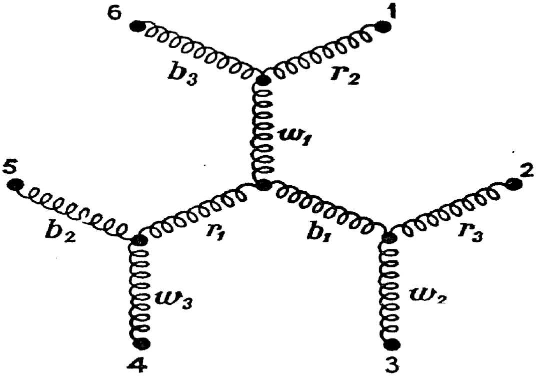


Fig. 11.12.—SIX-PHASE BRANCHED STAR CONNEXION.

11. Twelve-Anode Connexions

With 12 anodes a 6-phase connexion may be used, but each anode should have a separate secondary winding to ensure proper sharing of the load. Twelve anodes are used when the current exceeds the safe carrying capacity of six (see Art. 1). But a number of true 12-phase connexions are used too. Their chief drawback is that if the A.C. wave contains considerable 5th and 7th harmonics the rectifier is liable to revert to 6-phase working. Moreover, although the 6th and 18th harmonics in the D.C. voltage are reduced, the 12th and 24th harmonics are as large with 12-phase working as with 6-phase. And these latter harmonics cause telephonic interference.

The three 12-phase connexions mainly used are (*a*) the quadruple zig-zag, with three interphase transformers, giving simultaneous operation of four anodes (Art. 12); (*b*) the triple four-phase with a three-limb interphase transformer, giving simultaneous operation of three anodes (Art. 13); and (*c*) the double branched star (or double triple-star), with no interphase transformer, but with two sets of primary windings connected in series Y-delta (Art. 14). This too gives simultaneous operation of three anodes.

12. Quadruple Zig-Zag Connexion

Each phase has eight secondary windings, four with a large number of turns and four with a small number, connected as shown in Fig. 11.13. Neglecting drop, the higher pressure secondary windings should give maximum voltages each equal to 0·816 V_m, and the lower pressure ones maximum voltages equal to 0·299 V_m.

With these values the voltage applied to anode No. 1 is $V_m \{j\,(0{\cdot}816 + 0{\cdot}299/2) + 0{\cdot}866 \times 0{\cdot}299\}$, *i.e.* $V_m \{0{\cdot}259 + j\,.\,0{\cdot}966\}$, *i.e.* $V_m \{\sin 15^\circ + j\,.\cos 15^\circ\}$. This is a voltage of maximum value

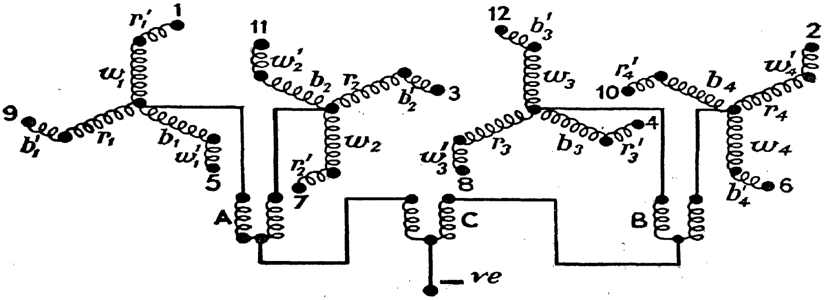


Fig. 11.13.—QUADRUPLE ZIG-ZAG CONNEXION.

equal to V_m and 15° short of its maximum. Similarly the voltage applied to anode No. 12 has its maximum equal to V_m, but is 15° past its maximum. Again, the voltage applied to anode No. 2 (see Fig. 11.13) is $V_m \{j\,(0{\cdot}299 + 0{\cdot}816/2) + 0{\cdot}866 \times 0{\cdot}816\}$, *i.e.* $V_m\{0{\cdot}707 + j.0{\cdot}707\}$, *i.e.* $V_m\,\{\sin 45° + j.\cos 45°\}$. This is again of maximum value equal to V_m, but is 45° short of its maximum. By treating the voltages applied to the other anodes in the same way it can be shown that they are all of maximum values equal to V_m, and differ in phase by successive angles of 30°.

The interphase transformer A (Fig. 11.13) causes anodes Nos. 1 and 3 to operate simultaneously (and later Nos. 3 and 5), and interphase transformer B causes Nos. 2 and 4 to do the same. Similarly C causes the two pairs to operate simultaneously. Since four operate together, each continues for $\frac{1}{3}$rd of each period, hence $V_D = 0{\cdot}827\,V_m$ as for a 3-phase rectifier (Art. 3). Each anode carries a quarter of the D.C. amperes for $\frac{1}{3}$rd period, so the R.M.S. value is $\sqrt{(I_D/4)^2/3}$, *i.e.* $0{\cdot}144\,I_D$. Thus the total secondary rating is $12\,(0{\cdot}966 + 0{\cdot}299)\,\{V_D/(0{\cdot}827 \times \sqrt{2})\}\,0{\cdot}144\,I_D$, *i.e.* $1{\cdot}65\,V_D I_D$.

If the maximum value of the primary winding voltage is $m.V_m$ the primary current's values are the sums at each instant of $0{\cdot}816\,I_D/4m$ and $0{\cdot}299\,I_D/4m$ for each inner and outer secondary operating at that instant. This gives a primary current of $0{\cdot}558 I_D/m$ maximum value (when four secondaries of one phase are operating) and of $0{\cdot}394\,I_D/m$ R.M.S. value. Hence the primary rating is $3m.\{V_D/(0{\cdot}827 \times \sqrt{2})\} \times 0{\cdot}394\,I_D/m$, *i.e.* $1{\cdot}01\,V_D I_D$.

The interphase transformers A and B have each to produce the same voltage as for the 6-phase double-star but carry only $I_D/4$ instead of $I_D/2$; so their combined rating is $0{\cdot}089\,V_D I_D$ (cf. Art. 7). The third interphase transformer carries $I_D/2$, but its voltage is only half the difference between the maximum and minimum voltages of the 6-phase double-star (see Fig. 11.09, and Art. 7). This value is $V_m\,(0{\cdot}866 - 0{\cdot}750)/2$, *i.e.* $0{\cdot}058\,V_m$, *i.e.* $0{\cdot}070\,V_D$. So the rating of C is $0{\cdot}035\,V_D I_D$; and the total for the three interphase transformers is $0{\cdot}124\,V_D I_D$. Adding this to the mean of the ratings of the primary and secondary of the main transformers gives a utilization factor of 0·688.

When the load is too small to excite the interphase transformers the rectifier will operate as 12-phase with no overlap, and so give a D.C. voltage of $0{\cdot}989\,V_m$. This is nearly 20 per cent. above the light load value.

The primaries may be connected in Y or delta, but usually the latter is preferred (cf. Art. 7). It can be shown that in this case the line current is $\sqrt{3}$ times the primary current.

13. Triple Four-Phase Connexion

The secondary windings and three-limb interphase transformer are connected as shown in Fig. 11.14. The outer windings, such as A—3 produce a voltage 0·816 V_m, and the centre windings such as AB a voltage 0·598 V_m, with a centre point tapping at C. This the resultant voltages from C to each of the terminals 3, 6, 9, 12 is equal to V_m (cf. Art. 12), with 90° phase differences.

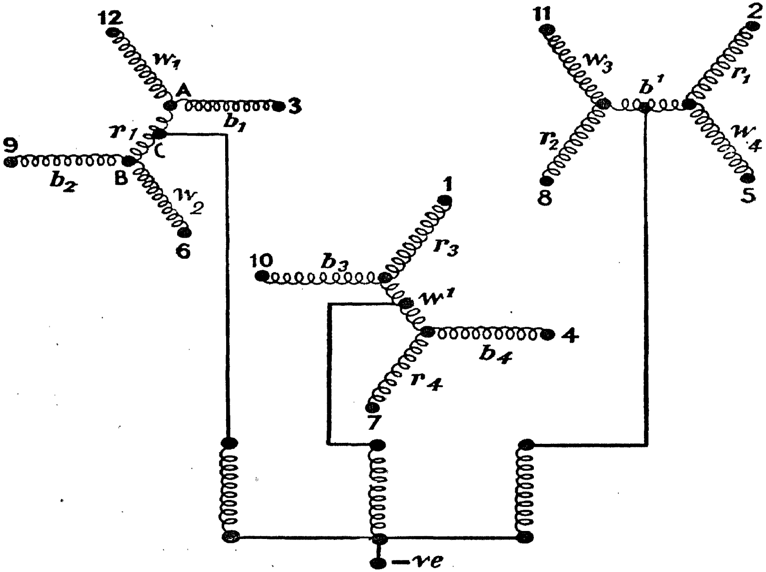


Fig. 11.14.—TRIPLE FOUR-PHASE CONNEXION.

The three-limb interphase transformer equalises the voltages applied to three anodes, one from each group, such as Nos. 1, 2, and 3. So each anode carries $I_D/3$ for a quarter of each period, giving a R.M.S. value of $\sqrt{(I_D/3)^2/4}$, *i.e.* 0·167 I_D. And V_D has the same relation to V_m as for a four-phase rectifier without overlap: thus, $V_D = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} V_m \sin\theta \,.\, d\theta/(\pi/2) = 0{\cdot}900\ V_m$. The central windings, such as ACB (Fig. 11.15), carry in each half $I_D/3$ for half of each period, giving a R.M.S. value of $\sqrt{(I_D/3)^2/2}$, *i.e.* 0·236 I_D.

Thus the total secondary rating is $\{V_D/(0{\cdot}900 \times \sqrt{2})\} \times \{12 \times 0{\cdot}816 \times 0{\cdot}167 + 3 \times 0{\cdot}598 \times 0{\cdot}236\} I_D$, *i.e.* $1{\cdot}61\ V_D I_D$. The primary current (cf. Art. 12) has a maximum value of $0{\cdot}644\ I_D/m$ and a R.M.S. value of $0{\cdot}433\ I_D/m$. So the total primary rating is $3\{mV_D/(0{\cdot}900 \times \sqrt{2})\} \times 0{\cdot}433\ I_D/m$, *i.e.* $1{\cdot}02\ V_D I_D$.

Each winding of the interphase transformer has to produce a voltage varying between $+ 0{\cdot}173\ V_m$ and $- 0{\cdot}089\ V_m$, and of four times normal frequency. Owing to its distorted wave-form the advantage of increased frequency may be neglected and the rating made on its voltage range, giving a rating as a transformer of $3/2 \times 0{\cdot}131\ V_D/({\cdot}900 \times \sqrt{2}) \times 0{\cdot}236\ I_D$, *i.e.* $0{\cdot}036\ V_D I_D$.

The transformer utilization factor is equal to the reciprocal of $(1{\cdot}02 + 1{\cdot}61)/2 + {\cdot}036$, *i.e.* $0{\cdot}740$. The primary windings must be delta-connected, the line current has a greatly distorted wave with a hollow top, and its R.M.S. value is only a little over 1 per cent. greater than that of the primary winding current.

At no load the voltage rises to $0{\cdot}989\ V_m$ (see Art. 3), *i.e.* about 10 per cent., so that in this respect the connexion is better than the quadruple zig-zag (Art. 12).

14. Double Branched-Star Connexions

This employs two transformers with their secondary windings connected in the 6-phase branched-star method (Art. 10). One set of primary windings is delta-connected; the other set is connected as for Y-connexion, but the ends which would normally form the star point are connected to the corners of the delta (see Fig. 11.15). This series star-delta connexion of the primaries makes a 30° phase difference between the primary currents (and voltages) in the two transformers. The corresponding phase difference between the two sets of secondaries makes the two branched stars produce 12-phase voltages. Another name for this connexion is "cascade 12-phase fork".

This method causes three anodes to operate simultaneously (two in one branched-star, and one in the other) without any interphase transformer. The phase voltage of the delta-connected set of primaries is $\sqrt{3}$ times the phase voltage of the Y-connected set. Hence if the turns ratio of the latter is m, that of the former is $\sqrt{3}m$. This, combined with the series connexion of the two sets, makes the load current divide itself between the three operating anodes in the ratios $1 : \sqrt{3} : 1$. The R.M.S. anode current is therefore $0{\cdot}173 I_D$ instead of $0{\cdot}167\ I_D$ (see Art. 13). But the inner secondaries, viz. those connected to the two neutral points, N_1, N_2, carry current during 5/12ths of each period. This amounts to $I_D/(2+\sqrt{3})$, $\sqrt{3}\,I_D/(2+\sqrt{3})$, $2I_D/(2+\sqrt{3})$, $\sqrt{3}\,I_D/(2+\sqrt{3})$ and $I_D/(2+\sqrt{3})$ in successive twelfths of a period, giving 0·268 I_D as R.M.S. value.

The R.M.S. voltage per secondary is 0·445 V_D. So the total secondary rating is 0·445 $V_D\{12 \times 0{\cdot}173\, I_D + 6 \times 0{\cdot}268\, I_D\}$, *i.e.* 1·64 $V_D I_D$. The primary rating is the same as for the triple four-phase connexion, viz. 1·01 $V_D I_D$. The slightly higher rating for the transformer secondaries is more than balanced by the absence of any interphase transformer; the transformer utilization factor being 2/(1·01 + 1·64), *i.e.* 0·755. It should be remembered, however, that the two separate transformers will cost more than a single transformer of their combined rating.

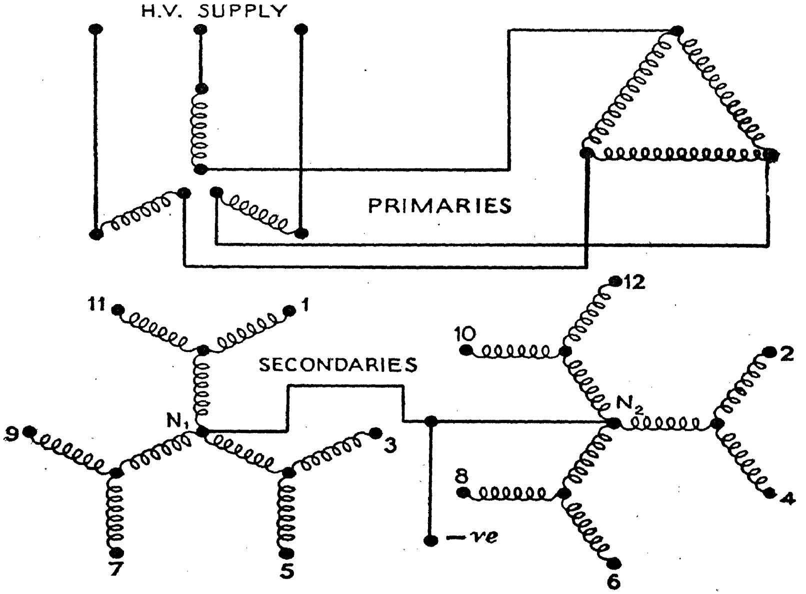


Fig. 11.15.—Double Branched-Star Connexion.

The line current is the same as the current in the star-connected primaries, and the current in the delta-connected primaries is $1/\sqrt{3}$ of these.

15. Grid Control

Grids, similar to those of thermionic valves (see Chapter X., Art. 3), are used sometimes in mercury rectifiers to control the voltage. Their action, however, is somewhat different. As long as the grid is maintained at a sufficient negative potential it prevents the initiation of the arc. But when the arc has started, the subsequent application of a negative potential to the grid cannot stop it.

This difference from the action of the valve grid is due probably to the rectifier grid rapidly becoming covered by a thin film of positive ions. These neutralise the grid field, and so the arc continues.

The voltage required for grid control is not critical. It usually lies between twice and six times the arc drop. The value adopted depends partly on the voltage and current of the rectifier, and partly on the method used for varying the amount of grid control.

The effect of grid control is to reduce the average D.C. voltage, since the arc continues on an anode to which a lower voltage is applied until the grid control is removed and permits it to transfer to the anode to which a higher voltage is applied.

For instance in a 6-anode rectifier without overlap the effect of grid control maintained on each anode for a phase angle α is to make the operation of anode No. 2 (see Fig. 11.06, Art. 3) last from $\pi/3 + \alpha$ to $2\pi/3 + \alpha$. The mean D.C. voltage is therefore

$$\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_m \sin\theta d\theta/(\pi/3), \quad i.e.\ 2 \cos\alpha \sin(\pi/6) V_m/(\pi/3),$$

i.e. $0{\cdot}955\ V_m \cos\alpha$. Thus the effect is to reduce the voltage in the ratio $1 : \cos\alpha$. And the same ratio applies to other cases.

16. Methods of Grid Control

Several methods are in use for varying the extent of grid control, and so furnishing a means of varying the D.C. voltage. One method is to use a phase-shifting transformer. This is similar to an induction regulator (see Chapter IX., Art. 13), except that for a 6-anode rectifier it requires six secondary windings. These windings are connected between the cathode (forming a star point) and the corresponding grids.

By altering the position of the movable secondary the phase of all the secondary voltages is altered by the same angle. This changes the time during which the grids delay the starting of the arc on each successive anode. Sometimes a constant grid bias is used in addition.

The drawback of this method, and of others in which the grid potential is altered gradually, is that it makes the regulation of the rectifier worse. As the load increases the arc drop increases. Hence the positive voltage needed on the grid to permit arc transfer increases, and so the instant of transfer becomes later. This lowers the D.C. voltage, and so increases the reduction of this caused by a given increase of the load. Consequently this type of control is confined to the smaller rectifiers.

The alternative principle is to apply a sudden positive impulse to each grid at the desired instant of arc transfer; during the remainder of each period the grids are kept at a negative potential, sufficient to prevent arc transfer. This is more difficult to apply, but has the advantage of making the instant of transfer definite, and so does not make the regulation worse.

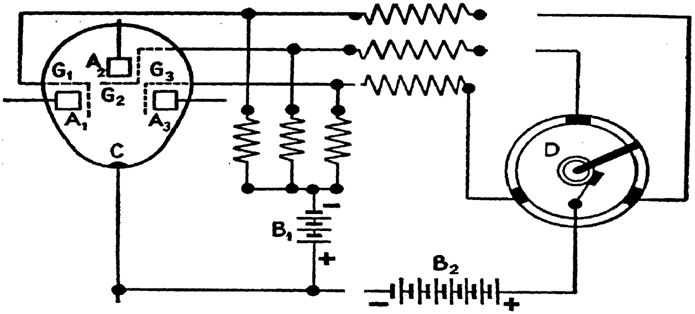


Fig. 11.16.—Grid Control: Connexions for Three-Anode Rectifier.

One method of applying the impulse system is to use a rotating distributor, with a segment for each grid, driven by a small synchronous motor. The connexions for a 3-anode rectifier are shown in Fig. 11.16; but the same method can be used for 6 or for 12 anodes. The grids (G_1, G_2, G_3) are normally kept at a negative potential to the cathode C, by the battery B_1. The distributor, D, consists of a contact arm rotated synchronously over a stationary ring which has a short contact piece for each grid. When the arm touches the contact the corresponding grid is made positive by the battery B_2. The two sets of resistances are to prevent short-circuiting the batteries.

By a suitable choice of the two batteries' voltages, and of the two sets of resistances, the grids can be made positive almost instantaneously, and the current broken by the distributor arm can be kept small. Alteration of the amount of delay in arc-transfer is produced by changing the position of the ring of the distributor over which the rotating arm moves.

In some cases grid control is used only (or in addition) for dealing with a "back-fire." This consists in a reversal of the direction of flow of power in one anode. The frequency of occurrence of this has been reduced greatly by the use of shields, protecting the anodes from the direct blast of mercury vapour and spray from the cathode. Care in the selection of anode material, and improvements in methods of evacuation have assisted in this.

The method employed for using grid control for this purpose is to apply negative potential to all the grids. This prevents the arc restriking on all the other anodes (which otherwise would feed into the backfiring anode), leaving only the D.C. component of the back-fire to be interrupted by a reverse-current cut-out. The application of this negative potential may be controlled by a quick-acting relay, energised either from the A.C. supply or from the D.C. side. Radio valves are sometimes employed for this purpose.

QUESTIONS ON CHAPTER XI

1. Discuss the relative advantages of 3-anode and 6-anode mercury rectifiers, and explain the advantages of using an interphase transformer in the latter case.

2. Draw connexion diagrams for three methods of supplying a 12-anode rectifier from 3-phase mains. For each case determine the voltages of the various secondary windings to give 500 D.C. volts, neglecting arc drop.

3. For one of the methods described in question No. 2 find the ratings of the transformer windings, both primary and secondary, of the reactors, if used, and of the supply cables. Neglect transformer losses in these calculations, and take an output current of 400 A.

4. Explain the operation of grid control in mercury rectifiers, and state the relative advantages of the various methods used in applying this.

CHAPTER XII

ALTERNATING CURRENT TRANSMISSION

1. Line Inductance

In D.C. transmission the p.d. at the receiving end (load) is less than the p.d. at the generating end by the "drop"; and the "drop" is the product of the current transmitted and the total resistance of the line (lead and return) (see Volume I., Chapter XVIII., Art. 2).

In A.C. transmission the voltages have to be combined vectorially which makes the problem more complex. Moreover the line has both inductance and capacitance, not concentrated at a point but distributed along the line. The consequences are that it is possible for the load p.d. to be higher than the generator p.d., and that the p.d. at an intermediate point may be higher than the p.d. at either end.

In order to deal with the problem it is necessary to know the line constants, viz. its resistance, inductance, and capacitance. The leakage through the insulation between the lines affects transmission too, but, as in D.C. transmission, its effect is so small that it can be neglected, or at worst considered separately.

The resistance of the line is the same as for D.C. except that a small increase may have to be made for conductors of large diameter (see Chapter V., Art. 16).

To find the inductance consider a point, P, outside a wire carrying a current of I amperes. The lines of force are concentric circles with the centre of the wire (C) as common centre. Let CP $= x$ cm. Then for non-magnetic surroundings

$$\mathbf{B}_x = \frac{4\pi}{10}\,\mathrm{I}/2\pi x = 2\mathrm{I}/10x.$$

The total flux inside a cylindrical shell, of thickness δx and 1 cm. length, concentric with the wire is $\mathbf{B}_x\,.\,\delta x$. Hence if the radius of the wire is r cm. the total flux surrounding each centimetre of the length of the wire and within a cm. of its centre is:—

$$\int_r^a \mathbf{B}_x\,.\,dx = \int_r^a (2\mathrm{I}/10x)\,dx = (2\mathrm{I}/10)\log_e(a/r).$$

Hence with the return wire parallel to the lead and with their centres s cm. apart the inductance due to the external flux is given by

$$\begin{aligned} \mathbf{L} &= (2/10^9)\log_e(s/r) \text{ henries per cm. of conductor} \\ &= (4{\cdot}61/10^9)\log_{10}(s/r) \text{ henries per cm. of conductor.} \end{aligned}$$

The integration is extended to s, not to $(s - r)$, to allow, to a very close approximation, for the flux, due to the current in one wire, which passes through the other one.

When the size of the wire is considerable in comparison with the distance between the conductors an addition must be made for the flux in the conductor itself. Assuming uniform current density the current inside a circle of radius y is Iy^2/r^2 (when $y \not> r$);

$$\therefore \quad \mathbf{B}_y = \mu\,(4\pi Iy^2/10r^2)/2\pi y = \mu.2Iy/10r^2,$$

where μ = the permeability of the conductor.

But this flux links with only (y^2/r^2) of the total current, and it is therefore equivalent to a flux $\mathbf{B}_y'$ linked with the whole current, where

$$\mathbf{B}_y' = (y^2/r^2)\,\mathbf{B}_y = \mu.2Iy^3/10r^4.$$

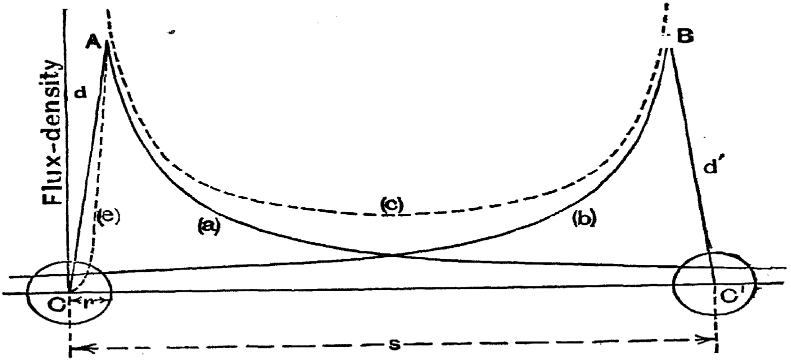


Fig. 12.01.—FLUX DUE TO TWO PARALLEL CONDUCTORS.

(a) Flux-density due to left-hand wire. (b) Flux-density due to right-hand wire.
(c) Resultant external flux density. (d) Internal flux due to left-hand wire.
(d^1) Internal flux due to right-hand wire. (e) Equivalent internal flux due to left-hand wire.

Hence the total equivalent flux is equal to

$$\int_0^r (\mu.2Iy^3/10r^4)\,dy = \mu I/20,$$

and the additional inductance is given by

$$\mathbf{L}' = \mu/(2 \times 10^9) \text{ henries per cm.}$$

For copper, aluminium, or other non-magnetic material $\mu = 1$; and in this case the total inductance

$$= \{4{\cdot}61 \log_{10}(s/r) + 0{\cdot}5\}/10^8 \text{ henries per cm. of conductor.}$$

Fig. 12.01 shows the flux-density due to each of two wires (lead and return), and the resultant flux-density, along the line joining their centres.

2. Line Capacitance

When there is a p.d. between two parallel conductors the lines of stress in the dielectric are similar to the lines of force between two magnetic poles of the same size as the conductors (see Fig. 12.02). By symmetry the plane midway between the conductors and perpendicular to their plane is always at a potential midway between those of the two conductors. It may, therefore, be called the **neutral plane.**

In the same way as for the corresponding case for current and magnetic field strength (see Art. 1), it may be shown that the dielectric field intensity, due to one conductor at a point such as A, x cm. from its centre, is $2q/x$ electrostatic units, where $q =$ the total electrostatic charge per cm. of conductor. If some other material, of permittivity k, fills the space between the wires the intensity is $2q/(kx)$.

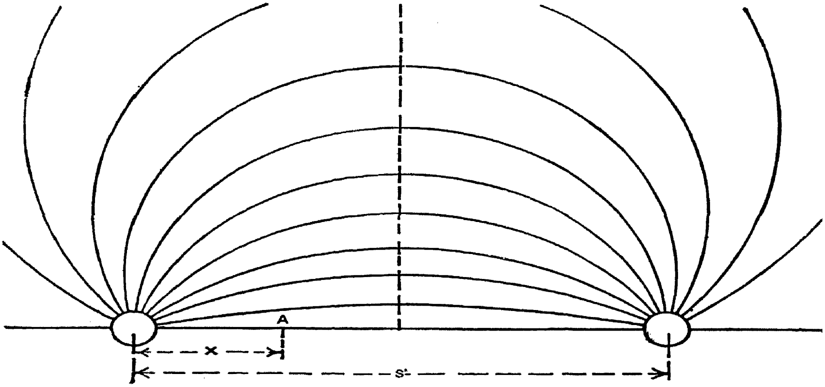


Fig. 12.02.—LINES OF ELECTRIC STRESS FOR TWO PARALLEL WIRES.
— — — Neutral plane.

At a point A on the line joining the centres of the two conductors the total field intensity due to both is $2q\,\{1/x + 1/(s - x)\} = K_x$: where $s =$ the distance between centres in cm.

The potential difference between two points is $\int K_x . dx$, just as m.m.f. $= \int H . dl$ (see Volume I., Chapter IV., Art. 12). Therefore the total potential difference between a conductor and the neutral plane is:—

$$\int_r^{\frac{s}{2}} K_x . dx = \int_r^{\frac{s}{2}} 2q \{1/x + 1\,(s - x)\}\, dx = 2q\,\Big[\log_e x - \log_e (s - x)\Big]_r^{\frac{s}{2}}$$

$$= 2q \log_e \left(\frac{s - r}{r}\right) \text{ E.S.U. per cm.}$$

The capacitance (C) = flux/(p.d.)

$$= 1 \Big/ \left\{2 \log_e \left(\frac{s - r}{r}\right)\right\} \text{ E.S.U. per cm.}$$

$$= 1 \quad \left\{2c^2 \log_e \left(\frac{s - r}{r}\right)\right\} \text{ E.M.U. per cm.}$$

where c = the velocity of light = 3×10^{10} cm. per sec.

Changing into microfarads and common logarithms, and neglecting r in comparison with s,

$$C = 0{\cdot}241 \Big/ \left(10^6 \log_{10} \frac{s}{r}\right) \text{ microfarads to neutral plane per cm.}$$

The capacitance between the lines is *half* this value, since it is that of two such capacitances in series. This is seen alternatively by noting that for a given charge the p.d. between lines is double the p.d. from line to neutral plane.

It may be noted that when r is small compared with s, the product $C\mathbf{L}$ in electromagnetic units is equal to $1/c^2$. Therefore, using the microfarad and the henry as units,

$$(C/10^{15}) . (\mathbf{L} \times 10^9) = \{1/(3 \times 10^{10})\}^2;$$

$$\therefore\ C\mathbf{L} = 1/(9 \times 10^{14}) = 1{\cdot}11/10^{15}.$$

This relation may be used to calculate approximately C or **L** for a line for which one of these two has been found by actual measurement.

For a cable with a core of circular section, radius r, surrounded by a hollow cylindrical conductor of internal radius R, *i.e.* a single or a concentric cable, if Ψ = electric flux per cm. of cable:—

$$\text{P.d.} = \frac{1}{k} \int_r^R (2\Psi/x)\, dx = (2\Psi/k) \log_e (R/r) \text{ E.S.U. per cm.};$$

$$\therefore\ C = \Psi/(\text{p.d.}) = k \Big/ \left(2 \log_e \frac{R}{r}\right) \text{ E.S.U. per cm.}$$

$$= k \Big/ \left(2c^2 \log_e \frac{R}{r}\right) \text{ E.M.U. per cm.}$$

$$= 0{\cdot}241k \Big/ \left(10^6 \log_{10} \frac{R}{r}\right) \mu\text{F. per cm.}$$

The inductance of cables can be worked out in a similar way. It is much lower than that of overhead lines unless the return cable is laid a long distance from the lead. The higher the voltage, *i.e.* the thicker the insulation, and the larger the conductor, the greater the inductance. The increase of the impedance of British standard single unarmoured cables due to this cause at 50 cyc. is as follows:—2 200 volts; under 1 per cent. up to ·06 sq. in. section (0·32″ dia.), rising to 4.6 per cent. for 0·2 sq. in. section (0·58″ dia.): 6 600 volts; 1·3 per cent. for 0·06 sq. in., 7·3 per cent. for 0·2 sq. in.: 11 000 volts; 2.0 per cent. for 0·06 sq. in., 11 per cent. for 0·2 sq. in. For 25 cyc. currents the increases are one half of the above. All these will be greater if the go and return cables are not in contact.

Example 1. *Calculate the reactance and the capacitance of a single-phase line with copper conductors of 3 S.W.G., 90 cm. apart; and the line drop and generator current when transmitting at a generated p.d. of 2 200 volts 50 cycles per sec., and a load of 120 A. at a power-factor of 0·6 for a transmission distance of 2 km.*

3 S.W.G. has a diameter of 0·64 cm.;

$$\begin{aligned}\therefore \text{ Inductance of the two lines} &= 2\,\{4{\cdot}61 \log_{10} (90/0{\cdot}32) + 0{\cdot}5\}/10^9\\ &= \{9{\cdot}22\,(1{\cdot}9542 - \bar{1}{\cdot}5052) + 1\}/10^9\\ &= 23{\cdot}6/10^9 \text{ henries per cm.}\\ &= 2{\cdot}36/10^3 \text{ henries per km.}\end{aligned}$$

The total inductance is, therefore, $4{\cdot}72/10^3$ henries;

$\therefore$ Reactance $= 2\pi \times 50 \times 4{\cdot}72/10^3 = 1{\cdot}483$ ohm.

Resistance of 3 S.W.G. hard-drawn copper at 15° C. = ·536 ohm/km.;

$\therefore$ Resistance at 55° C. (to allow for heating by the current)

$= {\cdot}536 \times 1{\cdot}16 = {\cdot}622$ ohm/km.;

$\therefore$ Total resistance of line $= 4 \times {\cdot}622 = 2{\cdot}488$ ohms;

$\therefore$ Impedance of line $= \sqrt{\{(2{\cdot}488)^2 + (1{\cdot}483)^2\}} = 2{\cdot}90$ ohms.

The voltage required for the line is, therefore, 120 × 2·90, *i.e.* 348 volts. But this is not, as a rule, in phase with the load p.d., and so they must be added vectorially, as shown in Fig. 12·03. In this, OA is the load p.d., and OB the load current. AC is the resistance drop, parallel to OB, and CD is the reactance drop, perpendicular to OB. Thus AD is the impedance drop, and OD is the p.d. at the generator end.

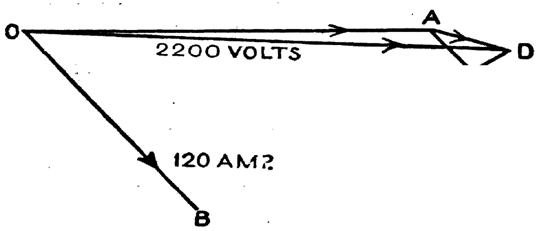


Fig. 12.03.—VECTOR DIAGRAM OF LINE DROP.

The problem is similar to that of the regulation of transformers and of alternators, and so the same method of calculation can be used.

Therefore approximately (see Volume I., Chapter XVII., Art. 13),

$$\text{Line drop} = 120\,\{2 \cdot 488 \cos \phi + 1 \cdot 483 \sin \phi\} \text{ volts}$$
$$= 120\,(1 \cdot 493 + 1 \cdot 186) = 321 \text{ volts.}$$

This is not much less than the impedance drop; but at unity power-factor the line drop would be only $120 \times 2 \cdot 488$, *i.e.* 299 volts approx., *i.e.* the resistance drop alone (or 305 volts by the more accurate formula).

The capacitance has been neglected so far.

$$C = 0 \cdot 241/10^6 \log (90/0 \cdot 32) = 0 \cdot 241/(10^6 \times 2 \cdot 448);$$
$$\therefore\ C = 0 \cdot 098/10^6\ \mu\text{F to neutral plane per cm.}$$

[Note that $CL = (11 \cdot 810^9) \times (0 \cdot 098/10^6) = 1 \cdot 16/10^{15}$, *i.e.* $1 \cdot 11/10^{15}$ approx.]

$$\therefore\ \text{Total capacitance } \textit{between lines}$$
$$= \tfrac{1}{2} \times (0 \cdot 098/10^6) \times 2 \times 10^5\ \mu\text{F} = \cdot 0098\ \mu\text{F};$$

$\therefore$ Capacitance current $= 2\pi \times 50 \times \cdot 0098 \times 2200/10^6$ A. $= \cdot 0068$ A. approx.

This is too small to make any appreciable difference, and this is always the case on overhead lines of moderate length.

Example 2. *If a line, as in Example 1 but 60 km. long, is used to transmit half the same load current at a generated p.d. of 33 000 volts, 50 cycles/sec., calculate the line drop and the generator current.*

$$\text{Total inductance} = 2 \cdot 36/10^3 \times 60 = 0 \cdot 142 \text{ henry};$$
$$\therefore\ \text{Reactance} = 44 \cdot 6 \text{ ohms.}$$
$$\text{Resistance} = 2 \times 60 \times \cdot 622 = 74 \cdot 06 \text{ ohms};$$
$$\therefore\ \text{Impedance} = 86 \cdot 9 \text{ ohms.}$$

Neglecting capacitance, maximum line drop $= 60 \times 86 \cdot 9 = 5\,210$ volts;
Line drop for a power-factor of $0 \cdot 6 = 60\,\{74 \cdot 6 \times 0 \cdot 6 + 44 \cdot 6 \times 0 \cdot 8\} = 4\,830$ V.

$$\text{Total capacitance} = \tfrac{1}{2} \times (0 \cdot 098/10^6) \times 60 \times 10^5\ \mu\text{F} = 0 \cdot 294\ \mu\text{F}.$$

Assuming this to be concentrated at the centre of the line, where the p.d. is $33\,000 - \tfrac{1}{2} \times 4\,830 = 30\,600$ volts,

$$\text{Capacitance current} = 2\pi \times 50 \times 0 \cdot 294 \times 30\,600/10^6 \text{ amperes} = 2 \cdot 83 \text{ A.}$$
$$\text{The power component of the load current} = 60 \times 0 \cdot 6 = 36 \text{ A.}$$
$$\text{The reactive component of the load current} = 60 \times 0 \cdot 8 = 48 \text{ A.};$$
$$\therefore\ \text{Generator current} = \sqrt{\{36^2 + (48 - 2 \cdot 83)^2\}} = 57 \cdot 8 \text{ amperes,}$$

neglecting the phase difference between the p.d.s at the middle of line and at the load.

Thus the effect of the line capacitance is to reduce the generator current and to increase its power-factor (cf. Volume I., Chapter V., Art. 12, Example 5).

This will modify the line drop to some extent since the current in half the line is altered in magnitude and in phase. Thus in Fig. 12·04, not drawn to scale, OA = load p.d.; OB = p.d. at generator end of line; C is the middle point of AB; and so OC = p.d. at centre of line. Again, OD = load current, OE = resistance drop, EF = reactance drop; therefore OF = total drop, neglecting capacitance, and so is parallel and equal to AB.

The capacitance current, OG, leads OC by 90°. The total current, OH, is the vector sum of OD and OG.

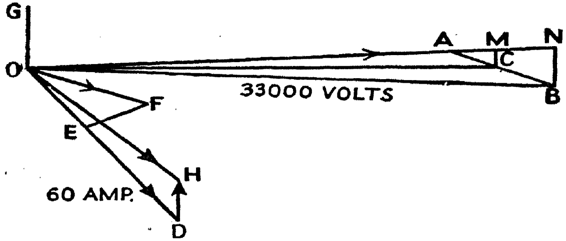


Fig. 12.04.—LINE DROP WITH CAPACITANCE CURRENT.

In the calculation of the generator current the phase difference between OA and OC has been neglected. If a vector diagram, similar to Fig. 12.04, is drawn *to scale* it will be found that the difference is negligible in this example.

Hence the power-factor for the generator half of the line is approximately 36/57·8, *i.e.* 0·62;

$$\therefore \text{ The drop on this part} = 57\cdot8\,\{74\cdot6 \times 0\cdot62 + 44\cdot6 \times \cdot78\} \times \tfrac{1}{2}$$
$$= 28\cdot9 \times 81\cdot1 = 2\,340 \text{ volts;}$$

$$\therefore \text{ Total drop} = 2\,340 + (\tfrac{1}{2} \times 4\,830) = 4\,760 \text{ volts.}$$

Although the two drops are not exactly in phase they may be added arithmetically when using the approximate formula, which neglects components perpendicular to the voltage.

The method of allowing for the phase difference is worked out below, although it is unnecessary in the actual problem.

$$\text{The angle AOD} = \cos^{-1} 0\cdot6 = 53\cdot1^\circ,$$
$$\text{The angle FOE} = \tan^{-1} (44\cdot6/74\cdot6) = 31\cdot0^\circ;$$
$$\therefore \text{ Angle AOF} = 53\cdot1 - 31\cdot0 = 22\cdot1^\circ.$$

Therefore if BN is drawn perpendicular to OA produced,

$$\text{BN} = \text{AB} \sin \text{BAN} = 5\,210 \sin 22\cdot1^\circ = 1\,960 \text{ volts;}$$
$$\therefore \text{ ON} = \sqrt{\{\text{OB}^2 - \text{BN}^2\}} = \sqrt{\{33\,000^2 - 1\,960^2\}} = 32\,940 \text{ volts.}$$
$$\text{And AN} = \text{AB} \cos \text{BAN} = 5\,210 \cos 22\cdot1^\circ = 4\,870 \text{ volts;}$$
$$\therefore \text{ OA} = 32\,940 - 4\,870 = 28\,070 \text{ volts;}$$
$$\therefore \text{ Line drop, neglecting capacitance} = \text{OB} - \text{OA} = 33\,000 - 28\,070$$
$$= 4\,930 \text{ volts,}$$

compared with the value of 4 830 found above.

To allow for capacitance, draw CM perpendicular to AN. Then—

$$\text{CM} = \tfrac{1}{2}\text{BN} = 980 \text{ volts,}$$
$$\text{MN} = \text{AM} = \tfrac{1}{2}\text{AN} = 2\,435 \text{ volts;}$$
$$\therefore \text{ Angle COM} = \tan^{-1} \{980/(28\,070 + 2\,435)\} = 1\cdot9^\circ$$

and $$\text{OC} = \sqrt{\{(32\,940 - 2\,435)^2 + (980)^2\}} = 30\,530 \text{ volts,}$$

whence capacitance current is 2·82 A. (instead of 2·83 A.). The component in phase with OA is 2·82 sin 1·9°, 0·09 A., and the component at 90° to OA is 2·82 cos 1·9°, *i.e.* 2·82 A.

On combining these with the load currents it will be seen that in this case the value of the generator current is not altered appreciably.

3. Corona Effect

When the p.d. between two smooth conductors, such as those of a transmission line, is increased gradually a voltage is reached at which a hissing noise is heard. In the dark a pale violet light can be seen surrounding the conductors. This is known as (visual) corona. Unless the conductors have been polished recently the violet light is accompanied by reddish beads of light at definite points along the conductors. Stroboscopic examination shows* that these beads occur when the conductor is negative and the violet light when it is positive.

Corona is due to a breakdown in the insulation of the air near the conductor. Unless the two conductors are very close the increase in the virtual diameter of the conductor reduces the dielectric stress on the remaining air between them. If the p.d. is increased the diameter of the corona is increased until a point is reached at which the whole of the intervening air breaks down and " spark-over " occurs. If the spacing is small (*i.e.* not more than $5\frac{1}{2}$ times radius of wire) the spark occurs without previous corona.

Corona is always accompanied by the formation of ozone (O_3) through the splitting up and re-combination of ordinary oxygen (O_2). The dissociated oxygen (O_1) and the extra atom from the ozone combine with the metal of the conductor and with other substances in the neighbourhood. They may combine with the the nitrogen of the air, especially when the dielectric stress is high.

The light, heat, and chemical actions which accompany corona imply a loss of electrical energy, and this loss can be measured by a wattmeter (see Art. 4).

The voltage gradient (viz. p.d. per cm.) or dielectric stress, between two parallel wires is greatest at the surfaces, and has there a value (provided the spacing is large) $= V_n/(r \log_e s/r)$, where $V_n =$ p.d. to *neutral* (see Art. 2).

If the spacing of the wires is altered the voltage gradient at which corona starts remains constant for a given size of wire.

But if wires of different sizes are compared, though the corona p.d. for a given spacing is less for smaller wires the voltage gradient is greater, and is given approximately for dry air by:—

$$g_v = 30\,(1 + 0{\cdot}3/\sqrt{r}) \text{ max. kilovolts per cm. to neutral.}$$

This is equivalent to saying that the gradient to start corona has a constant value of 30 (max.) kilovolts per cm. at a distance from the wire equal to $0{\cdot}3\sqrt{r}$ cm.

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* See *Dielectric Phenomena in H.V. Engineering* (F. W. Peek).

The effect of various other factors on the p.d. at which corona starts may be summarised as follows:—

(*a*) Air density. The value of the constant (30) varies directly as the density. The distance from the wire at which this gradient occurs varies inversely as the square root of the density; *i.e.*

$$g_v = 30\delta\,(1 + 0{\cdot}3/\sqrt{r\delta})$$

where δ = air-density expressed as a fraction of its density at 25° C. and 760 mm. barometric pressure. Hence:—

$$V_n = 30\delta\,(1 + 0{\cdot}3/\sqrt{r\delta})\,r\log_e(s/r)\ \text{max. kilovolts.}$$

(*b*) Humidity of the air has no appreciable effect.

(*c*) Frequency has no appreciable effect. Very high frequencies (say 30,000 cycles/sec.) may lower the voltage slightly.

(*d*) State of surface. If the conductor is wetted by a fine water spray the corona voltage is lowered greatly. The figure 30 is reduced to about 9, and this is compensated for only partly by the 0·3 being increased to about 0·8. An oil film has a similar but smaller effect, 30 being replaced by 19, and 0·3 by 0·65. In this case with very small wires the voltage is increased, owing to the increased diameter over the oil more than compensating for the reduction in the voltage gradient necessary.

Roughness of the surface reduces the corona p.d. due to local increases of voltage gradient. A similar result is produced by using a stranded conductor. In this case there are two critical voltages for corona, (i) that at which corona starts, the "local corona point"; (ii) that at which it increases rapidly, the "general corona point."

For cables with seven or more strands the former occurs at 72 per cent., and the latter at 82 per cent. of the critical voltage for a single wire of a diameter equal to the overall diameter of the stranded conductor. For equal *cross-sections* solid and stranded conductors give nearly the same result, the solid being slightly better.*

For three-phase lines of triangular spacing the maximum gradient is $(2/\sqrt{3} =)$ 1·155 times that of a monophase line with the same line volts and the same conductor size and spacing. For the same voltage to neutral the gradients are equal.

4. Corona Loss

Whenever visual corona occurs there is a waste of energy, and even at smaller voltages there is some waste due to a similar cause.

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* Ch. III., *Dielectric Phenomena in H. V. Engineering* (F. W. Peek).

The reason for the latter is that visual corona is always accompanied by an ultra-violet corona which is invisible but can be detected photographically. At voltages below but close to the starting point of visual corona the invisible corona is present alone, and so some loss occurs. Even below this, some loss occurs due to irregularity in the surface causing local corona.

The loss varies as the square of the excess of the voltage over the voltage (V_0) at which the invisible corona starts, except close to V_0,

$$\textit{i.e.}\ \text{corona loss} = (V - V_0)^2 \times \text{constant} \quad \dots\dots\dots\dots (a)$$

For round conductors the value of V_0 is that which makes the maximum voltage gradient at the surface equal to about 30 kilovolts per cm. (more when the diam. is under 1 cm., but it does not vary like the visual corona gradient, see Art. 3),

$$V_0 = 29{\cdot}8r \log_e (s/r) \div h \quad \dots\dots\dots\dots (b)$$

where h = peak factor of the voltage.

For air densities other than the standard (see below) the value of V_0 must be varied in direct proportion to the density.

For stranded conductors, if r is taken equal to the overall radius, the value of V_0 is about 15 per cent. lower for seven strands, the reduction being less with larger numbers of strands, up to 61. In all cases the value will be lowered, and so the loss increased, if the surface of the conductor is rough or irregular. For weathered wires the reduction is 2 per cent. to 7 per cent.

The constant in equation (a) depends on the size of the conductors and on their spacing. F. W. Peek* has shown that it varies as $\sqrt{(r/s)}$ over the range of sizes and spacings usual in practice. He has shown further that an increase of frequency increases the loss, but not in direct proportion. The change can be represented by making the constant proportional to $(f + 25)$, where f = frequency in cycles per sec.

Combining these results with measurements on the loss under various conditions leads to the relation:—

$$\text{constant } (a) = 2{\cdot}41\ (f + 25) \sqrt{(r/s)} \quad \dots\dots\dots\dots (c)$$

if the loss is measured in watts per kilometre of conductor, and the voltages in R.M.S. kilovolts to neutral.

The effect of variation of air density is that the loss constant varies inversely as the density. The figure given in (c) is for 25° C. and 760 mm. mercury barometer. For other temperatures and pressures the air density factor (δ) is given by $0{\cdot}392b/(273 + t)$, where b = barometer height in mm., and t = air temperature in ° C.

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* *Dielectric Phenomena in H. V. Engineering*, Ch. v.

Later experiments have shown this formula to give inaccurate results for high-voltage lines with conductors over 1 inch dia. A paper by Carroll and Rockwell published in *Electrical Engineering* (N. York), Volume 56, page 558, gives details of an alternative method for such cases.

For symmetrical 3-phase lines the same formulae apply if the voltage to neutral (= line volts/$\sqrt{3}$) is used (see Arts. 6 and 7).

Example 3. *Calculate the corona loss on a single-phase transmission line 40 km. long with 00 S.W.G. (·884 cm. diam.) conductors placed 150 cm. apart and operated at 110 kilovolts (R.M.S.) 40 cycles per sec. at normal air density.*

Repeat the calculation for storm conditions assuming this to be represented by lowering the critical voltage (V_0) to 80 per cent. of its fair weather value.

By equation (*b*):—

$$\begin{aligned} V_0 &= 29{\cdot}8 \times {\cdot}442 \log_e (150/{\cdot}442) \div \sqrt{2} \\ &= (13{\cdot}17 \times 5{\cdot}83)/1{\cdot}414 \\ &= 52{\cdot}8 \text{ kilovolts (R.M.S.) to neutral} \end{aligned}$$

say 52 kV., allowing about 4 per cent. for roughness of wire.

By equation (*c*):—

$$\begin{aligned} \text{The constant} &= 2{\cdot}41\,(40 + 25)\,\sqrt{({\cdot}442/150)} \\ &= 2{\cdot}41 \times 65 \times {\cdot}054 = 8{\cdot}46 \end{aligned}$$

110 kV. (R.M.S.) between lines = 55 kV. (R.M.S.) to neutral;

$$\begin{aligned} \therefore \text{Loss} &= 8{\cdot}46\,(55 - 52)^2 \text{ watts per km. of conductor} \\ &= 0{\cdot}076 \text{ kilowatt per km. of conductor} \\ &= 0{\cdot}152 \text{ kW. per km. of line} \\ &= 6{\cdot}1 \text{ kW. in whole line.} \end{aligned}$$

For storms $V_0 = 52 \times 0{\cdot}8 = 41{\cdot}6$ kV. (R.M.S.);

$$\begin{aligned} \therefore \text{Loss} &= 8{\cdot}46\,(55 - 41{\cdot}6)^2 \times 2 \text{ watts per km. of line} \\ &= 3{\cdot}04 \text{ kW. per km. of line} \\ &= 122 \text{ kW. in whole line.} \end{aligned}$$

The above line is thus suitable only for voltages up to about 100 kV. (R.M.S.) between lines. It would still have a considerable storm loss at this pressure, but the storm is unlikely to occur simultaneously along the whole line. Moreover the lower temperature during a storm would increase the air density and so reduce the loss.

When such cases occur the best way of reducing the corona loss is to increase the size of the conductor. This may be done:—

(i) By using aluminium; for equal conductivity this requires 64 per cent. greater area than copper and so 28 per cent. greater diameter, hence V_0 is increased by about 20 per cent.

(ii) By using a combination of aluminium and steel wires.

(iii) By using copper-clad steel (Monnot metal) or aluminium-clad steel.

Method (ii) is the most usual, as this gives good strength as well as the necessary conductance. The aluminium carries most of the current, and the steel supplies most of the strength. The following are examples of British standard cables of this type (see B.S.S. 215 for full details). Six aluminium wires each 0·118 inch diameter surrounding a centre steel wire of the same section, making the diameter 0·354 inch. This has very nearly the same resistance as a nominal 0·04 sq. in. copper cable (19/·052 inch) whose diameter is 0·260 inch. This type of construction is used for all sizes below that equivalent to 0·1 sq. in. copper.

For larger sizes the centre core is made of seven equal-sized steel wires. With few exceptions the outer part consists of 30 aluminium wires in two rings, or 54 aluminium wires in three rings, in each case of the same section as the steel ones. Examples are (*a*) 30 Al. and 7 steel, each 0·102 inch diameter, conductor diameter 0·714 inch; copper equivalent 0·15 sq. in. nominal, 37/·072 inch, diameter 0·504 inch; (*b*) 54 Al. and 7 steel, each 0·139 inch dia., conductor dia. 1·251 inch; copper equivalent 0·5 sq. in. nominal, 61/·103 inch, dia. 0·927 inch.

For unusually long spans the proportion of steel is increased. An example of this is the overhead cables for the River Forth crossing of the British Grid. This has a centre of 19 steel wires, each 0·125 inch dia., surrounded by a ring of 18 aluminium wires of the same section.

The strength of the aluminium wires used for overhead conductors is about 25 000 lb. per sq. in. (higher for very small wires, and less for large ones), and that of the steel about 180 000 lb. per sq. in., varying in a similar way. In calculating the strength of a strand it is usual to take 85 per cent. of the sum of the strengths of the steel wires, plus 98 per cent. of the sum of the strengths of the aluminium wires.

5. Maxwell and Orlich Capacitance Coefficients

In a system containing more than two conductors, such as a three-phase line, it is not possible to define *the* capacitance, since several capacitances exist.

Maxwell's method of dealing with this problem is by means of a system of "capacitance coefficients," which are defined as follows.

The self-capacitance coefficient (K_1) of conductor No. 1 is equal to (charge on No. 1)/(potential of No. 1), when all the other conductors are at zero potential. It will be seen that if there are only two conductors K_1 coincides with the capacitance of the two. The self-capacitance coefficients of the other conductors are defined in the same way.

Further a set of mutual-capacitance coefficients are defined in the following way. $_2K_1$ is the ratio (charge on No. 2)/(potential of No. 1), when all the conductors except No. 1 are at zero potential. And $_1K_2$ = (charge on No. 1)/(potential of No. 2), when all the conductors except No. 2 are at zero potential. It can be proved that $_1K_2 = {_2K_1}$, hence the name mutual. The proof depends on the fact that the electrostatic flux between Nos. 1 and 2 depends on (p.d.)/(permittance of path).

It is evident that the mutual coefficients will all be negative. For if No. 1 has a positive charge and a positive potential, and all the rest are at zero potential, they will all have negative charges, since all the lines of electric force from No. 1 must end on one of the other conductors. Further, for the same reason,

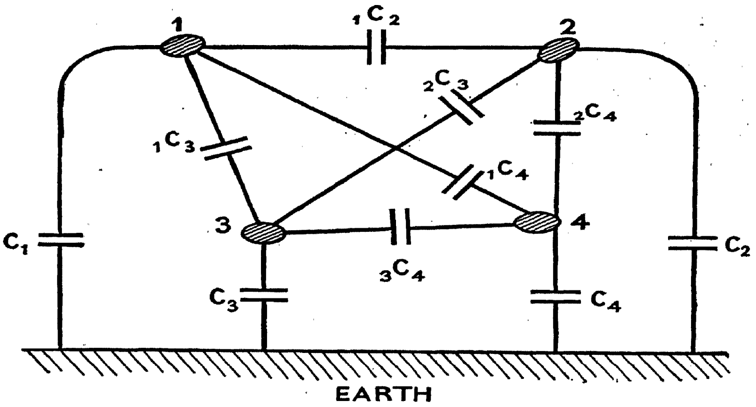


Fig. 12.05.—ORLICH'S METHOD OF "CAPACITANCE COEFFICIENTS."

$$K_1 + {_2K_1} + {_3K_1} + \ldots \text{ etc.} = 0,$$

since the positive charge on No. 1 must be numerically equal to the sum of the negative charges on all the rest.

Orlich's method is to represent the capacitances:—

(*a*) by a set of condensers, C_1, C_2, etc., each connected between the corresponding conductor and a distant one (" earth "), and:—

(*b*) a set of condensers, $_1C_2$, $_1C_3$, $_2C_3$, etc., each connected between two conductors.

Consider such sets of condensers for four conductors (Fig. 12.05). If No. 1 is at potential V, and all the others are at zero ("earth") potential, the charge on No. 1 is

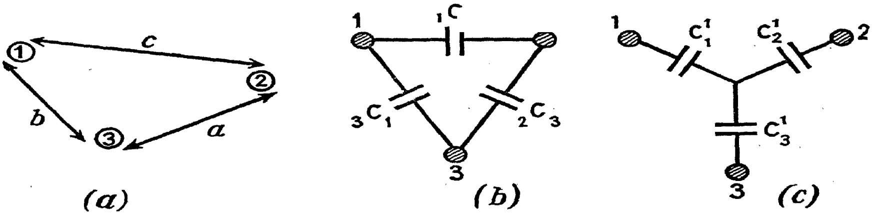
$$C_1V + {}_1C_2 . V + {}_1C_3V + {}_1C_4 . V$$

So, by definition, $K_1 = C_1 + {}_1C_2 + {}_1C_3 + {}_1C_4$.

And under the same conditions the charge on No. 2 is $- {}_1C_2 . V$, and therefore ${}_2K_1 = {}_1K_2 = - {}_1C_2$; and similar relations hold for the other coefficients. By combining these results the following relation is obtained:—

$$C_1 = K_1 + {}_1K_2 + {}_1K_3 + {}_1K_4.$$

The Maxwell coefficients are the easier to calculate from the dimensions and spacings of the conductors, but the Orlich ones are more convenient to use in circuit calculations.



(a) (b) (c)

Fig. 12.06.—CAPACITANCE OF THREE-PHASE OVERHEAD TRANSMISSION LINE

6. Capacitance of Three-Phase Overhead Line

Let r cm. be the radius of each conductor of a 3-conductor overhead line, spaced a, b, c cm. apart as shown in Fig. 12.06. If conductor No. 1 has a potential V and a charge q per cm., when Nos. 2 and 3 are at zero potential and with charges $-q_2$ and $-q_3$ per cm., then $q_2 + q_3 = q$. If $a = b = c$, $q_2 = q_3 = \frac{1}{2}q$. (The general case will be considered later.)

Considering the work done on unit charge in passing from No. 1 to No. 2 along the line of centres (cf. Art. 2), since no net work is done by the charge on No. 3:—

$$\begin{aligned} V &= \int_r^{c-r} \frac{2q}{x}dx + \int_{c-r}^{r} \frac{-2q_2}{c-x}dx \\ &= 2q \log_e \frac{c-r}{r} + 2q_2 \log_e \frac{c-r}{r} \\ &= 3q \log_e \frac{c-r}{r} = 6q_2 \log_e \frac{c-r}{r}; \end{aligned}$$

$$\therefore \text{ (see Art. 5) } K_1 = 1\bigg/\left(3 \log_e \frac{c-r}{r}\right) \doteqdot 1\bigg/\left(3 \log_e (c/r)\right);$$

$$\text{and } {}_2K_1 \doteqdot -1\bigg/\left(6 \log_e (c/r)\right);$$

and by symmetry, $K_2 = K_3 = K_1$; and ${}_3K_2 = {}_1K_3 = {}_2K_1$.

Hence $C_1 = K_1 + {}_1K_2 + {}_1K_3 = 0$, which is evidently correct since the earth is supposed to be too far away to affect the capacitances. And:—

$${}_2C_1 = -{}_2K_1 = 1\bigg/\left(6 \log_e (c/r)\right);$$

and the same for the other mutual capacitances.

Thus the capacitances of a symmetrical 3-conductor overhead line may be represented by 3 condensers of this value connected in delta. Denoting this capacitance by C, the delta-connected ones may be replaced by three Y-connected condensers, each of capacitance C′.

The joint capacitance between two conductors, say No. 1 and No. 2, in the delta-connected case [Fig. 12.06 (*a*)] is $C + \frac{C}{2}$, *i.e.* $1\frac{1}{2}$ C; whereas in the Y-connected case [Fig. 12.06 (*b*)] it is $\frac{1}{2}$ C′. So for equivalence $\frac{1}{2}$ C′ = $1\frac{1}{2}$ C, i.e. C′ = 3 C. Hence $C' = 1\bigg/\left(2 \log_e (c/r)\right)$, which is the same expression as for the capacitance to neutral for a 2-conductor overhead line (Art. 2). Consequently the line capacitance current in a 3-phase symmetrical overhead line is the same as that for a 1-phase line with equal conductor sizes and spacings and equal voltages to neutral.

Under these conditions the maximum charge on each of the 3-phase and 1-phase lines is the same. And since the stress at the surface of a conductor is $2q/r$ (see Art. 2) the maximum stresses will be equal in the two cases, as will be their R.M.S. values (cf. (Art. 3).

7. Unsymmetrical Spacing

In this case q_2 and q_3 may be unequal. Moreover to the expression for V in Art. 6 must be added a term expressing the work done by $-q_3$. When r is small this term is $-\int_b^a \frac{2q_3}{x}dx$ (where x is distance from No. 3), *i.e.* $-2q_3 \log_e (a/b)$, or $2q_3 \log_e (b/a)$;

$$\therefore\ 1/K_1 = 2\,\{\log_e (c/r) + (q_2/q) \log_e (c/r) + (q_3/q) \log_e (b/a)\}$$

Since under the assumed conditions V is also the p.d. between No. 1 and No. 3, another expression can be found for K_1 by considering the work done on unit charge moving from No. 1 to No. 3. This gives:—

$$1/K_1 = 2\,\{\log_e (b/r) + (q_3/q) \log_e (b/r) + (q_2/q) \log_e (c/a)\}.$$

By equating these two expressions the values of q_3/q and q_2/q can be found with the assistance of the further relation $q_3/q + q_2/q = 1$ (see Example 4). Further:—

$$-\,1/{}_2K_1 = 2\,\{(q/q_2) \log_e (c/r) + \log_e (c/r) + (q_3/q_2) \log_e (b/a)\}$$

and $$-\,1/{}_3K_1 = 2\,\{(q/q_3) \log_e (b/r) + \log_e (b/r) + (q_2/q_3) \log_e c/a\},$$

both of which can now be evaluated.

Expressions for the remaining coefficients can be obtained in the same way, and they can be combined to find the Orlich values as before. It will be found again that C_1, C_2, and C_3 are all zero, but the mutual capacitances are no longer equal.

To find the equivalent capacitances to neutral, proceed as follows. The capacitance between No. 1 and No. 2 [see Fig. 12.06 (*a*)] is

$${}_1C_2 + \frac{{}_3C_1 \cdot {}_2C_3}{{}_3C_1 + {}_2C_3} = \frac{{}_1C_2 \cdot {}_3C_1 + {}_1C_2 \cdot {}_2C_3 + {}_3C_1 \cdot {}_2C_3}{{}_3C_1 + {}_2C_3} = \frac{\Sigma\,(C.C)}{{}_3C_1 + {}_2C_3},$$

where $\Sigma\,(C.C)$ = sum of products of pairs of capacitances.

For equivalence with the Y-connected condensers [Fig. 12.06 (*b*)], the reciprocal of this, viz. $\frac{{}_3C_1 + {}_2C_3}{\Sigma\,(C.C)}$ must equal $\frac{1}{C_1'} + \frac{1}{C_2'}$. Similarly, $\frac{{}_1C_2 + {}_3C_1}{\Sigma\,(C.C)} = \frac{1}{C_2'} + \frac{1}{C_3'}$, and $\frac{{}_2C_3 + {}_1C_2}{\Sigma\,(C.C)} = \frac{1}{C_3'} + \frac{1}{C_1'}$.

From which it can be seen that

$$C_1' = \Sigma\,(C.C)/{}_2C_3; \quad C_2' = \Sigma\,(C.C)/{}_3C_1; \text{ and } C_3' = \Sigma\,(C.C)/{}_1C_2.$$

When the three mutual capacitances are equal these reduce to the previous relationship $C' = 3\,C$ (see Art. 6).

The above method of replacing delta-connected capacitances by equivalent Y-connected ones may be applied to resistances by writing $\frac{1}{R}$ in place of C; so that $\frac{1}{R_1'} = \Sigma\left(\frac{1}{R} \cdot \frac{1}{R}\right) \Big/ \frac{1}{{}_2R_3}$. Hence, since $\Sigma\left(\frac{1}{R} \cdot \frac{1}{R}\right) = \frac{\Sigma\,(R)}{{}_1R_2 \cdot {}_2R_3 \cdot {}_3R_1}$ (where $\Sigma\,(R) = {}_1R_2 + {}_2R_3 + {}_3R_1$) we have $R_1' = \frac{{}_1R_2 \cdot {}_3R_1}{\Sigma\,(R)}$; and two similar expressions give the other two Y-connected resistances.

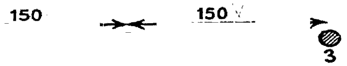


Fig. 12.07.

Exactly the same relation holds if the three are pure reactances, or if they are impedances with equal ratios of reactance to resistance.

Example 4. *For 3 conductors, each 1·2 cm. dia. with their centres in line and 150 cm. apart, calculate (a) the equivalent capacitances to neutral; (b) the stress at the surface of the centre conductor with a 3-phase line voltage of 90 kV.*

Let No. 1 conductor be at potential V and have a charge q per cm., and Nos. 2 and 3 at zero potential with charges of $-q_2$ and $-q_3$. Then by considering the movement of unit charge from No. 1 to No. 2,

$$V = 2q \log_e \frac{150 - 0{\cdot}6}{0{\cdot}6} + 2q_2 \log_e \frac{150 - 0{\cdot}6}{0{\cdot}6} + 2q_3 \log_e \frac{300}{150}$$

$$= 2\,\{(q + q_2) \log_e 249 + q_3 \log_e 2\}$$

$$= 11{\cdot}035\,(q + q_2) + 1{\cdot}386\, q_3.$$

Similarly in moving from No. 1 to No. 3:—

$$V = 2q \log_e \frac{300 - 0{\cdot}6}{0{\cdot}6} + 0 + 2q_3 \log_e \frac{300 - 0{\cdot}6}{0{\cdot}6}$$

$$= 12{\cdot}425\,(q + q_3).$$

Equating these gives $1{\cdot}390q - 11{\cdot}035q_2 + 11{\cdot}039q_3 = 0$, whence (since $q = q_2 + q_3$), $12{\cdot}429q_3 = 9{\cdot}645q_2$, which gives $q_2 = q\ 12{\cdot}429/(12{\cdot}429 + 9{\cdot}645)$ $= 0{\cdot}563q$ and $q_3 = 0{\cdot}437q$.

$$\text{Whence } V = 12{\cdot}425 \times 1{\cdot}437q = 17{\cdot}85q;$$

$$\therefore\ K_1 = 1/17{\cdot}85 = 0{\cdot}0560 \text{ E.S.U. per cm.,}$$

$${}_2K_1 = -\,0{\cdot}563K_1 = -\,0{\cdot}0315 \text{ E.S.U. per cm.,}$$

$$\text{and } {}_3K_1 = -\,0{\cdot}437K_1 = -\,0{\cdot}0245 \text{ E.S.U. per cm.;}$$

$\therefore\ {}_1C_2 = 0{\cdot}0315$, and by symmetry ${}_2C_3 = 0{\cdot}0315$; and ${}_3C_1 = 0{\cdot}0245$ (all in E.S.U./cm.). From these:—

$$\Sigma\,(C.C) = {\cdot}0315\,({\cdot}0315 + {\cdot}0245 + {\cdot}0245) = {\cdot}00254;$$

$$\therefore\ C_1' = C_3' = {\cdot}00254/{\cdot}0315 = {\cdot}0805 \text{ E.S.U./cm.,}$$

$$\text{and } C_2' = {\cdot}00254/{\cdot}0245 = {\cdot}1037 \text{ E.S.U./cm.}$$

Now 90 kV. = 300 E.S.U. When the potential of No. 2 is at its maximum to neutral, the voltages between No. 2 and both Nos. 1 and 3 are 0·866 of their maxima. Hence at this moment the charge per cm. on No. 2 is $0{\cdot}866 \times 300 \times h \times ({\cdot}0315 + {\cdot}0315)$, *i.e.* $16{\cdot}38 \times h$, where h = peak factor. Hence maximum stress at surface of conductor No. 2 (see Art. 2) is $2 \times 16{\cdot}38 \times h/0{\cdot}6$, *i.e.* $54{\cdot}6 \times h$, E.S.U./cm., and its R.M.S. value is 54·6, E.S.U./cm., *i.e.* 16·4 kV./cm.

8. Effects of Asymmetry

In a line with unsymmetrical spacing it is usual to "transpose" the conductors at ⅓rd and at ⅔rds of the length of the line. This makes each phase occupy each of the three positions in turn, so that the capacitances are equalised.

The chief factors in the three mutual capacitances are log (a/r), log (b/r), and log (c/r). The mean value of these is $\frac{1}{3}$rd of their sum, which is equal to log $(\sqrt[3]{abc}/r)$. Hence in a transposed line the asymmetry may be allowed for by using the formula for a symmetrical line with a spacing of $\sqrt[3]{abc}$. Moreover as the capacitances are equal they can be replaced by Y-connected capacitances three times as large.

Thus in example 4, Art. 7, the equivalent symmetrical spacing is $\sqrt[3]{150 \times 150 \times 300} = 189$ cm. Hence

$$C' \text{ (to neutral)} = 1\Big/\left(2 \log_e \frac{189 - 0{\cdot}6}{0{\cdot}6}\right) = {\cdot}0870 \text{ E.S.U./cm.};$$

whereas the average of the three separate values is ·0882 E.S.U./cm.

Transposition does not reduce the higher stress near the conductor whose two spacings are least (cf. Example 4, Art. 7). So that in considering the likelihood of corona, and in calculating corona loss, the actual spacings must be considered. The only effect of transposition in this regard is to divide the corona loss equally between the phases.

9. Inductance of 3-Phase Lines

Inductance of multi-conductor systems may be treated by means of self- and mutual-inductance coefficients on the same lines as capacitance. But for 3-phase lines the following simpler method is sufficient.

In a 3-phase symmetrical line the flux due to the current in the third conductor does not link with the loop formed by the other two. Hence the mutual (or loop) inductance is given by the same formula as for a single-phase line.

If these three equal loop inductances are replaced by equivalent Y-connected inductances the latter are only $\frac{1}{3}$rd as great. This results from the facts that the currents in the delta-connected loop inductances are only $1/\sqrt{3}$ of line currents, the voltages to neutral are only $1/\sqrt{3}$ of line voltages, and inductance is proportional to (induced voltage)/(rate of change of current).

As the capacitances to neutral are 3 times the mutual capacitances the approximate relation (see Art. 2) $CL = 1{\cdot}11/10^{15}$, holds good for three-phase provided both are taken to neutral, or both between lines.

For an unsymmetrical transposed line the single-phase formula may still be used, provided the equivalent spacing $\sqrt[3]{abc}$ (see Art. 8) is used.

10. Insulators

Insulators for overhead lines are nearly always made of porcelain. Glass is used sometimes but is more brittle and slightly inferior in other ways. There are two types of insulator employed, the " pin " and the " suspension."

Sections of **pin type** insulators are given in Fig. 12.08. The insulator is supported by a steel pin with a screw thread cemented into a threaded hole in the porcelain. The pin may be inserted during erection with a packing of hemp yarn served with either linseed oil or red lead, but cementing at the factory is preferable. An alternative is to use a soft metal liner (Fig. 12.08 C).

For large insulators type B (Twiss) is preferable to type A as the difference of expansion of the pin and insulator may cause the latter to split. Though A may be slightly stronger mechanically, B is amply strong enough for any load that can come on to it in use.

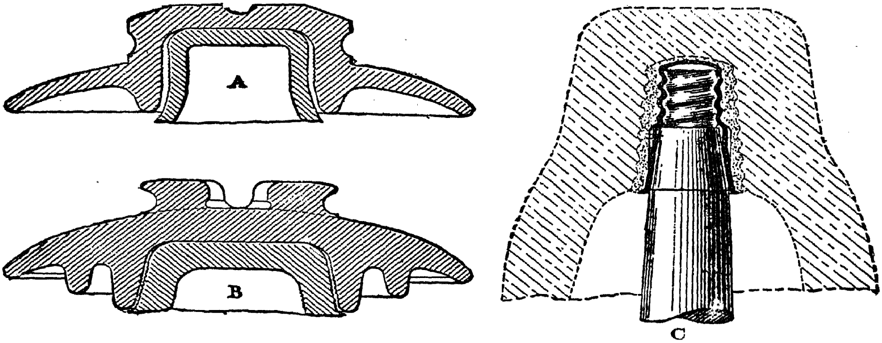


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Fig. 12.08.—PIN TYPE INSULATORS.

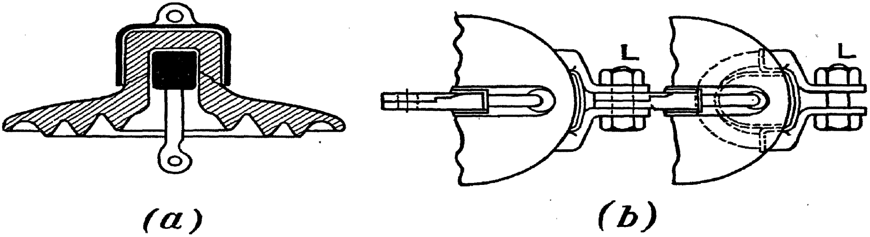
The upper surface of the insulator becomes leaky in wet weather, but the " petticoats " (usually three in number) provide a long dry path to prevent leakage from line to pin.

For very high pressure this type becomes very large and expensive. The cost can be reduced in such cases by making it in two or more pieces cemented together.

The **suspension type** insulator (Fig. 12.09) is suitable for very high pressures since a chain of any desired number of these can be used in series. The cost therefore increases about in proportion to the voltage, whereas that of the pin type increases more nearly as the square of the voltage. The weight of the pin type above 25 kilovolts increases approximately as the $1\cdot5$th power of the voltage.

Two distinct types of suspension insulator are used, the metal-capped type [Fig. 12.09 (*a*)], and the interlinked type [Fig. 12.09 (*b*)]. In the former by making the cap sufficiently deep as shown the stress on the porcelain can be made compressive, but this requires careful design and manufacture. In the latter the stress is naturally compressive, but there is some risk of excessive electrical stress between the nearest points of the links (L, L, L). An advantage of this type is that if one of a string of insulators breaks, the line does not fall.

The voltage is not distributed equally over the insulators in a string if they are all alike. The effect of the capacitances to earth is that the bottom insulator carries the largest capacitance current, and therefore has the largest p.d. across it. For this reason the porcelain should be made as thin as possible consistent with sufficient dielectric strength. This increases the (series) capacitance of the insulator and so diminishes the disturbance of the equality of the p.d.s due to the (shunt) capacitances to earth (see Example 5).



(*a*) (*b*)

Fig. 12.09.—SUSPENSION TYPE INSULATORS.

(*a*) = Metal capped. (*b*) = Twiss interlinked Insulator.

Close coupling of the units is desirable so that if an arc occurs it goes right across the whole string, and not in a series of arcs across the units. The former type of arc is much less liable to damage the porcelain (cf. "guard rings," Art. 12).

Example 5 *Find the voltages across a string of four similar insulators for a total p.d. of 40 kV., if their capacitances to earth are $\frac{1}{8}$th of the capacitance of an insulator.*

The equivalent circuit is shown in Fig. 12.10.

Let V = p.d. across top (earth end) insulator. The current in it is proportional to 8CV, and that in the first earth capacitance to CV. Therefore the current in the second insulator is proportional to 9CV, and the p.d. across it is 9CV/8C., *i.e.* $1\frac{1}{8}$V. Thus the p.d. across the second earth capacitance is

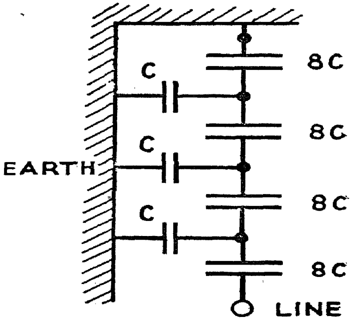


Fig. 12.10.

$2\frac{1}{8}$ V, its current is proportional to $2\frac{1}{8}$ CV, and that in the third insulator to $11\frac{1}{8}$ CV. So the p.d. across the third insulator is $11\frac{1}{8}$ CV/8C., *i.e.* 1·39 V, which makes the total p.d. across the third earth capacitance 3·515 V. Hence its current is proportional to 3·515 CV, that in the fourth insulator 14·64 CV, and the p.d. across fourth insulator 14·64 CV/8C., *i.e.* 1·83 V. This makes the total p.d. = 5·345 V. Therefore V = 40/5·345 = 7·48 kV = p.d. across top insulator. Whence p.d.s across the other three are respectively 8·42 kV, 10·40 kV., and 13·7 kV.

The string efficiency is 40/(4 × 13·7), *i.e.* 73 per cent.

The string efficiency can be increased by increasing the capacitance of the bottom insulator, *e.g.* by fitting a guard ring (Art. 12). In the above example the best result by this method is produced by increasing the bottom capacitance to 8C × 1·83/1·39, viz. to 10·53 C, which makes the p.d. across it equal to that across the third insulator, *i.e.* 1·39 V. Hence the total p.d. becomes 4·905 V, and the new value of V is 40/4·905, *i.e.* 8·15 *k*V. The p.d.s across the other three insulators become 10·17 *k*V, 11·34 *k*V, and 11·34 *k*V; and the string efficiency becomes 40/(4 × 11·34), *i.e.* 88 per cent.

Alternatively, if the maximum p.d. across an insulator is kept at its former value of 13·7 *k*V, the string can now withstand a total p.d. of 40 × 13·7/11·34, *i.e.* 48·3 *k*V.

11. Poles

Poles for moderate pressure lines are usually wooden. Unless timber is very cheap the wood is treated protectively. In Britain this is done by **creosoting** the poles, generally by the Rüping process in which air at high pressure is used to drive the excess creosote out of the pores of the wood. In Europe the copper sulphate process was almost universal at one time, but **kyanising** has supplanted this. The latter process employs a solution of sublimate of mercury for protection. Its main drawback is the poisonous nature of the solution.

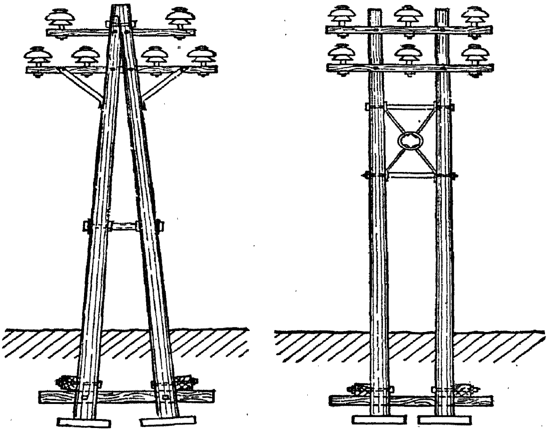
When a simple pole is not strong enough the A-type [Fig. 12-11 (*a*)] is used. Two wooden poles are bolted together at the top and stiffened by an iron tie-bar about a third of height up. The spread should be at least ⅛th of height. A stronger but considerably more expensive pattern is the H-type [Fig. 12.11 (*b*)]. This consists of two vertical wooden poles connected together by two struts, and diagonal tie-bars, all of steel. For heavy lines a third strut and another set of tie-bars are added.

For high pressure lines, especially with long spans, lattice-work metal masts are usual (Fig. 12.12). For crossing rivers, etc., these have been built up to heights of 250 ft. The foundations are made of concrete. Ferro-concrete masts are used too, especially where wood is expensive.

12. Lightning Protectors

Overhead lines and the apparatus connected thereto are liable to damage by lightning. The damage may be due to either (*a*) the lightning striking the overhead line and passing thence to earth through some part of the insulation of the system, or (*b*) the lightning striking something near the line, and the consequent sudden change of voltage inducing momentary high voltages which break down the insulation of the system. In both cases the disturbances caused is one of high pressure, high frequency, and moderate energy, but the power is very large owing to the rapidity of the discharge.

Long distance transmission lines are often provided with a **ground wire,** viz. an extra conductor carried on the tops of the poles and connected to earth at frequent intervals, preferably at every



(*a*) A-POLE. (*b*) H-POLE.

Fig. 12.11.—WOODEN LINE-POLES.

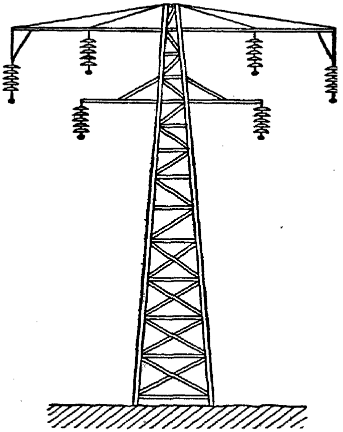


Fig. 12.12.—STEEL LATTICE MAST.

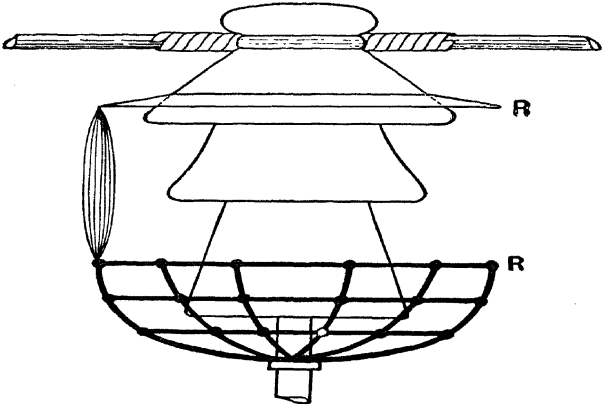


Fig. 12.13.—INSULATOR WITH GUARD RINGS.

pole. This reduces the probability of lightning striking the line and still more important, diminishes the disturbances due to lightning as they travel along the line to the generating and sub-stations.

A somewhat similar device is to fit the insulators with one or two metal **guard rings** (R, R, Fig. 12.13), placed so as to reduce the spark-over voltage below the value for a dry insulator without a guard ring, but not below its value when wet. The advantage of this is that if an arc is caused by lightning or otherwise the path of the arc is largely kept away from the insulator, and so damage is prevented or reduced.

The simplest type of lightning protector is the **horn gap arrester** (Fig. 12.14). The arms are set at such a distance apart that the normal line voltage will not start an arc. The high voltage due to lightning sets up an arc between the two nearest points of the gap. The normal voltage can *maintain* an arc across this distance, but the arc is forced upwards, partly by magnetic forces and partly by the rising of the air heated by the arc. Thus it is lengthened until it becomes too long for the line voltage to maintain, when it breaks. The line is disconnected from earth as soon as the excess voltage has been dissipated.

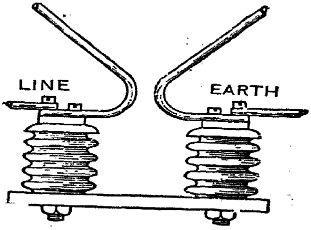


Fig. 12.14.—HORN GAP ARRESTER.

These are generally employed in conjunction with an **electrolytic arrester.** This consists of a number of cone-shaped trays of aluminium held short distances apart by insulating material. These spaces are filled with a solution of tetra-borate of aluminium, mixed with tartrate of aluminium if freezing is likely to occur.

About 300 volts per tray is allowed, and this pressure is insufficient to break down the thin film formed on the aluminium by electro-chemical action. But when a rise of pressure occurs the film breaks down and allows a discharge to earth to occur until the voltage falls to normal, when the film reforms.

A similar action occurs in the **oxide film arrester,** which consists of layers of pellets of lead peroxide between insulated brass plates. A high voltage breaks down the oxide film, but it re-forms when the voltage falls to normal.

The horn-gap and electrolytic arresters are connected in series when both are used. In this case the horn-gap may be fitted with a short-circuiting device (see Fig. 12.15), to " charge " the electrolytic arrester, *i.e.* to form the non-conducting film. This has to be done about every twenty-four hours as the electrolyte slowly dissolves the film.

The spark-over voltage of a gap is altered widely by the frequency, a high frequency requiring a higher voltage. This difference is particularly great with needle points for electrodes, *e.g.* with a spacing of 10 cm. the spark-over voltages are 76 kV. (max.) at 50 cyc., 99 kV. at 100 000 cyc,, and 132 kV. at 350 000 cyc. With rounded electrodes this increase of voltage is smaller, and with spheres of sufficiently large radius it becomes inappreciable. An ordinary horn-gap of round wire behaves somewhat like the spheres when the spacing is small, but approaches the needle-point characteristic as the spacing increases. A horn-gap fitted with sphere electrodes, therefore, gives better protection against high frequency impulses, especially on high pressure lines.

In 1918 the Westinghouse Co. introduced the " impulse gap " which makes the voltage to cause high frequency discharge *lower* than that for normal frequency. The auxiliary electrode (Fig. 12.15) at normal frequency is kept at a potential intermediate between those of the spheres corresponding with its position, and so has then no effect on the discharge voltage. This is done by means of insulators, acting as condensers of small capacitance, and the " unbalancing resistance." Usually the auxiliary electrode is placed in the middle of the gap, and the capacitances and resistance so chosen that its potential is midway between those of the spheres.

On high frequency the capacitance impedances are reduced inversely as frequency, and so the main part of the pressure is across the resistance, and therefore, across the gap between the auxiliary electrode and the right-hand sphere. If this gap breaks down nearly the whole pressure is transferred to the left-hand gap. which breaks down in turn. Thus the effective gap width for high

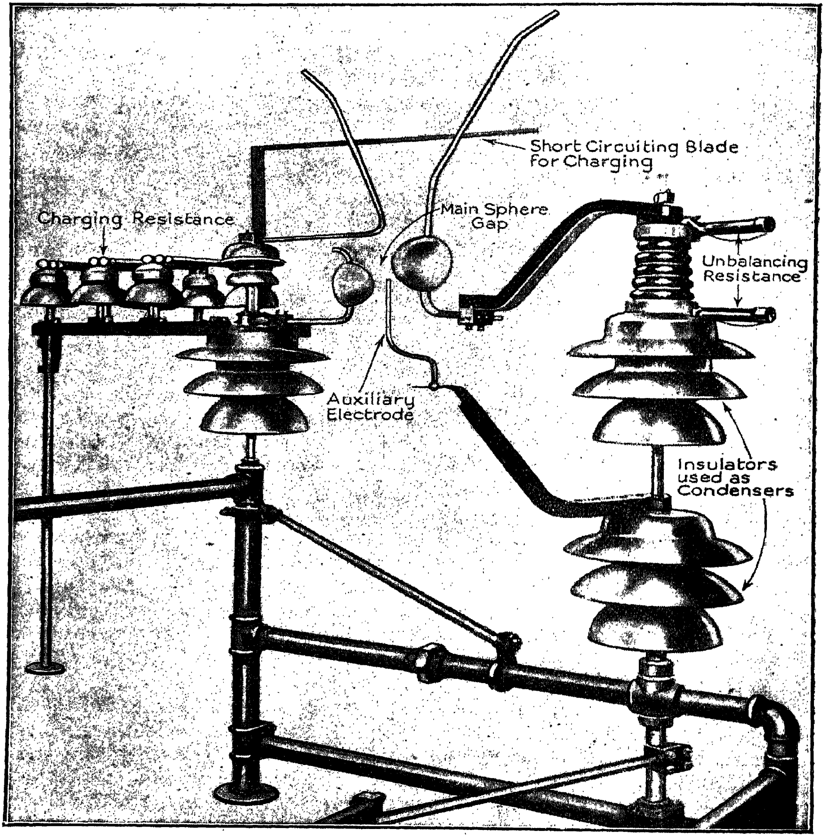


Fig. 12.15.—METROPOLITAN VICKERS CO.'S IMPULSE GAP WITH SPHERICAL AND AUXILIARY ELECTRODES AND CHARGING DEVICE.

frequency is reduced to little more than half its value at normal frequency.

In the construction shown in Fig. 12.15 the sphere and horn parts of the left-hand electrode (earth side) are separated, and the charging resistance is connected between them. This results in this resistance being normally out of circuit and in its coming into circuit when the horns are bridged for charging the electrolytic arrester.

In Fig. 12.16 is a side view of the whole arrangement when installed out of doors, the preferable method. For three-phase lines four electrolytic arresters are used, one connected to each line and their other ends connected together to form a star point, and the fourth connected between this star point and earth. If the neutral of the system is earthed with no resistance in the earth connexion the fourth electrolytic arrester may be omitted.

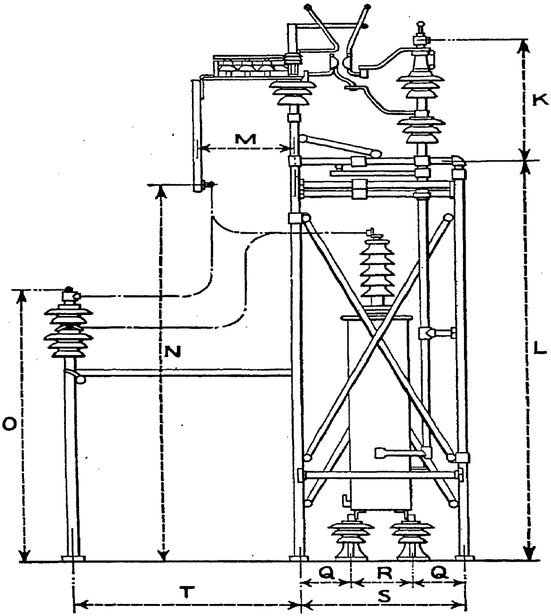


Fig. 12.16.—METROPOLITAN VICKERS CO.'S IMPULSE GAP, ELECTROLYTIC ARRESTER, AND CHARGING DEVICE

For voltages up to 50 kV. the dimensions in metres in Fig. 12.16 are:—K = 1·12; L = 3·2; M = 0·81; O = 2·86; P = 0·464; Q = 0·425; R = 0·413; T = 0·81; overall height of electrolytic arrester = 2·08; height of base above ground = 0·4. The distance between centres of the four electrolytic arresters is 0·76; and the distance between centres of the three-horn-gaps is 1·79.

Another method is to use cylinders of one of the non-arcing metals (usually an alloy of zinc, *e.g.* brass) which were discovered by Wurtz to possess the property of suppressing an arc when the pressure reverses. The cylinders are knurled (*i.e.* grooved in two directions at right angles) and are set in a row with their axes parallel, with clearances of about 1 mm. By themselves these are suitable only for voltages up to 2 200. By placing a high resistance in series to limit the discharge current their use can be extended to ten times this pressure, and this combination has operated satisfactorily even on 33-kilovolt lines.

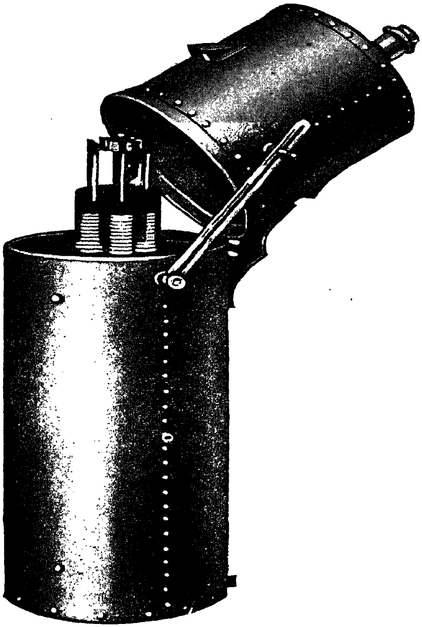
For still higher voltages two sets of gaps in series with a high resistance are used, one set being shunted by another high resistance. The effect of this arrangement is that when a large rise of pressure occurs discharge takes place through both sets of gaps and the series resistance. As the pressure falls the shunted set of gaps become able to suppress their arcs because they do not carry the whole discharge. Both resistances are now in series with the unshunted set of gaps and the discharge current is diminished so that these in turn can suppress their arcs. Thus without interposing so much resistance in the discharge path as to make the protection of the line insufficient the gaps are enabled to suppress their arcs. It is called the " low equivalent " gap arrester.

The drawback of this type of arrester is that the voltage is not distributed equally between the gaps for the same reason as in the case of suspension insulators (Art. 10). Therefore the electrolytic type is preferable on extra high voltage lines.

The use of **condensers** for protection against surges depends on the fact that the impedance of a condenser varies inversely as the frequency. Hence if a small capacitance is connected between line and earth it takes very little current at line frequency and yet permits a large current to flow at the high surge frequency. Since the line is not connected electrically to earth as in the other types of arrester, the normal condition of small condenser current is restored when the surge has been dissipated. On the other hand the energy of the surge has to be dissipated in the line resistance (see further Art. 21).

A condenser suitable for this purpose is that designed by Moscicki (see Fig. 12.17). It consists of a glass tube (A) with silver deposited chemically on both inside and outside, and copper electroplated on the silver. The tube is thickened at the neck to withstand the greater dielectric force at this point. The neck is enclosed in an insulator, and the space between filled with an insulating compound which prevents surface discharge between the edges of the two metallic coatings. The condenser is mounted in an iron tube (B), and the space between condenser and tube is filled with water mixed with glycerine. This is to prevent local heating which might cause a breakdown.

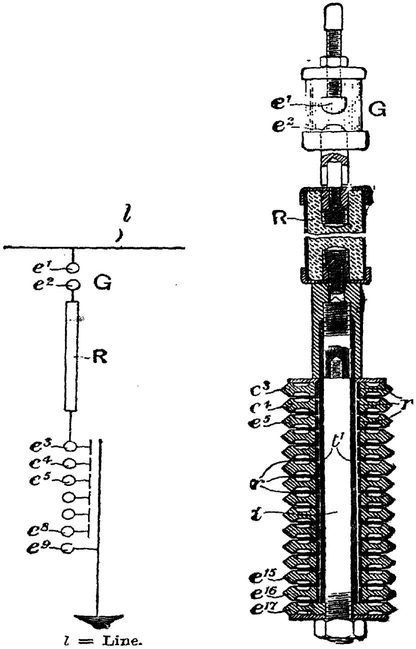
Each condenser can withstand a pressure of 35 kilovolts (max.) and two or more are connected in series for higher pressures. To increase the capacitance a number are connected in parallel. It must be remembered that series connexion diminishes the capacitance, *e.g.* six in parallel and in series with another set of six connected in parallel have a capacitance only three times that of one.



(*a*) Section. (*b*) Battery of Condensers in Outdoor Mounting.

Fig. 12.17.—Moscicki Condenser.

The Giles Valve is a combination of non-arcing gaps with condensers. It consists of a main gap [G, Fig. 12.18 (*a*)], in series with a resistance, and in series with a number of subsidiary gaps. All but the last of the pieces between which the gaps are formed (e^3 to e^9), are connected to earth through condensers. The result is that the full line voltage to earth is normally across the main gap which is set for (say) 25 per cent. over-pressure. Should this occur an arc is set up across G, but only the current which the first condenser can pass flows. The p.d. across the next gap then rises nearly to the full voltage and it breaks down and additional current flows to earth through the next condenser.



(*a*) CONNEXIONS. (*b*) SECTION.

Fig. 12.18.—GILES VALVE.

This continues until the whole of the gaps break down and discharge occurs through them to earth. Each half cycle the arcs are extinguished, this action being assisted by the shunting effect of the condensers.

Fig. 12.18 (*b*) shows the actual arrangement in section, the parts being lettered to correspond with Fig. 12.18 (*a*).

13. Reactors

Coils possessing reactance are used to prevent a disturbance, due to lightning or a short-circuit, from spreading over the whole system. For instance, they are installed where overhead lines enter generating or sub-stations. An arrester (Art. 12) is connected to the line close to the reactor to discharge to earth any over-pressure produced in the line. If reactors or other protective devices are not used the end turns of the transformers or generators connected to the line require extra heavy insulation to withstand the transient high pressures produced.

The effect of the reactor is to reflect most of the disturbance back along the line, and to cause its energy to be dissipated either by discharge through the arresters or by losses in line resistance. Since the disturbance is of high frequency the reactance chokes back its passage to a much greater extent than it opposes the normal line current (see further Art. 20).

The simplest type consists of a few turns of wire wound in a coil which is made of hour-glass shape [Fig. 12.19 (*a*)] for greater stiffness. When more reactance is required a cylindrical coil is used [Fig. 12.19 (*b*)], sometimes with insulating distance pieces to hold the turns in place.

For still higher reactances a flat (" pancake ") coil is used, often in an oil tank. This type is not suitable above 25 kilovolts. It has, however, other uses, *e.g.* for sectionalising bus-bars with a number of alternators in parallel, so that a breakdown on one section has less effect on the other alternators; and for limiting the short-circuit current in alternators of small intrinsic reactance. Reactors with concrete cores are used for these purposes too. The windings are of bare cable, fastened to the cores by wooden or concrete supports with non-magnetic bolts.

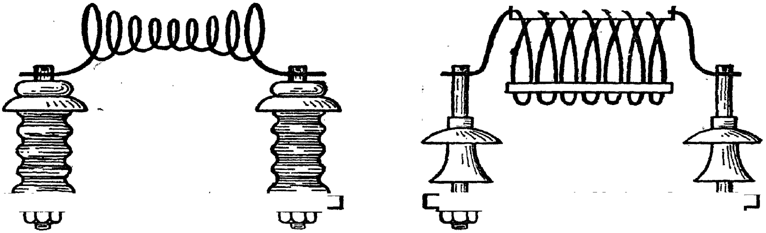

(*a*) HOUR-GLASS TYPE. (*b*) CYLINDRICAL TYPE.

Fig. 12.19.—REACTORS.

A Swiss committee* recommends the use on overhead lines of reactors shunted by carborundum resistances of 600 ohms to 1 000 ohms. Up to about 15 kV. the shunted reactor is to be placed in series with the apparatus to be protected and a horn gap with a liquid series resistance used to allow a discharge on rise of pressure

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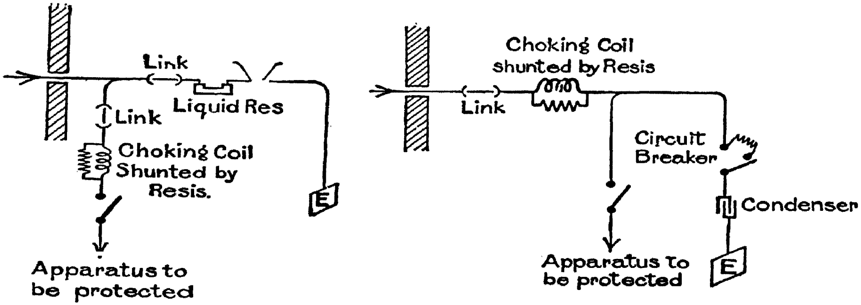
* See *Bulletin No.* 6 of 1916 of the *Schweiz. Elektrotech. Verein.*

[Fig. 12.20 (*a*)]. For higher voltages the shunted reactor is placed in the line, and a condenser [Fig. 12.20 (*b*)] used to diminish the pressure rise due to the part of the surge which passes the reactor.

They recommend inductances varying from 0·15 millihenry per phase for 10 kilovolts up to 0·80 mH. per phase for 100 kV. When several coils in series are used, only the one nearest to the apparatus to be protected is shunted.

14. Static Dischargers

Under certain atmospheric conditions a static electric charge will accumulate on an overhead line. This may increase to such an extent that a discharge occurs, which is followed by a discharge due to the line pressure. Even if this occurs through horn-gaps or other lightning protectors (see Art. 12) a surge is caused in the line which may cause breakdown at other points. It is better therefore to dispose of the static charge before it can accumulate to a dangerous extent.



(*a*) MODERATE VOLTAGES. (*b*) HIGH VOLTAGES.

Fig. 12.20.—PROTECTION BY SHUNTED REACTORS.

The two best methods of doing this are by water jets and by iron-cored reactors. The former method is suitable up to about 11 kV., and the latter for higher pressures without any limit.

A dropping water jet is cheapest, but is liable to break up into separate drops when electrically charged. A rising jet is therefore much safer. The jet may be free in air (see Fig. 12.21) or guided by insulating tubes. The drawback of the latter is that deposit may occur in the tubes, and of the former that the jet may be diverted by wind. The size of the jet should be readily adjustable to suit the resistivity of the water used.

With iron-cored reactors the only special point is that separate ones should be used for each phase, a common magnetic circuit being undesirable. At least two sets should be used, one near each end of the line.

15. Line Surges

When a charge on a line is produced suddenly by its being struck by lightning; or when a charge induced by an oppositely charged cloud is released suddenly, by the discharge of the cloud by lightning near the line, this charge tends to travel along the line at very high velocity. Such a travelling charge constitutes a current, and its presence connotes a potential on the corresponding parts of the line. Under certain conditions these travelling waves move without change of shape, *i.e.* if the graph of current against position on line is plotted at different times the shape remains unchanged though the position of the wave as a whole has altered.

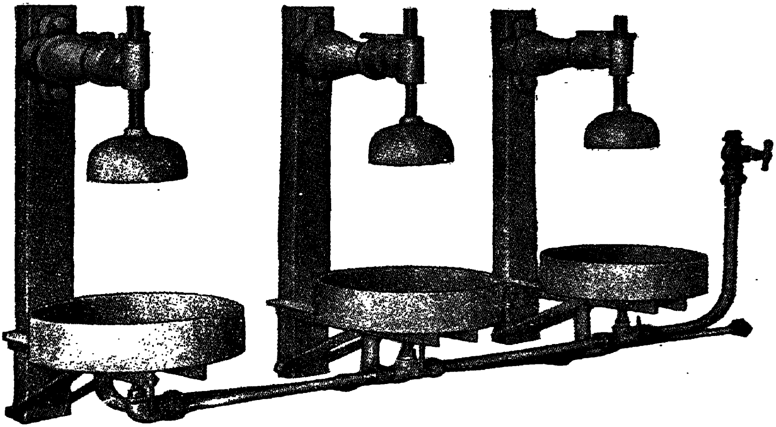


FIG. 12.21.—G.E. CO.'S WATER JET STATIC DISCHARGER.

The same applies to the graph of charge per unit length against position on line. And this latter graph is the same shape as that of potential against position if the line is uniform, since charge per unit length = potential × (capacitance per unit length).

The effect of resistance is to diminish the size of these waves exponentially, without any change of shape.

16. Surge Impedance

If part of a uniform line has a denser charge, and therefore a higher potential than the neighbouring part it tends to discharge into the latter, *i.e.* to produce a current. Conversely if the current in the first part is greater than in the second, the latter rises in potential since it receives more charge than it loses.

If the wave is travelling without change of shape these two conversions must balance, since the original p.d. must produce a current equal to the original, and the original difference between the currents must produce a potential equal to the original. Neglecting resistance no energy is lost in these conversions; therefore the original static energy due to charge must be equal to the original (kinetic) energy due to current, otherwise the division of energy between the two is reversed, *i.e.* the wave shapes are changed.

Thus if C is the capacitance of unit length of line, and L is the inductance of unit length of line,

$$\tfrac{1}{2}\,\mathrm{C}v^2 = \tfrac{1}{2}\,\mathrm{L}i^2 \text{ (see Volume I., Chapter V., Art. 14);}$$

$$\therefore\ v^2/i^2 = \mathrm{L/C}; \quad \therefore\ v/i = \sqrt{\mathrm{L/C}}.$$

v/i is the ratio of the voltage to the current at any point in the travelling surge, and so is called the "surge" (or "natural") impedance of the line. Note that this is quite different from the ordinary impedance of the line to alternating currents; in particular it is independent both of the length of the line, and of the frequency used for transmission.

17. Surge Velocity

A simplified mathematical proof of the value of surge impedance can be obtained by considering a rectangular surge travelling along a uniform line. At any instant the same graph (Fig. 12.22) may be used to represent a uniform charge spread over a length l and giving rise to a potential $\mathrm{V_0}$ over this length of line, accompanied by a constant current $\mathrm{I_0}$ in the same part of the line, with no current or

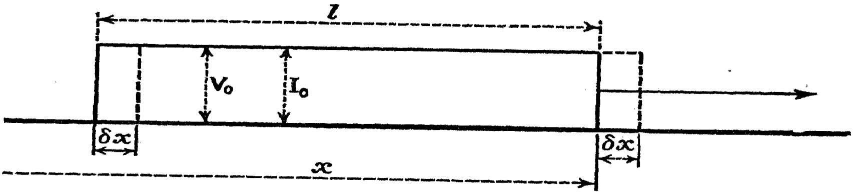


Fig. 12.22.—RECTANGULAR SURGE.

voltage on the rest of the line. After a short time, δt, all this has moved a short distance, δx, to the right.

Consider the piece of line into which the wave has moved. Its capacitance is $C\delta x$, therefore it has gained a charge $V_0C\delta x$. This has been produced by I_0 flowing into it for a time δt, therefore $V_0C\delta x = I_0 . \delta t$. Its inductance is $L\delta x$ and the p.d. across it V_0, therefore the rate of increase of current is $V_0/L\delta x$. This brings the current up to I_0 in time δt. Therefore $V_0\delta t/L\delta x = I_0$, or $V_0\delta t = LI_0\delta x$.

Equating the two values of $\delta x/\delta t$ obtained in these ways gives $I_0/V_0C = V_0/LI_0$. Whence, $V_0^2/I_0^2 = L/C$, *i.e.* surge impedance $= \sqrt{L/C}$. Substituting this in either equation shows that the velocity of the surge $(u) = \delta x/\delta t = 1/\sqrt{LC}$.

As has been shown in Art. 2 the latter expression is nearly equal to the velocity of light in any overhead line. Thus all surges in overhead lines travel at a velocity nearly equal to that of light (3×10^{10} cm./sec.). This enormous velocity justifies the neglect of the alternations of the ordinary power voltage and current when dealing with the effects of surges.

At the rear of the wave the actions proceeding are opposite to those at the front. The current flowing out of the small length δx lowers its potential to zero, and the p.d. across it reduces the current to zero; both these actions being completed in the same short time, δt, as before. Thus the wave shape remains unchanged, but moves continually to the right.

If the current were negative the movement would be reversed. Similarly a negative voltage and a positive current makes the velocity negative, for a negative charge moving to the left is equivalent to a current flowing to the right (cf. Chapter X., Art. 2). But if both are negative the velocity is positive, *i.e.* a negative charge moving in positive direction is equivalent to a negative current.

18. Effects of Change of Surge Impedance

If the wave comes to a point in the line where the surge impedance changes, a wave is transmitted and another one is reflected. During the time of arrival of the original wave at the transition point the reflected wave and the original overlap. Thus the shapes of the voltage and current graphs in this part of the line are changing, and (total voltage)/(total current) is not equal to the surge impedance. But the ratio voltage/current for each component still remains equal to the surge impedance, except that the sign of either the voltage or of the current is reversed in the reflected wave.

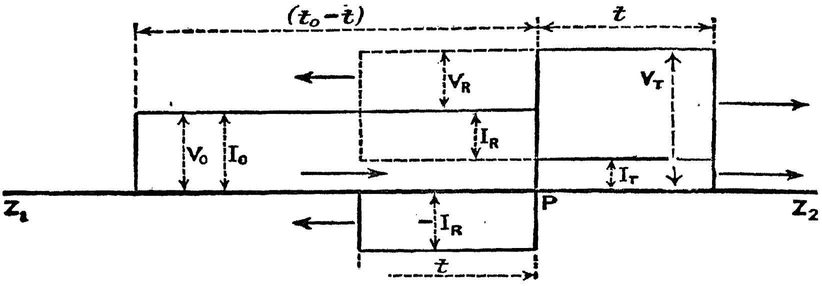


Fig. 12.23.—CHANGE OF SURGE IMPEDANCE.

Consider a rectangular incident wave of length t_0 (*i.e.* it takes a time t_0 to pass a fixed point) at a time t after reaching a transition point, P, to a line of higher surge impedance.

The voltage, V_T, of the transmitted wave will be higher than the original voltage V_O. And since the voltages at the two points on each side of P must be equal (except instantaneously) the voltage, V_R, of the reflected wave must equal $(V_T - V_O)$.

The reflected current must be negative since the velocity is reversed, and if its value is $- I_R$ the equality of the currents on the two sides of P gives:—

$$I_R = I_O - I_T \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Since each voltage equals the product of the corresponding current and the surge impedance, the former relation gives:—

$$I_R Z_1 = I_T Z_2 - I_O Z_1,$$

$$\text{whence:—} \quad I_R = I_T \frac{Z_2}{Z_1} - I_O \quad \ldots\ldots\ldots\ldots (2)$$

Equating the values of I_R found in (1) and (2) gives

$$I_T \left(\frac{Z_2}{Z_1} + 1\right) = 2I_O,$$

$$\text{whence:—} \quad I_T = \frac{2Z_1}{Z_1 + Z_2} I_O$$

And $V_T = I_T Z_2$, and $I_O Z_1 = V_O$, so that

$$V_T = \frac{2Z_2}{Z_1 + Z_2} V_O; \text{ and } V_R = V_T - V_O = \frac{Z_2 - Z_1}{Z_2 + Z_1} V_O$$

If Z_2 is made infinite $I_T = 0$, $V = 2 V_O$, and $V_R = V_O$ (whence $I_R = I_O$). Thus a surge reaching an open circuit is reflected completely, the reflected wave having the original voltage, and a reversed current equal in magnitude to the original current. During the arrival the voltage at the open circuit is double the original voltage; and this doubled voltage extends backwards to a distance having a maximum value of half the length of the incident wave.

When Z_2 is less than Z_1, V_T is less than V_0, and so the reflected voltage is negative $(-V_R)$. The reflected current is therefore positive, and the transmitted current is larger than the original. Working out the relationship as above gives the following results:—

$$V_T = \frac{2Z_2}{Z_1 + Z_2}V_0; \quad I_T = \frac{2Z_1}{Z_1 + Z_2}I_0 \text{ (as before);}$$

$$\text{and } V_R = \frac{Z_1 - Z_2}{Z_2 + Z_1}V_0; \quad I_R = \frac{Z_1 - Z_2}{Z_2 + Z_1}I_0.$$

(These are the same as before with the sign changed, *i.e.* both cases are given by $V_R = \frac{Z_2 - Z_1}{Z_2 + Z_1}V_0$, $I_R = \frac{Z_1 - Z_2}{Z_2 + Z_1}I_0$, if V_R and I_R are actual values: the signs being settled by the relative values of Z_1 and Z_2.)

Ultimately the original wave is divided into a transmitted wave of length t_0, and a reflected wave of the same length, in both cases.

If Z_2 is made zero, $V_T = 0$; $I_T = 2\,I_0$; $V_R = -V_0$; and $I_R = I_0$. This means that a surge reaching a short-circuit to earth is reflected as a wave having the original current flowing in the original direction (which is now opposite to the direction of travel) at a reversed voltage equal in magnitude to the original. During arrival the current at the short-circuit is double the original, and the maximum length covered by this is half the original wave (cf. the case of open circuit, considered above).

19. Energy in Surges

In the last article the lengths were stated as times. A surge of length t_0 has an actual length ut_0, where u is its velocity. So the capacitance of the part of the line occupied by the surge is $ut_0 . C$, and its inductance is $ut_0 . L$. Hence the static energy of the surge is $\frac{1}{2}(ut_0 . C)\,V^2$, and its inductive energy $\frac{1}{2}(ut_0 . L)\,I^2$. Since in a wave travelling without change of shape these are equal, the total energy is $ut_0 . CV^2$, or $ut_0 LI^2$. But $u = 1/\sqrt{CL}$, and $V = ZI = I\sqrt{L/C}$, so that each of these expressions for the total energy is equal to $VI . t_0$.

In the case of a wave of other shape than the rectangular it can be shown by dividing it up into small lengths that total energy $= \int v . i . dt$, where v and i are the voltage and the current at the same point, and the integration is extended over the whole length of the wave.

Note that the integration is a space integral at a fixed instant, although the symbol dt has been used. This is because the lengths have been expressed in the form of the times taken by the wave to travel over them.

Applying the above expression to the case of change of surge impedance (Art. 18) gives:—energy in transmitted wave $= \frac{4\,Z_1Z_2}{(Z_1+Z_2)^2}V_0I_0t_0$; and energy in reflected wave $= \left(\frac{Z_2-Z_1}{Z_2+Z_1}\right)^2 V_0I_0t_0$. For instance if $Z_2 = 4\,Z_1$, 64 per cent. of the energy in the original surge is transmitted, and 36 per cent. reflected. The same proportion holds good for $Z_2 = \frac{1}{4}\,Z_1$.

In the case of a rectangular surge reaching an open-circuit (Art. 18), when half the surge has arrived half the original length has been raised to double voltage. The capacitance energy per unit length is therefore quadrupled, so the total capacitance energy has been doubled. But the current has been reduced to zero at every point, so the inductance energy (originally half the total) has disappeared. Thus the total energy is unchanged in amount, but has become capacitance energy entirely.

20. Effects of a Reactor

When a surge reaches a reactor (see Art. 13) installed in a line it cannot at first pass on, since it requires a finite time to produce any current in an inductance. The initial action is that of an open-circuit, *i.e.* the voltage on the incident side is doubled and the

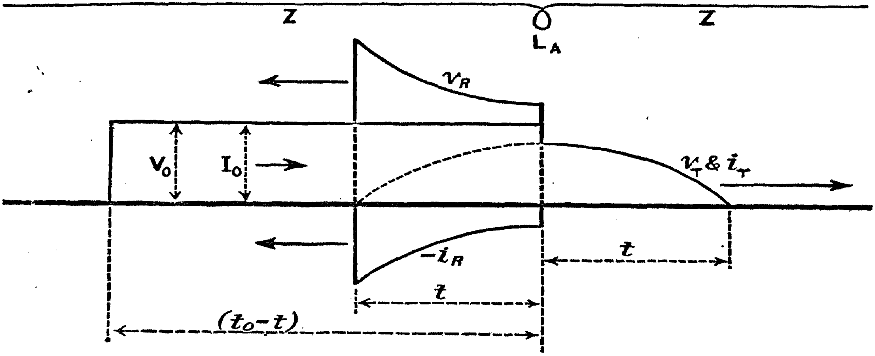


Fig. 12.24.—SURGE PASSING A REACTOR.

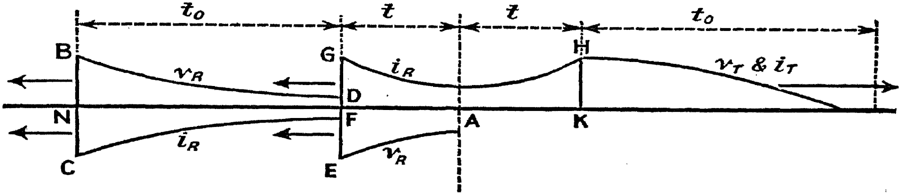


Fig. 12.25.—WAVES AND TAILS.

current reduced to zero. As the current in the reactor is built up the increase of voltage becomes less.

Consider an originally rectangular wave of length t_0 at a time t (less than t_0) after first reaching a concentrated inductance L_A; the line on each side of this having the same surge impedance. The current flowing into the inductance is the same as that flowing out, so $I_0 - i_R = i_T$, where i_R is the magnitude of the negative reflected current. Multiplying by the surge impedance gives $V_0 - v_R = v_T$.

The p.d. across L_A is $(V_0 + v_R) - v_T$, which by substituting for v_R from the previous relation is seen to be equal to $2\,(V_0 - v_T)$, *i.e.* $2\,V_0 - 2\,Zi_T$,

$$L_A\frac{di_T}{dt} = 2\,V_0 - 2\,Zi_T: \; i.e. \; L_A\frac{di_T}{dt} + 2\,Zi_T = 2\,V_0.$$

This is of the same form as the equation for current when a steady voltage ($2\,V_0$ in this case) is switched on to an inductive circuit ($2\,Z$ having replaced R), so the solution is of the same form. The final steady value of the current (by putting $\frac{di_T}{dt} = 0$) is $\frac{V_0}{Z}$

$$\therefore \; i_2 = I_0\left(1 - e^{-\frac{2Z}{L}t}\right)$$

Whence $i_R = I_0 - i_T = I_0 e^{-\frac{2Z}{L}t}$ and similar expressions give v_T and v_R.

When more than half of the original surge has arrived the general appearance is as in Fig. 12.24, except that there is no longer any point at double voltage since the front of the reflected wave has passed beyond the back of the incident wave. But after the whole wave has reached L conditions change. Current continues to flow in L_A in the same direction, so the reflected wave is followed by a tail of positive current and therefore negative voltage. Let Fig. 12.25 represent conditions at a time t after completion of arrival of surge (i.e. $t + t_0$ after arrival of front), and let V_1 and I_1

be the values of v_T and i_T when this time is zero. V_1 and I_1 will be very close to V_0 and I_0 except when the original surge is very short.

Now $i_R = i_T$, so v_R is equal but opposite to v_T. Hence the p.d. across L_A is $- v_R - v_T = - 2\,v_T = - 2\,Zi_T$;

$$\therefore\ L_A \frac{di_T}{dt} + 2\,Z\,.\,i_T = 0,$$

which is similar to the case of a current dying away in an inductive circuit, and has as solution, $i_T = I_1 e^{-\frac{2Z}{L_A}t}$. The value of i_R is the same, and $v_T = v_R = V_1 e^{-\frac{2Z}{L_A}t}$.

Note that BN and DE each equal V_0, *i.e.* the *change* of voltage at the start of the tail is V_0. Similarly NC = GF = I_0.

Example 6. *A rectangular surge of 150 k V. and 18 microseconds long, travelling in an overhead line of surge impedance 300 ohms reaches a reactor having an inductance of 0·002 henry. Find the initial rate of rise of the transmitted current, and the values of the transmitted and reflected waves at various points.*

It is convenient to use the microsecond as the unit of time in surge calculations. If the microhenry and microfarad are likewise used as units the formula in the above and the following Arts. are unchanged.

Thus $L_A = 0{\cdot}002 \times 10^6 = 2\,000$ microhenries: $I_0 = (150 \times 10^3)/300 = 500$ A.: $2Z/L_A = 2 \times 300/2\,000 = 0{\cdot}3$;

$$\therefore\ i_T = 500\,(1 - e^{-0\cdot 3t})\ \text{A.};\ v_T = 150\,(1 - e^{-0\cdot 3t})\ k\text{V.};$$

$$i_R = -\,500\,e^{-0\cdot 3t}\ \text{A.};\ v_R = 150\,e^{-0\cdot 3t}\ k\text{V.}$$

Whence, for $t =$	1;	2;	3;	4;	6;	8;	10;	14;	18	
$i_T =$	130;	226;	297;	349;	417;	455;	475;	493;	498	A.
$v_T =$	39;	68;	89;	105;	125;	136;	143;	148;	149	kV.
$-\,i_R =$	370;	224;	203;	151;	83;	45;	25;	7;	2	A.
$v_R =$	111;	82;	61;	45;	25;	14;	7;	2;	1	kV.

Note that all the above reflected currents are negative.

The initial rate of rise of i_T is that due to 2×150 kV. applied to L_A, and so equals $2 \times 150 \times 10^3/2\,000$, *i.e.* 150 A. per microsecond.

For the wave-tails, $i_R = i_T = 498\,e^{-0\cdot 3t}$, where t is time from completion of arrival. The values are therefore nearly the same as those given above for $-\,i_R$; and the values of v_T and $-\,v_R$ in the tails are nearly the same as those given above for v_R.

21. Effects of a Condenser

When a surge reaches a condenser (see Art. 12) connected between line and earth the first effect is to reduce the voltage to zero, *i.e.* it acts like a short-circuit (see Art. 18). Thus the reflected wave front has the original magnitudes of current and voltage, but the latter is reversed, *i.e.* it is the converse of the case of the reactor (Art. 20). As the condenser charge and voltage rise, so do the values of v_T and i_T, accompanied by a fall in i_R and v_R.

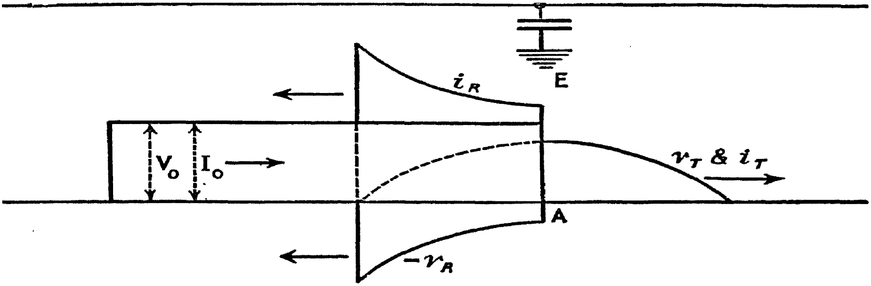


Fig. 12.26.—SURGE PASSING A CONDENSER.

In this case the voltages on each side of A (Fig. 12.26) must be equal, so $V_o - v_R = v_T$, or dividing by Z, $I_o - i_R = i_T$, whence $i_R = I_o - i_T$.

The current flowing into the condenser, C_A, is $(I_0 + i_R) - i_T$, *i.e.* $2\,(I_o - i_T)$, and this is equal to $C_A\frac{dv_T}{dt}$, *i.e.* $C_A Z\frac{di_T}{dt}$. Equating these gives $C_A Z\,\frac{di_T}{dt} + 2\,i_T = 2\,I_o$, whence the final steady value of i_T is I_o, and at time t, it is $i_T = I_o\left(1 - e^{-\frac{2}{C_A Z}t}\right)$. Hence:—

$$i_R = I_o - i_T = I_o e^{-\frac{2}{C_A Z}t}$$

$$\text{and current into condenser} = 2\,I_o e^{-\frac{2}{C_A Z}t}.$$

The last is the same as the charging current of a condenser with a resistance $Z/2$ in series, to which a steady voltage is applied giving an initial charging current $2\,I_o$.

Note that both in this case and in that of the reactor the surge impedances of the two lines act like resistances, in parallel in this case, and in series in the former.

After the whole of the surge has arrived the conditions change. The results of working out the formulae on the same lines as for the reactor gives the following results:—

$$i_T = -\,i_R = I_1 e^{-\frac{2}{C_A Z}t};\quad v_T = v_R = V_1 e^{-\frac{2}{C_A Z}t} = \text{condenser volts};$$

$$\therefore\ \text{Condenser discharge current} = 2\,I_1 e^{-\frac{2}{C_A Z}t};$$

that is, it is the same as that of a condenser charged to a p.d. V_1 when discharging through a resistance equal to $Z/2$.

The shapes of the tails are similar to those for the reactor (see Fig. 12.25), except that for the reflected wave the voltage and current are interchanged.

The advantage of the condenser is that it does not increase the voltage on the incident side as the reactor does. This may cause a flash-over at the nearest pole, and a flash-over is likely to produce fresh surges. On the other hand, for a given flattening of the wave-front a condenser usually costs more.

22. Surge Absorbers

Neither the condenser nor the reactor dissipates any appreciable power. So all the energy of a surge has to be dissipated in line resistance, or in the apparatus reached by the transmitted portion. Hence in some cases resistances are added so as to dissipate some of the energy.

In the case of a condenser the resistance is connected in series with it (cf. Giles valve, Art. 12). If the value of the series resistance is R_A the circuit is equivalent to a condenser with $Z/2$ and R_A in series. Consequently at the first instant the voltage does not drop to zero, but to $R_A/(R_A + \frac{1}{2} Z)$ of V_0. From this it rises towards the value V_0 with a "time constant" equal to $C\ (R_A + \frac{1}{2} Z)$. Thus if b is the reciprocal of this time constant, $v_T/V_0 = 2\ R_A/(2\ R_A + Z) + \{Z/(2\ R_A + Z)\}\{1 - e^{-bt}\}$. Thus part of the steep front is transmitted, and only the rest of it rounded off. The higher the resistance (up to $\frac{1}{2} Z$) the greater the amount of energy absorbed.

Similarly with a reactor in the line the addition of a resistance in parallel gives some absorption of energy. In this case the steep part of the transmitted wave is $2\ Z/(R + 2\ Z)$ of the height of the incident wave: so that the lower the resistance the greater the transmitted steep part, and (down to $R_A = 2\ Z$) the greater the amount of energy absorbed.

So in both cases a compromise has to be made to obtain both energy absorption and front-flattening to a reasonable extent.

The Ferranti Surge Absorber consists of an inductive coil surrounded by a metal sheath insulated from it and from earth. When a surge produces a rapid change of current in the coil this induces corresponding currents in the sheath by transformer action. These produce heat in the sheath and so dissipate some of the energy.

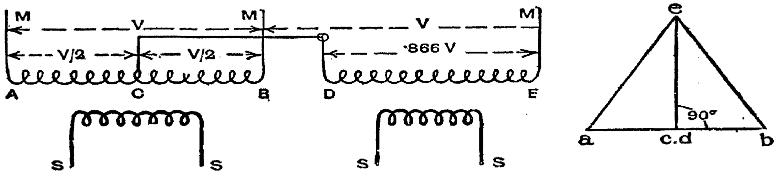
23. Phase Transformation

It is possible by the use of transformers connected to a polyphase supply to obtain power with a different number of phases, or monophase power.

One of the most usual of such arrangements is the Scott three-phase to two-phase transformation (patented in 1894) shown in Fig. 12.27.

Two transformers are used: one primary, AB, has a tapping at its middle point, C, and is connected across two of the three-phase mains. The second primary, DE, has ·866 (= $\sqrt{3}/2$) times as many turns as AB, and is connected between the third main and the centre tapping. The two secondaries have equal numbers of turns.

The vector diagram shows that these secondaries give voltages differing in phase by 90°; and that the greater ratio of secondary volts to primary volts in the second transformer makes the two secondary voltages equal. Moreover, if the two secondary loads are equal in magnitude and in angle of lag (or lead), the same is true for the three three-phase currents.



(*a*) DIAGRAM OF CONNEXIONS. (*b*) VECTOR DIAGRAM.

Fig. 12.27.—SCOTT METHOD FOR THREE-PHASE TO TWO-PHASE TRANSFORMATION.

One application of this is for the supply to electric furnaces each requiring monophase current. The load can thus be balanced with two similar furnaces, instead of with the three necessary with straightforward three-phase transformation.

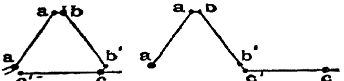
The arrangement can be used the other way round to transform two-phase to three-phase.

The simplest way of obtaining monophase supply from a two-phase or three-phase system is to connect a monophase transformer between two of the high-voltage leads. The drawback of this is that the remaining lead (or leads) carries no current.

An alternative method for three-phase is to use an open △- (*i.e.* V-) connexion with the two secondaries connected in series to the monophase load. This gives equal currents in the three leads apart from the no-load components. With a non-inductive load the current will lead one of the secondary voltages by 60°, and will lag behind the other by the same angle: hence the leakage effects will be opposite in the two transformers. The output is only ·577 of that of the same transformers used with the ordinary V-connexion, and $\frac{1}{3}$ of that of three similar transformers connected in $\triangle$. The output obtained is thus only half of the sum of the transformer ratings. This is evident also from the fact that the total secondary e.m.f. is equal only to that of each separate secondary.

A similar method is to use the Scott connexion (Fig. 12.26) with the secondaries connected in series. Similarly, with a two-phase high voltage supply the secondaries may be put in series for a monophase load. Both these arrangements give ·707 of the output of the same transformers on a two-phase load, and so are superior to the V-connexion for this purpose.

Yet another way for three-phase is to alter the connexions of three $\triangle$-connected secondaries as shown in Fig. 12.28 by disconnecting one secondary (cc'), and connecting one end (c') of the disconnected secondary to the end (b') to which the other was connected before. The voltage between a and c is then double that of any one secondary. The load current is equal to that in each secondary, and so the output is $\frac{2}{3}$ of that of the same transformers connected in $\triangle$*. It is, therefore, better than the V-method in this respect, but requires three transformers as against two (cf. Art. 25). It is not quite so good in output as the Scott arrangement if a transformer of the correct ratio is used in the latter; but if two similar transformers are used, one with an 86·6 per cent. tapping, the output of the Scott arrangement is very slightly inferior (see No. 32, page 158). A further alternative is given in Q. No. 18.



(a) $\triangle$-connected (b) Monophase

Fig. 12.28.—MONOPHASE LOAD ON THREE-PHASE SUPPLY.

None of these arrangements can balance the power since this necessarily fluctuates in a monophase load, while it is constant in a balanced polyphase load (see Vol. I., Chapter VI., Art. 7).

24. Frequency Transformation

This is not in general possible without the employment of rotating machinery. But with a three-phase supply it is possible to obtain with static transformers a secondary voltage having a frequency three times that of the primary supply. This is effected by connecting the secondaries as for delta, but opening the delta at one point, *e.g.* by disconnecting a and c' as in Fig. 12.28*a*.

---

* The primary currents are equal but the secondary currents are not.

The transformers are designed to work at a much higher flux-density than in normal working, and this results in a pronounced third harmonic in the secondary voltages. The secondary voltages of the same frequency as the primary give zero resultant voltage between *a* and *c′*, hence the possibility of using the ordinary △-connexion.

For the triple-frequency components the phase differences of one-third of the original periodic time become a complete period, *i.e.* they are in phase with each other. Consequently a voltage is produced between *a* and *c′* equal to three times the third harmonic of each secondary, and having three times the frequency of the primary supply.

This is advantageous when lighting is required from a low-frequency supply, *e.g.* one at 15 cycles per sec., such as is often used for traction. But the output is much less than normal.

25. Connexions of Transformers on Three-Phase System

The primary windings may be connected either Y or delta, and the same applies to the secondary ones. With either Y — Y or △ — △ connexion the ratio of the voltages is the same as the ratio of the turns. But with

$$\text{Y} - \triangle \text{ the ratio } V_1/V_2 = \sqrt{3}\,\mho_1/\mho_2;$$

and with

$$\text{Y the ratio } V_1/V_2 = \frac{1}{\sqrt{3}}\,\mho_1/\mho_2.$$

Thus the same transformer gives three different ratios, the highest being three times the lowest.

The same applies when three monophase transformers are used on a three-phase supply.

The advantage of the Y-connexion, particularly on high voltage windings, is that the insulation has to withstand only ·577 ($= 1/\sqrt{3}$) of the line pressure. This, together with the smaller number of turns per phase, results in less of the winding space being occupied by insulation. The advantage of the △-connexion is that if one phase breaks down supply can be continued on all three phases of the load up to a total of ·577 of the full output. This method of connexion is sometimes used as the regular one for small amounts of power and is called V-connexion (see Fig. 12.29).

Its disadvantage is that the total capacity of the transformers required is over 15 per cent. greater than with three transformers connected in either Y or △. This is because the transformer current is equal to the line current instead of only $\left(\frac{1}{\sqrt{3}} = \right)$ ·577 of it, as in delta connexion. The advantage is that only two transformers (or two phase-windings) are required instead of three, which effects a saving in cost, particularly for small outputs.

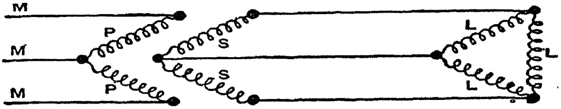


Fig. 12.29.—V-CONNECTED TRANSFORMERS.

M M M. Three-phase mains. P P. Primaries. S S. Secondaries. L L L. Three-phase load.

When transformers are to be connected in parallel on a three-phase system only certain arrangements can be so connected. If all the transformers are connected the same way no difficulty arises. The same applies when some are connected Y — Y and others △ — △ as both sets of secondary e.m.f.s are then opposite in phase to the corresponding primary e.m.f.s. On the other hand it is impossible to connect either a Y — △ or a △ — Y in parallel with either a Y — Y or a △ — △, because the secondary e.m.f.s in the former two are 30° out of phase with the primary e.m.f.s.

A Y — △ and a △ — Y can be connected in parallel with each other, provided that the turns ratios of the two transformers (or of the two sets of monophase transformers) differ in such a way as to give the same secondary line voltages.

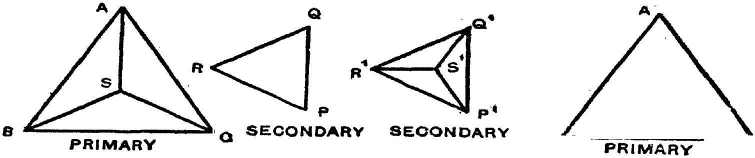


Fig. 12.30.—INTERCONNEXION OF Y — △ AND △ — Y-CONNECTED TRANSFORMERS.

The method of effecting this is illustrated in Fig. 12.30. AB, BC, CA are the line voltages on the primary side, and SA, SB, SC the primary voltages on the transformers in the Y — △-connected set. The secondary voltages on both transformers and lines of this set are given by PQ, QR, RP, in phase respectively with SA, SB, SC. In the $\triangle - Y$ connected set the line and transformer primary voltages are given by A′B′, B′C′, C′A′. The transformer secondary voltages are given by S′P′, S′Q′, S′R′, in phase respectively with A′C′, B′A′, C′B′ (note phase order and direction). The resultant secondary line voltages are therefore given by P′Q′, Q′R′, R′P′, and so the two sets of secondary terminals can be connected in parallel.

Example 7. *Three transformers are connected* $\triangle - Y$ *to a* 3000-*volt three-phase supply and are to deliver* 400 *volts between lines. Find the ratios of numbers of turns in primaries and secondaries, and of the primary line and winding currents to the secondary line currents. (Neglect no-load current effects, etc.)*

P.d. across each primary = 3 000 volts;

$$\text{and p.d. across each secondary winding} = \frac{400}{\sqrt{3}} = 231 \text{ volts.}$$

$$\frac{\text{No. of turns in each primary}}{\text{No. of turns in each secondary}} = \frac{3000}{231} \qquad 13{\cdot}0$$

Secondary line and winding currents are equal

$$\therefore \frac{\text{primary winding current}}{\text{secondary line current}} = \frac{231}{3000} \qquad {\cdot}077$$

$$\therefore \frac{\text{primary line current}}{\text{secondary line current}} = {\cdot}077 \times \sqrt{3} = {\cdot}133$$

This last ratio may be obtained alternatively by equating the input to the output, giving a value of $\frac{400}{3000} = {\cdot}133$.

QUESTIONS ON CHAPTER XII

1. Calculate the inductance and the capacitance per kilometre of single-phase lines with conductor centres 70 cm. apart for various diameters from $\frac{1}{2}$ cm. to $2\frac{1}{2}$ cm. Plot inductance and capacitance against diameter.

2. Calculate the inductance and the capacitance per kilometre of single-phase lines with conductors of $1\frac{1}{2}$ cm. diameter for various spacings of the conductors apart, from 30 cm. to 150 cm. Plot inductance and capacitance against spacing.

3. Show that the difference between the voltages at the transmitting and receiving ends of two overhead wires is given approximately by $I\,(X \sin \phi + R \cos \phi)$, where I is the current, X the reactance, R the resistance, and $\cos \phi$ the power-factor of the load.

4. In what circumstances is a corona produced on an overhead transmission line? What voltage can be used without producing corona on an overhead three-phase transmission line with the wires eight feet apart, each wire being half-an-inch in diameter?

5. Calculate the voltage gradient to start corona for wires of diameters from 0·6 cm. to 2 cm.

Hence determine for each case the spacing to prevent corona with a p d. of 100 kilovolts between the lines.

Plot the spacing against the diameter.

6. What physical, and what commercial, considerations limit the voltage which may be used for the long distance transmission of electrical energy?

Discuss the chief phenomena to be taken into account, and how they are affected by varying external conditions.

7. Assuming that the potential gradient within an insulating medium determines the tendency of the insulation to break down, show that, for a definite breakdown tendency, the thickness of the insulation required between the core and the metal sheath of a cable increases more rapidly than the voltage from core to sheath.

8. A concentric cable 2 miles in length is supplied with A.C. at 5 000 volts, 60 cyc., from an alternator giving a pure sine wave. It is found to take 1·2 A. The diameter of the inner conductor is 8 mm., the thickness of insulation 5 mm., and the insulation resistance may be taken indefinitely large. What is the permittivity (specific inductive capacity) of the dielectric?

9. A string is made up of five insulators, each having a capacitance five times the capacitance of each link to earth. Find the string efficiency.

Explain how the efficiency can be improved by increasing the capacitance of one insulator. If an increase of 50 per cent. is made in the above case, to what value is the string efficiency raised?

10. Describe a suitable form of protection from atmospheric agencies for a three-phase 66 000 volt overhead transmission scheme. Enumerate the causes likely to produce disturbing effects.

11. Explain carefully a good form of electrolytic lightning arrester. Illustrate your answer by a sketch showing approximate dimensions suitable for an assumed case. What are the advantages of this form of lightning arrester as compared with those of others?

12. Explain in detail one form of lightning arrester employed to protect an extra high-voltage plant where the line leaves the station. Give a diagram of connexions, and explain the principles involved.

13. Describe the advantages and disadvantages of (*a*) an air line, (*b*) a three-core cable, for transmitting three-phase power for a distance of 20 miles at a pressure of 20 000 volts. [C. & G., B.

14. How are travelling waves reflected at the end of a transmission line

(*a*) when the line is open-circuited,
(*b*) when the line is short-circuited?

15. An overhead line of surge-impedance 500 ohms feeds a transformer of surge-impedance 2000 ohms and the transition is protected against potential surges by a horn-gap and an ohmic resistance.

Sketch the arrangement and estimate values for the resistance such that the intensity of a steep-fronted potential surge would be reduced by 50 per cent., (*a*) compared with its value without the horn-gap and ohmic resistance; (*b*) compared with its initial value.

16. An overhead transmission-line of 375 ohms surge-impedance is connected to a cable of 75 ohms surge-impedance.

What happens to a high potential surge at the transition point, and what are the effects on these phenomena, when the two lines are connected through a reactor, or when the transition point is connected to earth through a capacitance?

17. Two 100 kVA. transformers, 2 000 volts to 200 volts are used to give L.V. two-phase current from a three-phase supply with 2 000 volts between mains by the Scott connexion. Draw a diagram of connexions and calculate the maximum output.

18. Two 60 kVA. transformers 3 000 V/200 V are V-connected but with one primary reversed. Find the secondary pressure and the maximum output on a monophase load.

CHAPTER XIII

PROTECTION

1. Overcurrent Circuit-Breakers

These circuit-breakers automatically interrupt a circuit if the current exceeds a certain value, and thus serve to prevent damage from overheating. They are also called **overload** or **maximum current** circuit-breakers.

The I.T.E. overload circuit-breaker is shown in Fig. 13.01. The current passes through a solenoid T, called the **tripping coil,** and when it exceeds a certain value pulls up an iron core E. This comes against a movable rod and so knocks out the catch: this is called **tripping** the switch. The switch being thus released is pushed out by a spring and plunger. Circuit is first broken at the main contacts AB, and afterwards at the auxiliary contacts CD. These latter consist of carbon blocks, one pair attached to the moving contact A by springs and copper strips, and the others each similarly attached to the two fixed contacts. Their object is to take the arc produced by interrupting a large current, and so prevent damage to the main contacts.

The iron core E is supported by an adjusting screw K. By raising or lowering it the current at which the circuit-breaker operates can be diminished or increased. Various values of this current are marked on the plate L supporting the screw K. M is a rubber stop and spring grip which stops the moving contact when circuit is broken and prevents it rebounding.

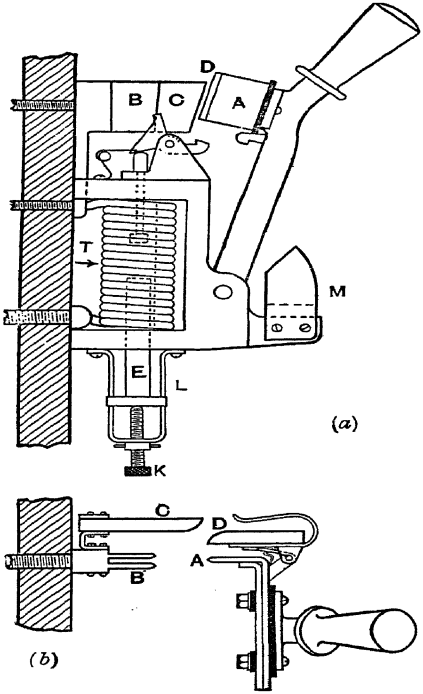


Fig. 13.01.—Overload Circuit-Breaker.

(a) Side view. (b) Plan of contacts.

In both views one fixed contact, the main spring, and part of the moving contact are omitted.

Another type is shown in Fig. 13.02. This has been designed for tramway work (up to 600 volts) or other cases where space is limited. The example shown is suitable for 100 amp. continuously and is of the free handle type (see Art. 2). The working parts are enclosed in a non-magnetic box with a removable iron cover. It is so arranged that the arc formed on breaking circuit is in a strong magnetic field which lengthens it and thus extinguishes it.

In many cases over-current circuit-breakers are actuated by relays. In this case the relay closes a switch when the current exceeds the desired value. This completes the circuit from a battery or other source of current through the "tripping coil" of the main switch, which then breaks the circuit. Alternatively the relay may open a switch which normally short-circuits the tripping coil (see Fig. 13.08). This permits the main current (or a fraction of it) to flow through the tripping coil.

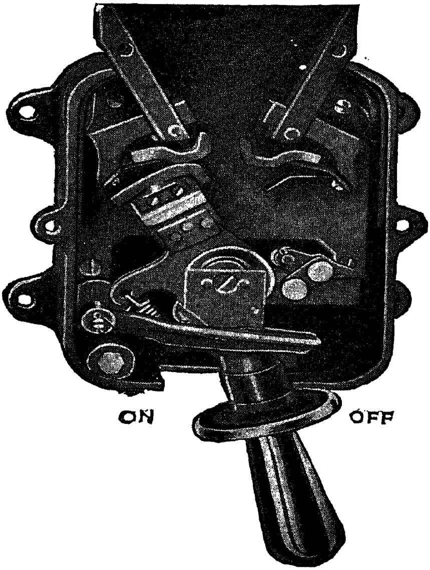


Fig. 13.02.—B.T.H. Circuit-Breaker with Magnetic Blow-Out.

2. Under-Current and Directional Circuit-Breakers

An under-current, or minimum current, circuit-breaker breaks the circuit if the current falls *below* a certain value. A directional one does not break circuit till the current has changed its direction, and sometimes not till it has reached a large value in the reversed direction. Either type may be combined with an overload circuit-breaker.

An under-current circuit-breaker is shown in Fig. 13.03. This breaks the circuit when the current falls below 3 per cent. of its normal value. By winding the coil with a fine wire and connecting it to the bus-bars or in series with the shunt-winding of the dynamo controlled, the switch can be used as an **under-current voltage** one ("no-volt release"). In this case it breaks the circuit when the voltage falls below, say, 10 per cent. of its normal value.

The action is as follows. As long as the current in the coil exceeds a certain value its magnetic action holds up the armature. When it decreases below this value the armature falls by gravity and knocks out a catch. This releases the upper moving contact of the switch and the lever which grips it to the fixed contacts, and its spring causes it to break the circuit (cf. Fig. 13.04). Auxiliary carbon contacts take the spark which occurs at break.

To close the switch the handle is moved upwards, which opens the circuit at D, D. On moving the handle down again the circuit is completed, but the upper contact is free to break circuit. It is thus impossible to keep the circuit closed when the conditions are wrong. This feature is a valuable one, especially in over-current circuit-breakers. It can be obtained in a variety of other ways, and is called the "free-handle." This term is applied to any device which can open the circuit automatically even if the operating handle is held.

The same type of switch is used as an overload circuit-breaker the "tripping" gear being then actuated by an iron armature attracted by a series-wound magnet (cf. Art. 1).

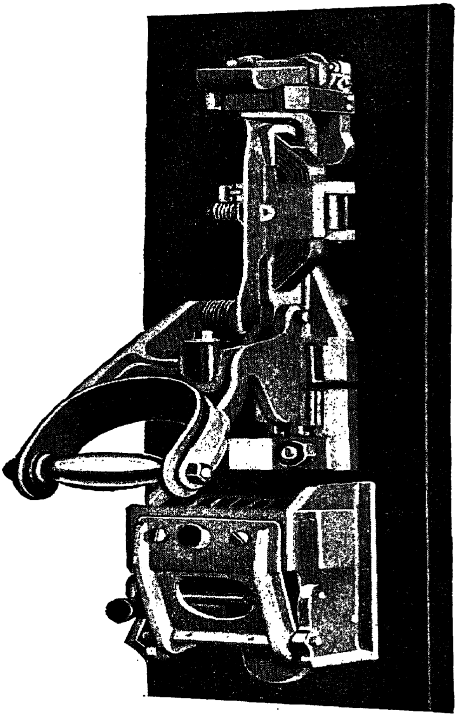


Fig. 13.03.—UNDER-CURRENT CUT-OUT.

A **directional circuit-breaker** breaks the circuit when the direction of the current reverses, and can be made to operate with a reversed current equal to 5 per cent. of the normal current or for any higher value desired. It consists of a modification of the over-current circuit-breaker. The soft-iron core or armature is replaced by a "polarized" one, *i.e.* one which is independently magnetized.

Thus as long as the current is in the normal (or " forward ") direction the armature is repelled instead of being attracted and the switch is held in. On reversal of the current the armature is attracted, and when the reversed current exceeds a certain value the tripping gear is actuated and the circuit broken.

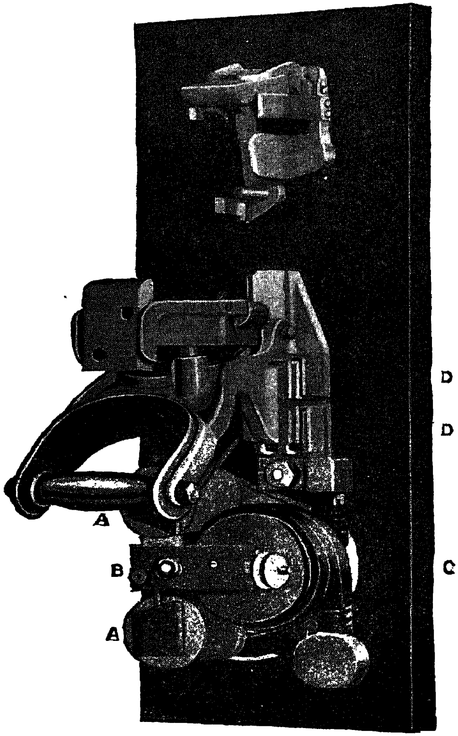


Fig. 13.04.—DIRECTIONAL CUT-OUT.

The " polarization " is effected by using a shunt coil connected across the mains to magnetize the armature. Thus if the polarity of the mains is reversed so is that of the armature: but the " forward " direction of the current is reversed at the same time. Consequently the circuit-breaker will do the work required of it without any change of connexions.

Fig. 13.04 shows a circuit-breaker of this type. A A are the coils which polarize the armature B. C is the series coil which repels B normally, but attracts it when the current in C reverses. When B is attracted it trips the switch in the same way as in the case of the under-current circuit-breaker described above.

Relays (see Art. 1) are frequently used for actuating directional circuit-breakers. The use of these relays has the special advantage that it enables the circuit-breaker to operate with a smaller reverse current.

3. Uses of Circuit-Breakers

Under-current circuit-breakers are used mainly in charging storage cells, since prolonged charging at a small current may harm them.

Directional circuit-breakers are used in central stations with a number of generators in parallel. Their use prevents a generator which has gone wrong from taking power from the others and thus adding to the load on the system. They are useful also in a sub-station supplied by two or more feeders in parallel. Suppose a fault develops on one of two feeders supplying a sub-station (see Fig. 13.05, in which the —ve feeders are omitted for clearness). The effect will be to open the over-current circuit-breaker on the faulty feeder at the central station. Unless directional circuit-breakers are used current will still flow to the fault from the sub-station. This will overload the sound feeder and open its circuit-breaker, thus interrupting the supply unnecessarily.

Over-current circuit-breakers are used on the feeders going out from a central station. Their use on the generator circuits is inadvisable, since the opening of one would increase the load on the generators still connected, and so cause their circuit-breakers to operate in turn. They are sometimes used on large motors instead of fitting the starter with an overload release.

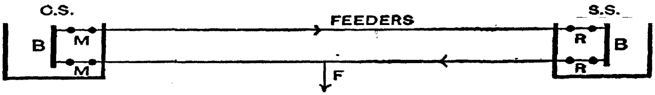


Fig. 13.05.—POSITIONS OF CIRCUIT-BREAKERS.

C.S. Central station. S S. Sub-station. B B. Bus-bars (+ve). F. Fault.
M M. "Maximum" cut-outs. R R. Directional cut-outs.

4. A.C. Over-Current Circuit-Breakers

These circuit-breakers for A.C. work may be of the same type as for D.C., viz. coil and core (see Art. 1). The only difference is that the core should be laminated in planes parallel to its axis, *e.g.* by being built up of iron wire. This reduces the eddy-currents therein, which heat the core and reduce the pull.

Separate calibration at the working frequency is necessary in any case for additional reasons. The *maximum* pull with A.C. is nearly double that with a steady current equal to the R.M.S. value, and the pull with a given current increases as the core rises. Hence a R.M.S. current smaller than the steady value may actuate the breaker. On the other hand if saturation of the core occurs, the mean pull with A.C. may be less than with an equivalent steady current.

Another type which is applicable only to A.C. uses a "shaded-pole" motor (Chapter VII., Art. 10). The coil carries a current proportional to the main current and the torque exerted on the copper or aluminium disc is opposed by a weight (Fig. 13.06). When the current exceeds the desired value the weight is lifted, and when it has risen to a certain point it closes a pair of contacts. These are in circuit with a battery, which then sends a current through the tripping coil which opens the main switch.

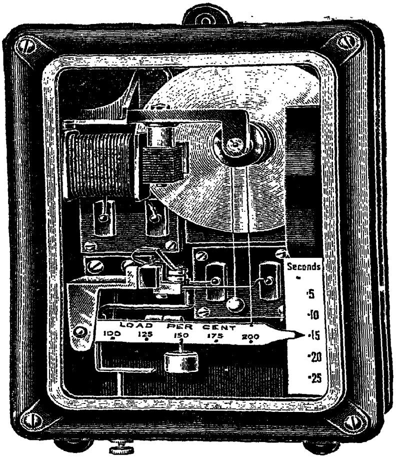


Fig. 13.06.—Nalder A.C. Over-Current Relay.

The load at which the breaker operates can be adjusted by altering the weight, or (as in the pattern shown) by altering the leverage at which the weight acts. The length of time between the commencement of the overload and the opening of the main switch can be adjusted by altering the distance the weight rises before closing the contacts. The heavier the overload the more rapidly the weight is raised against the opposing drag of the (permanent) damping magnet, and consequently the shorter the delay in opening the main switch. In other words, this relay has an adjustable **inverse time lag.**

This is illustrated in Fig. 13.07 which shows the results obtained with a similar relay made by Ferranti, Ltd. The left-hand set of curves are for a weight corresponding with an over-current of 25 per cent. The top one (A) gives the time-lags plotted against over-current with the longest distance of travel for the weight, the bottom one (C) the values with the shortest distance. Nine intermediate settings are possible of which one (B) is shown. When a larger weight is used the minimum over-current to actuate the relay is increased, but, as shown by the right-hand set of curves (*a*, *b*, *c*), the variations in the time-lag remain the same.

The advantage of an inverse time-lag is that it allows time for the fault to be cleared before the circuit is broken: and the less the

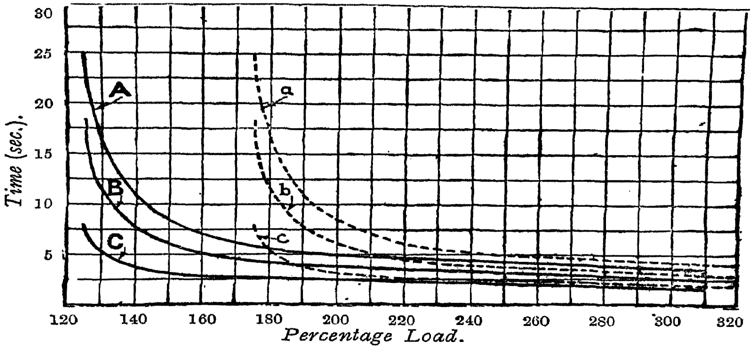


Fig. 13.07.—TIME LAGS FOR DIFFERENT SETTINGS.

over-current the longer the time-lag, which corresponds with the smaller likelihood of damage to the system. The adjustable nature of the time-lag permits of the use of such breakers in series with others controlling branch circuits set to shorter lags. Thus a fault on a branch actuates its breaker before the main breaker has had time to finish its travel. On the opening of the branch breaker the main-circuit relay returns to its initial position and the branch circuit alone is cut off.

Another time-lag device is to use a magnetic relay with a fuse in parallel with the tripping coil (Fig. 13.08). When the current increases sufficiently to attract the armature it opens the trip contacts. This puts the fuse and tripping coil into the circuit in parallel. Unless the over-current is a very heavy one the shunting effect of the fuse reduces the current through the tripping coil so much that it does not act until the fuse has melted. This provides the desired inverse time-lag, as the larger the current the more quickly does the fuse melt. Above a certain over-current the action becomes

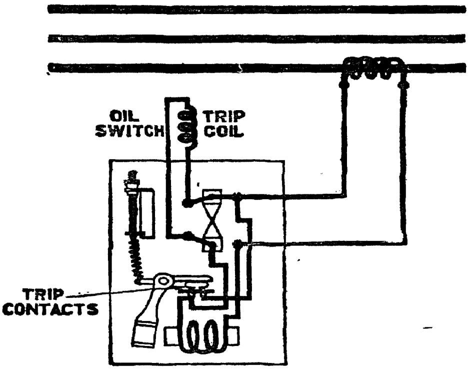


Fig. 13.08.—NALDER FUSED CIRCUIT-BREAKER.

"instantaneous," *i.e.* the circuit is broken as quickly as the magnetic relay and the tripping coil can act. The amount of lag can be altered by changing the size or the material of the fuse-wire: this alters also the over-current above which instantaneous action occurs.

The same relay can be used in conjunction with a battery for actuating the tripping coil. The magnetic device is then arranged to *close* the battery circuit through the tripping coil and fuse in parallel.

5. A.C. Directional Circuit-Breakers

Directional circuit-breakers are used in A.C. systems in the same way as in D.C. systems (see Art. 3), viz. for protecting alternators (or generating stations) connected in parallel, and at sub-stations supplied by two or more feeders in parallel. They are desirable, too, on transformer secondaries, rotary convertors, or motor generators, when any of these are in parallel.

In D.C. systems the meaning of " reverse " current is obvious. But since an alternating current reverses every half-period, the meaning of this type of protection is not quite so evident. The essential thing is the direction of flow of the mean power. For example, the current in an alternator becomes " reverse " when the machine absorbs more power in a cycle than it gives out. In other words, alternating current becomes " reverse " when it is more than 90° out of phase with the voltage. Hence one of the terms " reverse power " or " directional " is preferable to "reverse current."

Consequently the most straightforward method for an A.C. directional relay is to use a wattmeter movement. This may be of either the dynamometer or the induction type. In either case it is arranged so that as long as the mean flow of power is in the desired direction the moving part is pressed against a fixed stop. When the flow of power reverses, the moving part is moved away from the stop and closes contacts which actuate the tripping coil in the usual way.

The drawbacks of the wattmeter relay are that it is rather expensive, and that a reduction in the supply pressure increases the current required for operation at least in inverse proportion. This latter drawback can be diminished by the use of a voltage-fed auxiliary coil opposing the m.m.f. of the current coil (see Example 1).

A simpler but less satisfactory method is to use a **differential solenoid**, *i.e.* one with both series and shunt windings connected so as to oppose each other's magnetic effect. The weight of the core is such that the voltage coil alone cannot raise it. But when a reverse current occurs the two coils assist each other and the cut-out acts.

The defect of this method is that a heavy " forward " current will swamp the effect of the voltage coil and actuate the cut-out. This is overcome by the use of a **compound differential solenoid.** The principle of this is that one set of coils is wound as before and another cumulatively, *i.e.* with the voltage and current coils assisting normally. The magnetic pulls act on separate cores, or on different parts of the same core, and are opposed. As long as the current is " forward " the cumulative set of coils exerts the larger pull. When the current becomes " reverse " the differential set becomes the stronger and the relay is actuated.

This can be illustrated as follows: Let aV be the ampere-turns of each of the voltage coils, and bI those of each current coil (taking only the power component of the current). Neglecting saturation, the magnetic pull varies as the square of the ampere-turns. Thus the pull of the cumulative coils is proportional to $(aV + bI)^2$, and that of the differential coils to $(aV - bI)^2$. The net pull is the difference of these, and so is proportional to abVI, *i.e.* to the power. Therefore when the power reverses so does the pull.

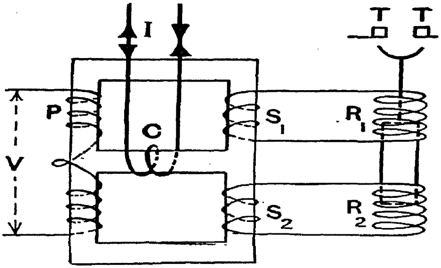


Fig. 13.09.—COMPOUND DIFFERENTIAL SOLENOID.

The method of applying this principle is shown in Fig. 13.09. A special transformer is used with both pressure (P) and current (C) primaries. It has two secondaries arranged so that with current and voltage in phase the two m.m.f.s assist in the lower secondary winding (S_2), and oppose in the upper secondary (S_1). When the phase of the current is reversed the opposite is the case. The current in R_1 then exceeds that in R_2, and the core is raised, closing the trip circuit at the terminals T, T.

Example 1. *A simple dynamometer type directional relay is converted into a compensated one by trebling the number of turns in its main coil, and adding an auxiliary coil which restores its normal operating current to the original value. Find the percentage operating currents of the original and compensated relays for various values of the supply pressure.*

In the original relay the operating torque is proportional to VI, where I = power component of the current. Hence the operating current varies inversely as the voltage: or $i = \frac{100}{v}$ per cent. of normal; where v = percentage voltage.

In the compensated relay the m.m.f. of the auxiliary coil at normal voltage must be twice the original operating m.m.f. The percentage combined field is thus equal to $(3i - 2v)$ where i and v are percentage values as before.

Since the moving-coil current is proportional to v, the torque is proportional to $v\,(3i - 2v)$. With v and i each 100 the torque is proportional to 10 000, so for operation at any voltage $3vi - 2v^2 = 10\,000$. Whence the values of i for various voltages can be found:—

$v =$	100;	80;	60;	40;	30;	25;	20
$2v^2 + 10\,000 =$	30 000;	22 800;	17 200;	13 200;	11 800;	11 350;	10 800
$3v =$	300;	240;	180;	120;	90;	75;	60
$i =$	100;	95;	96;	110;	131;	151;	180
$i_0 =$	100;	125;	167;	250;	333;	400;	500

The values of i_0, given in the last line, are the operating currents for the simple type.

6. Earthing Resistance

If a system is not permanently earthed at any point the pressures between the various lines and earth depend on their insulation resistances and capacitances to earth. Every line, therefore, must be insulated from earth so as to withstand the maximum voltage between this line and any other line. This increases the cost of the insulation with most systems of transmission (see Volume I., Chapter XVIII., Art. 11).

The advantage of not earthing is that it is possible to continue the supply even if a serious fault to earth develops. The advantage is rather illusory, especially if the fault is of variable resistance, *e.g.* an arcing " ground " (*i.e.* earth fault). For the changes of potential differences between the lines and earth cause capacitance charging currents to flow. These surge along the lines, and are likely to cause further breakdowns in the insulation (see Art. 7).

On the other hand, a direct connexion to earth throws a great strain on the overload circuit breakers if a " dead earth " occurs. It is, therefore, necessary to earth through a resistance if the circuit-breakers cannot deal safely with the maximum short-circuit current. The value of this resistance is chosen so that it limits the current to an amount with which the circuit breakers can deal safely on a dead earth, but does not prevent them acting if a serious fault occurs, short of a complete breakdown of insulation.

There is some advantage in making the resistance of carbon or other material with a negative temperature coefficient. The initial short-circuit current of an alternator is much larger than the steady value because armature reaction takes a little time to weaken the field. Hence for the first few cycles after the earth fault occurs the current is very large, and then it diminishes as the alternator field is weakened. The high resistance of the carbon resistance when cold reduces this initial rush. A more important advantage, due to the same cause, is that on a partial breakdown the action of the circuit-breakers is delayed, giving the fault time to clear itself. If it does not do so the reduction of the carbon's resistance as its temperature rises allows the current to increase and actuate the breakers.

7. Petersen Coil

The advantage of being able to continue supply with an earth fault on one line, and without the dangers due to arcing grounds, can be obtained by using one of these coils. They are highly reactive coils, connected between the star-point of a 3-phase supply and earth.

Such a coil is shown at P in Fig. 13.10, in which the capacitances to earth of the three lines (R, W, B) are represented by three condensers, C, C, C. Normally the star-point of the transformer secondaries (or other source of supply) is at earth potential, there is

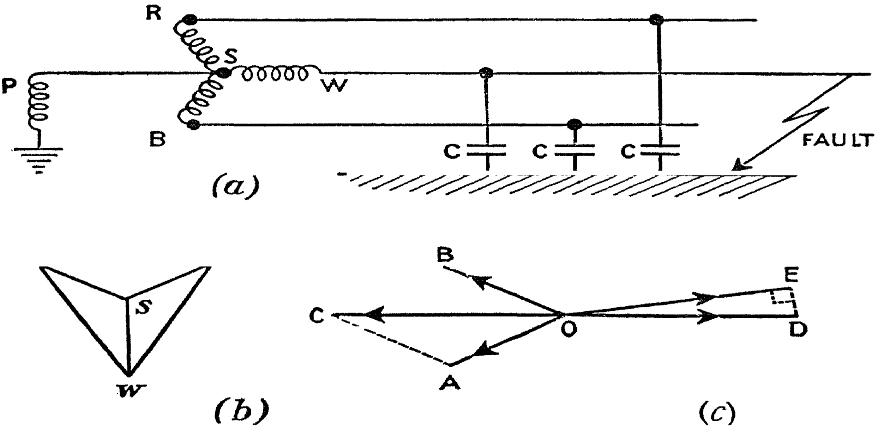


Fig. 13.10.—PETERSEN COIL.

no current in the Petersen coil, and the three earth capacitance currents give zero resultant.

When an earth fault occurs on line W, as indicated, its potential is brought to that of earth. The voltages across the other two earth capacitances become *bw* and *rw* in Fig. 13.10 (*b*), and the currents to earth through them ωCV, as shown by OA and OB in Fig. 13.10 (*c*); where $V =$ line voltage. Since these currents lead the corresponding voltages by 90°, $\angle AOB = 60°$, the resultant (OC) of OA and OB is $\sqrt{3}\ \omega CV$, and OC is perpendicular to *sw*.

If the inductance of the Petersen coil is L, and its resistance is neglected, the voltage across the coil is *sw*, which equals $V/\sqrt{3}$, the current in it is $V/\sqrt{3}\omega L$, and it is represented by OD which lags 90° behind *sw*.

The current flowing from earth through the fault is the vector sum of OC and OD. By adjusting the inductance to suit the line capacitances this sum can be made zero. For this result $\sqrt{3}\ \omega C = 1/\sqrt{3}\ \omega L$, *i.e.* $3\ \omega^2 LC = 1$. This means that the inductance should have the value which gives resonance with the three line capacitances to earth in parallel.

Actually the coil current will lag by less than 90°, like OE in Fig. 13.10. But the resultant of OC and OB can be kept small by " tuning " of the coil. Thus the fault current is kept small and so damage to the line is minimised. Supply can therefore be continued until a suitable opportunity to repair the fault occurs.

In practice it is found that the tuning need not be close for satisfactory operation. The coil is made with an iron-core with an air-gap. The iron-core is necessary to limit the rise of voltage on the coil which may occur during normal working. The coil in series with the three line to earth capacitances in parallel form a circuit resonant at normal frequency, so that a small voltage between earth and the star-point is liable to produce a high voltage on the coil.

Tuning is effected by altering the number of turns in circuit by means of tappings on the coil. If portions of the network are disconnected the reactance of the coil must be increased to maintain tuning.

8. Balanced Protection of Feeders

At any instant the sum of the currents is zero in the conductors of a feeder supplying any portion of the load, *i.e.* the current flowing towards the station is equal to that flowing away from it. This is true whether the supply is monophase or polyphase, and for all methods of connexion and all types of load. But if an earth fault

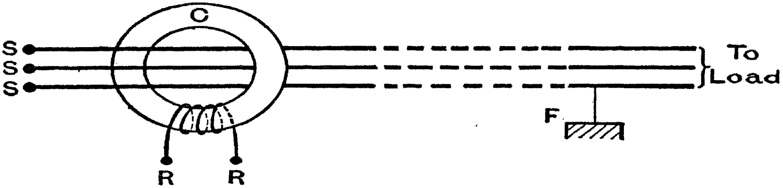


Fig. 13.11.—BALANCED FEEDER PROTECTIVE DEVICE.

C. Iron core. F. Earth fault. R R. Relay connections. S S S. Supply.

develops, part of the current completes its circuit through the earth connexion at the station. The sum of the currents *in the feeder* is then no longer zero, because the earth current has to be added to make the sum zero.

This fact is utilized for disconnecting a feeder when an earth fault develops on it. A laminated iron core (C, Fig. 13.11) surrounds all the cores of the feeder, whether contained in a single cable or in separate ones. A secondary coil wound on this is connected to the tripping coil on the feeder circuit-breaker. As long as there is no earth fault the magnetizing effects of the load currents balance, and no e.m.f. is produced in the secondary. If an earth fault develops, as at F, the fault current, completing its circuit through the station earth connexion, produces a net magnetizing effect. When this is large enough the feeder circuit-breaker is opened.

The advantage of this method is that it can operate before the fault current has increased so much as to operate through the overload coils. This reduces the shock on the circuit-breaker, and on the system, when breaking circuit.

With the simple arrangement shown in Fig. 13.11 there is considerable risk of the leakage fluxes, due to the currents in the separate conductors, actuating the relay. This is avoided in the Ferranti-Field arrangement (Fig. 13.12), by the use of a magnetic shield, *i.e.* a laminated iron ring with no windings on it. Nearly the whole of the leakage fluxes pass through the shield, as shown by *efgh* and the two similar paths. The flux along the path through the outer core, *abcd*, is due to the total current in all the cores, *i.e.* to the fault current.

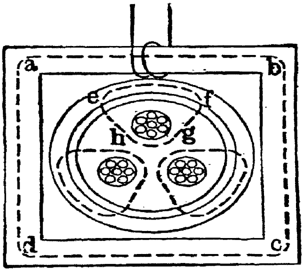


Fig. 13.12.
SHIELDED TRANSFORMER.

The complete core-balancing transformer is about 3 in. square and 18 in. long, and can be assembled round the cable without cutting or disturbing it. If the cable has a lead sheath, or metallic armouring, or both, the earthing of these must be arranged carefully so as not to interfere with the operation of the transformer. The best method is to insulate lightly from earth the cable-box in which the cable is terminated on the bus-bar side of the transformer. The lead sheath and armouring are bonded to this box, and an earthing wire is run from this *through* the transformer to a clamp on the sheath or armouring of the cable on the other side of the transformer. The connexion to the main earth bar is made from this latter clamp. Thus any of the leakage current which flows through the transformer returns through it again, and so does not link with its iron core.

An alternative is to interrupt the sheath and armouring at a point near the transformer (protecting the insulation by a waterproof tape) and to connect the two parts of the sheath and armouring by a bond *outside* the transformer. In this case the leakage current is prevented from passing through the transformer.

Another method is to use in a three-phase system three separate current transformers with their secondaries connected to a common point at each end, with the leakage relay (L, Fig. 13.13) connected between the two common points. The same transformers can be used for the over-current relays, M, M. There need not be more than two of these since the current in the third core is equal and opposite to the sum of the currents in the other two, provided there is no leakage to earth. Under this condition no current flows in L, but when an earth fault occurs the sum of the currents in the feeder cores is equal to the earth currents, and a corresponding current flows in L.

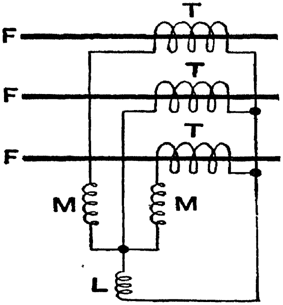


Fig. 13.13.—COMBINED OVERLOAD AND LEAKAGE PROTECTION.

A third method consists in letting each current transformer send a current through one of three coils wound on a common core. The total m.m.f. is then zero until an earth fault occurs. This method can be simplified by using only two windings on the core (Fig. 13.14). Two of the current transformers (T_1, T_2) are connected together, so that the sum of their currents passes through one winding, while that of the third transformer (T_3) passes through the other similar winding. This gives zero total ampere-turns until an earth fault occurs. When this happens the arm is attracted and closes the trip-coil circuit.

In these and other balanced methods of protection there is a risk of the relay acting on heavy loads if the current transformers are not quite alike. To obviate this the relays may be given a " bias," so that the want of balance has to exceed (say) 10 per cent. before the switch can be tripped.

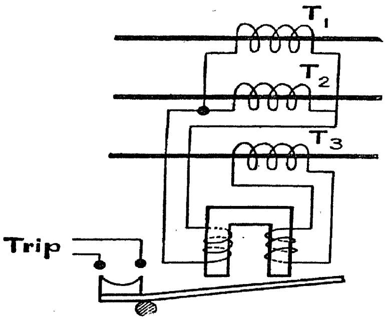


Fig. 13.14.—LEAKAGE RELAY.

Among the methods used for biassing are the following:—

(*a*) A small coil in series with the current transformer attracts a core which tends to hold relay off: in order to actuate the relay the out-of-balance force must overcome this as well as gravity.

(*b*) Each end of a lever is acted on by a coil, one (the operating coil) tending to move the lever, the other (the restraining coil) tending to hold it against a fixed stud. The bias is given by using different numbers of turns or different lengths of leverage or different methods of connexion.

(*c*) Operating and restraining coils with " shaded " poles act on a metal disc. The bias may be given as in (*b*), or by weighting the disc against the operating coil.

(*d*) A doubly-pivoted arm is used (see Fig. 13.15), with balanced coils pulling down at each end. To raise either end and actuate the corresponding trip, the coil at the other end must exert a pull greater than $(b + a)/a$ times the pull of the other coil, where a = length of arm from either core to the nearer pivot, and b = distance between pivots.

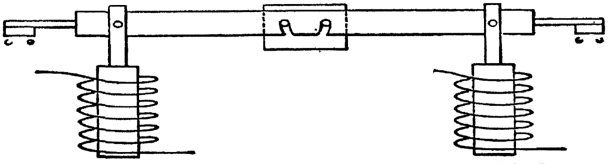


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Fig. 13.15.—MECHANICALLY BIASSED RELAY.

9. Merz-Price System

When two power-stations are interconnected the flow of power may be in either direction. Directional circuit-breakers are not applicable therefore, and the core-balancing system guards against earth faults only. The Merz-Price system with pilot wires affords protection against faults between cores in addition.

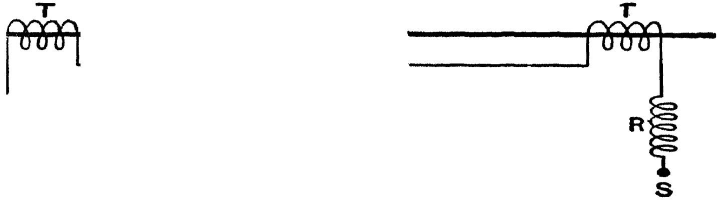

Fig. 13.16.—MERZ-PRICE SYSTEM.

The connexions for one phase are shown in Fig. 13.16, those for the other phases (or the other feeder in a single-phase supply) being exactly the same. Two similar current transformers, T, T, are connected to the line, one at each end. Their secondaries are connected together by the pilot wire, P, and through the relay coils, R, R, to the star points, S, S. The secondaries are in opposition, so that, as long as the currents at the two ends of the line are equal, the secondary e.m.f.s balance and no current is sent through the relays. When a fault occurs, the fault current may be supplied from either end or from both, but is in *opposite* directions at the two ends. Consequently a current is sent through the relays and the interconnector is disconnected at both ends.

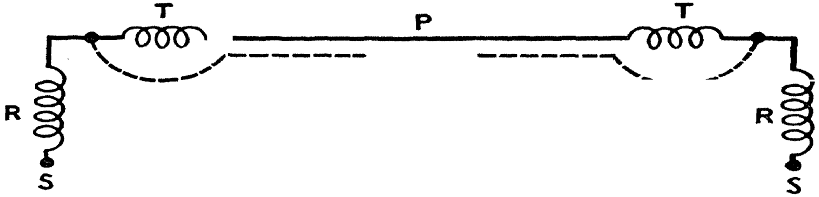

Fig. 13.17.—BEARD-HUNTER SHEATH.

R R. Relay coils. S S. Star points. T T. Secondaries of current transformers.
- - - Metallic sheath. P. Pilot wire.

The current transformers have to be of special design. Their magnetic circuit must be such that the full normal current in the feeder does not cause over-saturation in spite of there being no opposing secondary ampere-turns. This is best effected by using an iron core with an air-gap, as for choking coils. It is advantageous to distribute the air-gap required among a number (say 8) of short gaps, instead of using a single large one. This distribution keeps the flux-density in the iron more nearly uniform, and reduces the fringing.

A drawback of this system is due to the effect of the capacitance of the pilot cables. This capacitance permits currents (opposite in phase) to flow in each relay even when the transformer voltages balance. Consequently the relays have to be set so that the capacitance currents at full feeder load do not actuate the trip coils. Thus the sensitiveness of the protection is reduced, especially when the feeder load is light. But even with full feeder load a similar result is caused by the phase difference between the capacitance and fault currents, in consequence of which a considerable fault current produces much less increase in the total relay current.

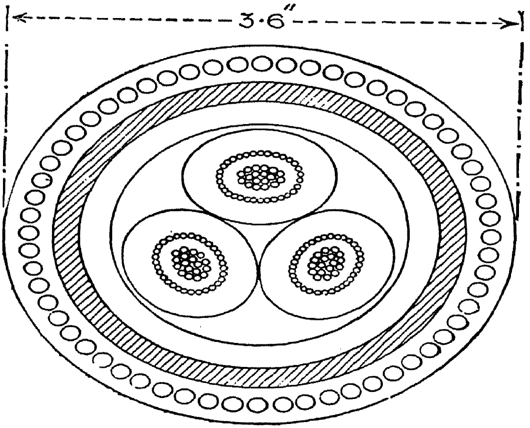

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Fig. 13.18.—THREE-CORE CABLE FOR SPLIT-CONDUCTOR SYSTEM.

The **Beard-Hunter** modification prevents the capacitance current from affecting the relay. Each core of the pilot cable has a metallic sheathing over its insulation. These sheathings are lightly insulated, and breaks are made in them, near the centre of the feeder if convenient. The connexions of one pilot cable are as shown in Fig. 13.17. The capacitance currents flow between the pilot cores and the two portions of the sheath, but do not flow in the relay coils (R, R).

10. The Split-Conductor System

This system does away with the pilot wires of the Merz-Price system (Art. 9) which are a troublesome addition to the line. Each conductor is divided into two of equal size and therefore of equal resistance. These two are insulated from each other, but this insulation need only be sufficient for a low voltage. In overhead lines the two halves of each conductor can be carried on separate insulators, or on the same insulator by providing it with an extra groove on top. In cables a *thin* layer of insulating material is placed between the two (see Fig. 13.18).

Under normal conditions each half of a conductor carries the same current and these induce equal currents in the secondaries of the two similar current transformers, T, T (Fig. 13.19). Consequently no current flows in the actuating coil (R) of the relay. When a fault occurs, either to earth or between phases, the balance is upset. The difference between the two induced currents now flows in R and the relay disconnects the feeder.

The connexions at the other end are the same if the supply may come from either end. In this case it is advisable to continue the split right up to the switch contacts (S) so that the halves of the conductors are disconnected when the switch opens. Otherwise if a fault occurs near one end, though the switch at that end will be opened, the one at the far end may remain closed. This is because the impedances of the two paths to the fault are in this case so nearly equal that the difference between the currents is insufficient to actuate the relay. With split contacts the whole fault current must travel by the faulty half-conductor.

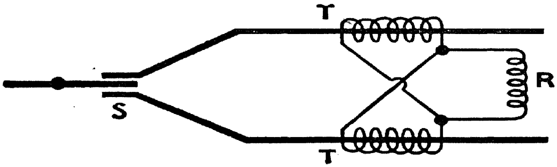

Fig. 13.19.—SPLIT-CONDUCTOR CONNEXIONS.

With two parallel feeders of the ordinary type (not split) supplied only at one end the same connexions can be used at the other end, except that two separate directional relays take the place of R in Fig. 13.19. If an earth or phase fault occurs on one the balance of currents is upset, and one of the relays is actuated and the faulty feeder disconnected. The sound feeder remains connected because the current in its directional relay is in the non-operating direction. If the two feeders are of unequal impedance the transformers are given different turns-ratios.

The advantage of this method compared with the straightforward directional protection is that it acts even if the fault resistance is so high that the current in the faulty feeder is not " reversed " at the sub-station end.

11. Transformer and Alternator Protection

Transformers in parallel should be protected by overload relays on the primary side, and by directional relays on the secondary side.

An alternative is to use current transformers on both sides choosing their ratios so that the secondaries carry approximately equal currents, opposite in phase, under normal conditions. The sum of the two secondary currents is sent through the relay coils, and being normally very small is not sufficient to actuate the relays. If a fault occurs the balance is upset and the transformer is disconnected on both sides.

In this method it is necessary to guard against the effect of the primary current rush which occurs as a rule when a transformer is switched into circuit. This can be done either by giving the relays sufficient time-lag, *e.g.* by a fuse connected in parallel, or by fitting a spring push-button to interrupt the trip-coil circuit temporarily when switching on.

An alternative is the Buchholz relay. It consists of a small tank connected by pipes near its lower end between the transformer tank and its conservator vessel. Normally the small tank is full of oil which holds up a pivoted float inside it. When a fault occurs in the transformer gas-bubbles are formed, and on their way up are trapped in the small tank. The float then drops, and operates a mercury switch in the tripping or alarm circuit.

To make the operation definite a ball is provided inside the float. Normally this is near the pivoted end. When the float moves below the horizontal the ball rolls to the other end and forces the float into its lowest position. Sometimes two floats are used. The upper one operates on light faults and rings an alarm. On persistent or heavy faults the lower float operates and trips the circuit-breaker.

Alternators for running in parallel are protected against loss of field or of driving power by directional relays. Protection against faults to earth can be given by the core-balancing system (Art. 8, Fig. 13.11) provided the connexion to the star-point, as well as the main leads, passes through the balancing transformer. They can be given further protection by balancing the currents at the two ends of each phase so as to detect a leakage to earth or between phases. This may be done by means of two separate current transformers, the difference between the secondary currents passing through the relay coil.

This is effected by connexions similar to those for split-conductor protection (Fig. 13.19), except that the secondaries are straight-connected instead of cross-connected, because the currents to the two ends of a phase are equal but opposite. This method is called the " circulating current " method. Alternatively the transformer secondaries and the relay coil may be connected all in series. The directions are selected so that normally the two transformer secondary e.m.f.s are opposed, as in the Merz-Price system for interconnectors (Art. 9).

Another alternative is Beard's system in which (see Fig. 13.20) three core-balancing transformers (cf. Art. 8) are used, one for each phase. The connexions to the star-point are brought through these in the reverse direction to the leads to the starting points of the phases. Normally the resultant m.m.f. in each transformer core is zero. When a fault occurs, either to earth or between phases, the difference between the currents in the two ends of the same winding produces a flux which induces an e.m.f. in the secondary of the transformer. This secondary is connected to the relay, and so the alternator is disconnected. The same method can be applied to transformers.

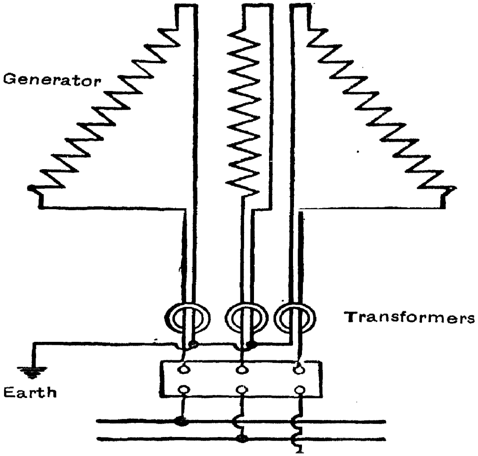

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Fig. 13.20.—BEARD'S ALTERNATOR PROTECTION SYSTEM.

A fault between turns of the same phase will not actuate the relays in the above system. The Mid-point protective gear, due to Kuyser and Beard* acts for faults of this sort as well as between phases or to earth. A reactance coil (H, I, or J, Fig. 13.21) is connected across each phase to the star-point. Current transformers (C_1, C_2, C_3) are connected between the mid-point (E, F, or G) of each phase and the mid-point (M, N, or O) of the corresponding reactance coils. The secondaries of these transformers are connected to the relay coils (d_1, d_2, d_3).

* "Protective Apparatus for Turbo-generators" (J. A. Kuyser), *J.I.E.E.*, vol. 60, p. 761.

Normally the corresponding mid-points, *e.g.* E and M, are at the same potential. When a fault occurs between turns, *e.g.* K and L, the voltage on that half (AE) is lowered to a greater extent than on the other half (ED). But the potential of M remains mid-way between those of A and D. Hence a current flows between E and M and actuates the relay.

A fault to earth or between phases likewise upsets the equality of the potentials across the two halves of a phase and so causes the relay to act.

To extend the protection to the generator leads these must each have a metallic shield (cf. Art. 12) insulated for half the phase voltage both from the conductors and from the lead sheath. These intermediate shields are connected to the mid-points of the phases at the alternator end, and are connected to the mid-points of the reactances at the switchboard end. A fault on the cables will then operate the gear, by causing current to flow in C_1, C_2, or C_3.

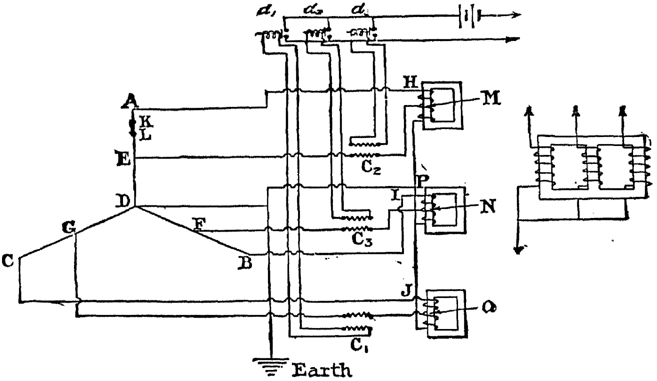

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Fig. 13.21.—MID-POINT PROTECTIVE SYSTEM.

12. Special Cables

Cables containing metallic shields, in addition to the conductors, have been devised for a variety of purposes. The simplest is that due to C. Beaver which contains a test sheath. This is a copper tape completely surrounding but insulated from the conductors. It is insulated to a smaller extent from the lead sheath so as to give an insulation resistance of about 50 megohm-miles. This does not increase the total thickness of insulation necessary. Its advantages are that the resistance of the outer part of the insulation can be measured while power is being transmitted; and that incipient breakdown of the insulation can be detected, *e.g.* by using an electrostatic voltmeter as an indicator. The cable can thus be removed from service and repaired before the fault has developed into a complete breakdown.

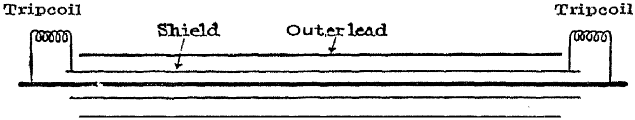

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Fig. 13.22.—WHITAKER'S CABLE PROTECTION SYSTEM.

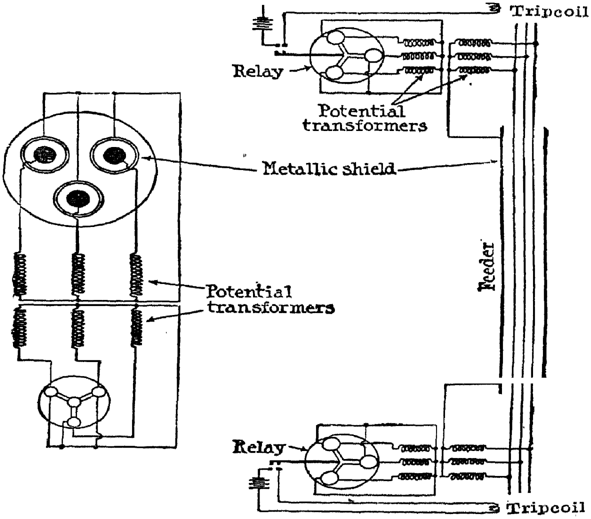

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Fig. 13.23.—BERGMANN CABLE PROTECTION SYSTEM.

E. G. Waters utilises this construction to disconnect the cable by connecting the trip coils between the shield and the lead sheath.

Hochstadter uses the same scheme with separate shields round each conductor. This has the advantage of protecting against faults between phases, as well as against faults to earth.

Whitakèr uses a similar device with the trip coils connected between the shields and the *cores* (see Fig. 13.22). In this case the insulation between the core and the shield is thin, while that between the shield and the lead sheath must be suitable for the full pressure. The trip coils must be insulated for the full pressure to earth.

The Bergmann Co. use potential transformers connected between the cores and the shields (see Fig. 13.23). The secondaries exert pulls on a three-arm relay, arranged so that the pulls balance when the voltages are equal. This is effected by using a central universal pivot and making the pulls act at right angles to the plane of the arms. When leakage occurs the voltages on the one or two faulty cores are altered, and the arms are pulled out of the horizontal position. The vertical contact arm is thus caused to connect together two ring-shaped contacts, which closes the trip circuit.

13. Bus-Bar Reactors

As mentioned in Chapter XII., Art. 13, bus-bars are sometimes sectionalised by being connected through reactors. The result of this is to reduce the current caused by a fault, since only the alternators on the faulty section feed directly into the fault. This does not constitute direct protection but reduces the kVA. to be dealt with by the circuit-breaker concerned, and so is an important part of any protective system.

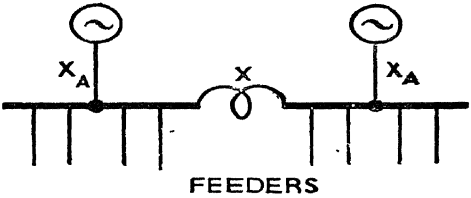


Fig. 13.24.—BUS-BAR REACTOR.

Reactors are usually specified as having a certain percentage reactance referred to a definite kVA. basis. The basis may be the full load of one alternator or any other convenient one. In any case the percentage is the voltage absorbed by the reactor when carrying the full load, expressed as a percentage of the normal voltage. Note that if the basis is altered the percentage value of a given reactor is altered in direct (not inverse) proportion.

The simplest case is that of two similar alternators of the same size connected to a bus-bar in two sections with a reactor between, as shown in single-line diagram in Fig. 13.24. Let X_A be the percentage reactance of each alternator (referred as usual to its own full-load), and X the percentage reactance of the bus-bar reactor referred to the same basis. Then if a complete (3-phase) short-circuit occurs close to one bus-bar the current fed into it by the corresponding alternator is $100/X_A$ times full load. But the other delivers to the fault only $100/(X_A + X)$.

So the total kVA. fed into the fault is $100\,\{1/X_A + 1/(X_A + X)\}$, *i.e.* $100\,(2\,X_A + X)/\{X_A\,(X_A + X)\}$; *e.g.* if $X_A = 10$, then for:—

X	0;	10;	20	30	40 %
$\left(\dfrac{\text{Fault current}}{\text{Full load}}\right) =$	20;	15;	13·3;	12·5;	12·0

14. Ring Connexions of Bus-Bar Reactors

A third section with another similar alternator may be added to the arrangement of Fig. 13.24, with an equal reactor connecting it to one of the original sections (see Fig. 13.25). A short-circuit on the centre section will receive $100/X_A$ from the centre alternator, and $200/(X_A + X)$ from the other two combined. But a short-circuit on C in addition to the direct feed of $100/X_A$ receives less from the other two. For the latter feed to B through $X_A + X$ in parallel with X_A. and thence through X to C. So the total reactance for this part is $X_A\,(X_A + X)/(2\,X_A + X) + X$, which is obviously greater than $(X_A + X)/2$, the value in the other case.

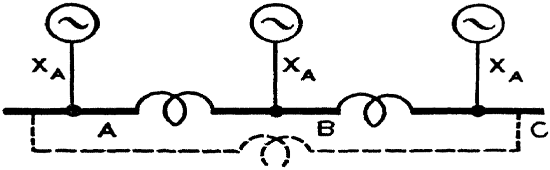


Fig. 13.25.—RING-CONNECTED REACTORS.

Taking $X_A = 10$ per cent., the direct feed is 10 times full load. If $X = 20$ per cent.; to a fault on centre section each of the outer alternators supplies $100/(10 + 20)$, *i.e.* $3\frac{1}{3}$ times full load, so the total fault current is $16\frac{2}{3}$ times full load. For a fault on C the reactance from the first two alternators is $10 \times 30/40 + 20$, *i.e.* $27\frac{1}{2}$, so that they supply to the fault $100/27\frac{1}{2}$, *i.e.* 3.64. Thus the total is only 13.64 times full load. The same value applies to a fault on A.

This want of symmetry can be overcome by using a third sectionalising reactor of the same value, connected between A and C as shown by the broken lines in Fig. 13.25. The effect is to bring the value of the fault current for faults on A or C up to the value for one on B. The increased cost is not in the ratio 3/2, since the maximum current in a reactor is reduced somewhat, from 3·64 to $3\frac{1}{3}$ times full load in the above example.

This latter ring arrangement can be extended to any number of sections, still retaining symmetry. And with more sections the extra cost of the symmetrical arrangement becomes relatively smaller, and without it the possible variation in the short-circuit current becomes larger.

15. Tie-Bar Connexion of Reactors

Instead of the mesh or ring connexion (Art. 14) a star method may be used by providing a tie bus-bar as shown in Fig. 13.26. With three sections the indirect feed reactance is $\frac{1}{2}(X_A + X) + X$, *i.e.* $(X_A + 3X)/2$, and so their feed to the fault is $200/(X_A + 3X)$. Thus with $X_A = 15$ per cent., the following relations hold:—

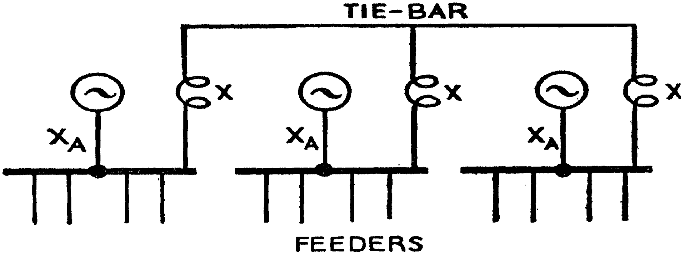


Fig. 13.26.—REACTORS CONNECTED TO TIE-BAR.

$$\begin{array}{lcllll} X & = & 0\ ; & 5\ ; & 10\ ; & 15\ \% \\ \dfrac{\text{Fault current}}{\text{Full load}} & \left\{= \begin{array}{l} (a) \\ (b) \end{array}\right. & \begin{array}{l} 13\frac{1}{3}; \\ 20\ ; \end{array} & \begin{array}{l} 6\frac{2}{3}; \\ 13\frac{1}{3}; \end{array} & \begin{array}{l} 4\frac{4}{9}; \\ 11\frac{1}{9}; \end{array} & \begin{array}{l} 3\frac{1}{3} \\ 10 \end{array} \end{array}$$

(*a*) is the indirect feed to fault; (*b*) is the total.

With this arrangement, too, any number of sections may be used and symmetry maintained. It is somewhat simpler to extend than the ring arrangement, since none of the existing connexions need be changed when this is done.

In most cases the rating of the reactors is less with the tie-bar than with the ring connexion (see Example 2). When a fixed basis is used the relative rating of reactors is given by X multiplied by (maximum current). For X settles the size of the reactor coil, and the maximum current settles the cross-section of the conductors. The latter can be determined from the maximum permissible temperature rise and the time a short-circuit may last, neglecting dissipation of heat since the time is very short (see Example 3).

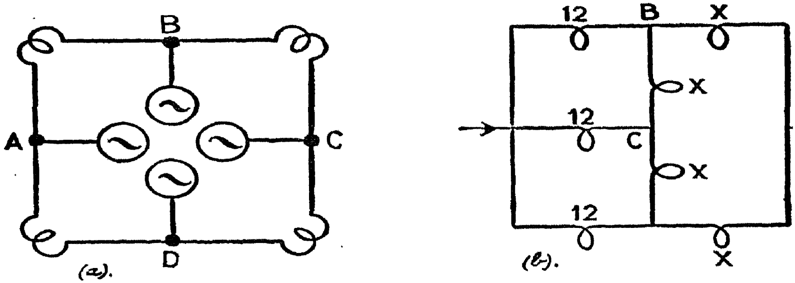


Fig. 13.27.—(*a*) CONNEXIONS, (*b*) EQUIVALENT CIRCUIT.

Example 2. *If 4 similar alternators of 12 per cent. reactance are connected to a sectional bus-bar with ring-connected reactors, find the maximum short-circuit current and the reactor rating if they are* (*a*) 10 *per cent.;* (*b*) 15 *per cent.; referred to one alternator.*

Find the percentage values of tie-bar reactors to give the same short-circuit currents, and their ratings.

(*a*) The direct feed to fault is 8·33 times full load current.

If the fault is on A the feed from the other alternators is as shown in Fig. 13.27 (*b*). Let c be the current to C, which by symmetry goes half to B and half to D; and b = direct feed to B = direct feed to D. Then:—

$$12c + \{\tfrac{1}{2} \times c + (b + \tfrac{1}{2}c)\}\, X = 100$$

which, when X = 10, reduces to

$$22c + 10b = 100 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\text{And } 12b + (b + \tfrac{1}{2}c)\, X = 100$$

$$\text{whence } 22b + 5c = 100 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\therefore\ c = 600/(242 - 25) = 2{\cdot}765$$

$$\text{and } b = 10 - 2{\cdot}2c = 3{\cdot}917.$$

$$\text{Total indirect feed} = 2b + c = 10{\cdot}60;$$

$$\therefore \text{ Total feed} = 18{\cdot}9 \text{ times full load.}$$

Rating of reactor = 10 × 10·60/2 = 53. Since the maximum current in a reactor is half the total indirect feed.

(*b*) When X = 15 the equations become:—

$$27c + 15b = 100; \quad \text{and } 27b + 7{\cdot}5c = 100$$

$$\text{whence } c = 400/(243 - 37{\cdot}5) = 1{\cdot}947;$$

$$\text{and } b = (100 - 27c)/15 = 3{\cdot}162;$$

$$\text{Total indirect feed} = 8{\cdot}27.$$

$$\text{Total feed} = 16{\cdot}6 \text{ times full load.}$$

$$\text{Rating of reactor} = 15 \times 8{\cdot}27/2 = 62.$$

With tie-bar connexion, similar to Fig. 13.26 but with an additional section, the reactance in the indirect feed to the fault is $(12 + X)/3 + X$, *i.e.* $(12 + 4X)/3$. The amount of this feed is therefore $300/(12 + 4X)$. Equating this to 10·60, viz. the indirect feed in (*a*) above, gives X = 4·75 per cent.

The whole of the indirect feed passes through one reactor. Thus rating of reactor $= 4{\cdot}75 \times 10{\cdot}60 = 50$.

Equating the above expression to 8·27, the indirect feed in (*b*), gives $X = 6.07$ per cent.

(Note that X is increased in a lower ratio than 10/15);

$\therefore$ Rating of reactor $= 6{\cdot}07 \times 8{\cdot}27 = 50$. In this case the increased reactance is balanced by the reduction in current.

Example 3. *Find the cross-section of conductor for a reactor which is to carry* 12 000 A. *short-circuit current for* 1·6 *sec. with a temperature rise of* 200° *C.*

The resistivity of copper at the average temperature on short-circuit is 2·4 microhm-cm.

If the current density used is $x \times 10^3$ A. per cm.^{2} the heat produced per sec. per c.c. $= (x \times 10^3)^2 \times 2{\cdot}4/10^6 = 2{\cdot}4x^2$ joules $= 0{\cdot}57x^2$ calories.

$$\text{Heat produced in } 1{\cdot}6 \text{ sec.} = 0{\cdot}92x^2 \text{ calories.}$$

The specific heat of copper = 0·095, and its specific gravity 8·9, so the heat required to raise 1 c.c. 200° C. = 169;

$$\therefore\ 0{\cdot}92x^2 = 169,\ i.e.\ x^2 = 184, \text{ and so } x = 13{\cdot}6.$$

$$\text{Hence current density } 13\,600 \text{ A./cm.}^2;$$

$$\therefore\ \text{Cross-section} = 12/13{\cdot}6 = 0{\cdot}88 \text{ cm}^2.$$

16. Normal Working with Bus-Bar Reactors

Under normal working conditions the load on a bus-bar section may be greater or less than the supply from the alternator connected to it. This entails the passage of the surplus or deficit through one or more of the bus-bar reactors.

With only two sections the conditions are similar to those of two alternators in parallel on load (see Chapter IV., Art. 21) except that an additional resultant voltage is required to drive the current through the reactor. If this current is a purely power one, viz. in phase with the bus-bar voltage, the reactor voltage is a leading one for the alternator which is transferring power to the other bus-bar section. The effect is therefore produced by this alternator's e.m.f. moving ahead of the e.m.f. of the other alternator.

If, however, the transferred current is to contain a lagging component it is necessary to raise the excitation of the first alternator and lower that of the other by suitable amounts. These changes in the e.m.f.s produce in the reactor a current lagging 90° behind the resultant e.m.f., and so lagging approximately 90° behind the bus-bar voltage. In these ways both the power and the reactive components of the load on the second bus-bar section can be supplied by the first alternator.

The division of the total power load between the two is settled by the setting of the governors of the two driving engines, since each alternator must run at synchronous speed.

The division of the total reactive load is settled by the excitations of the alternators. As a rule the most economical arrangement is to adjust these so that the power-factors of the two alternators are equal to each other, and therefore equal to the power-factor of the total load.

17. Short-Circuit Currents

In calculating the currents due to short-circuits near bus-bars (Arts. 15, 16) reactance alone has been taken into account. This is justifiable since the resistance of an alternator is always small compared with its reactance. The same is true to a lower degree of accuracy for transformers. But for a transmission line it may be necessary to include the effect of resistance. It is not always necessary since its importance depends on its ratio to the whole reactance in the circuit; and its neglect makes the calculated current larger than the true value, an error on the safe side.

Three-phase problems can be changed into equivalent ones in the following way. Let V = voltage between lines of a 3-phase supply, I = full load amperes, Z = ohms impedance per phase, including the line, to the point of short-circuit. Then:—

$$\text{Short-circuit current per line} = (V/\sqrt{3}) \div Z.$$

$$\text{Total VA of short-circuit} = \sqrt{3}\,V\,.\,(V/\sqrt{3})/Z = V^2/Z.$$

This is the same as for a single-phase line with the same voltage between lines, but with only one line and one phase of each winding included.

Moreover if Z includes an alternator or transformer of x per cent. reactance referred to P volt-amperes, this can be converted into X ohms as follows:—

$$I = P/\sqrt{3}\,V;\; IX = (x/100)\,V/\sqrt{3};\; \therefore\; X = (x/100)\,V^2/P.$$

And again the single-phase case gives the same result if X = total reactance of the apparatus in ohms.

Short-circuit calculations of all sorts in three-phase circuits, can therefore be reduced to single-phase circuit problems (see Example 4).

Note that though the voltage at the short-circuit falls to zero, the normal voltage is used in calculating the short-circuit kVA. This is because the circuit-breaker has to deal both with the short-circuit current, and with the normal voltage which is restored when the short-circuit is broken. So both these factors affect the size of the circuit-breaker.

Example 4. *A 3-phase generator 15 000 kVA., 12 per cent. reactance, feeds through a line 4 miles long a 5 000 kVA. step-down transformer of 8 per cent. reactance. Generator and transformer have each 1 per cent. resistance, and the line conductors have 0·60 ohm resistance and 0·40 ohm reactance per mile. Line pressure is 20 kV. Find the breaking capacities required by the high voltage and low voltage switches of the transformer.*

Treating this as single-phase.

Full load generator current $= 15\,000/20 = 750$ A.;

$\therefore$ Generator reactance $= (12/100)\ 20\,000/750 = 3{\cdot}20$ omhs.

Generator resistance $= 3{\cdot}20/12 = 0{\cdot}27$ ohm.

Line reactance $= 0{\cdot}40 \times 4 = 1{\cdot}60$ ohms.

Line resistance $= 0{\cdot}60 \times 4 = 2{\cdot}40$ ohms.

Thus impedance to high voltage side of the transformer $= \sqrt{\{(2{\cdot}67)^2 + (4{\cdot}80)^2\}} = 5{\cdot}49$ ohms;

$\therefore$ Short-circuit current $= 20\,000/5{\cdot}49 = 3\,640$ A.:

$\therefore$ Breaking capacity of high pressure switch $= 3\,640 \times 20$ kVA.
$= 72{\cdot}8$ MVA.

Transformer full load current $= 5\,000/20 = 250$ A.

Transformer reactance $= (8/100).20\,000/250 = 6{\cdot}4$ ohms.

Transformer resistance $= 6{\cdot}4/8 = 0{\cdot}8$ ohm;

$\therefore$ Impedance to low voltage side of transformer $= \sqrt{\{(3{\cdot}47)^2 + (11{\cdot}2)^2\}} = 11{\cdot}7$ ohms;

$\therefore$ Short-circuit current $= 20\,000/11{\cdot}7 = 1\,710$ A.;

$\therefore$ Breaking capacity of low pressure switch $= 1\,710 \times 20$ kVA. $= 34{\cdot}2$ MVA.

If it is worked in percentages, referred to the transformer full load of 5 000 kVA, then:—

Generator reactance $= 12 \times 5\,000/15\,000 = 4{\cdot}00$ per cent.

Generator resistance $= 4{\cdot}00/12 = 0{\cdot}33$ per cent.

Line reactance drop $= 250 \times 1{\cdot}60 = 400$ V.;

$\therefore$ Line reactance $= (400/20\,000) \times 100$ per cent. $= 2{\cdot}00$ per cent.

Similarly, line resistance $= 3{\cdot}00$ per cent.;

$\therefore$ Impedance to high pressure side of transformer $= \sqrt{\{(3{\cdot}33)^2 + (6{\cdot}0)^2\}} = 6{\cdot}86$ per cent.;

$\therefore$ Short-circuit current $= 100/6{\cdot}86 = 14{\cdot}56$ times full load;

$\therefore$ Breaking capacity $= 14{\cdot}56 \times 5\,000$ kVA $= 72{\cdot}8$ M.V.A.

Impedance to low voltage side $= \sqrt{\{(4{\cdot}33)^2 + (14{\cdot}0)^2\}} = 14{\cdot}6$ per cent.

Short-circuit current $= 100/14{\cdot}6 = 6{\cdot}84$ times full load.

Breaking capacity of low voltage switch $= 6{\cdot}84 \times 5$ MVA $= 34{\cdot}2$ MVA.

It will be seen that neglecting the resistances of the alternator and transformer would have little effect on the results; but that the line resistance has an appreciable effect, especially for the high voltage switch.

QUESTIONS ON CHAPTER XIII

1. Describe and explain the action of a "shaded-pole" overcurrent relay.

2. Describe, with sketches, one form of inverse time-lag relay circuit-breaker, and state the circumstances under which such an arrangement would be used.

3. Describe some form of time limit device that can be used in connexion with circuit-breakers. Explain how the time limits should be varied on the different parts of the system, in order to minimise disturbance due to short circuits. [C. & G., E.

4. Define "reverse current" in A.C. circuits. Describe the action of some directional circuit-breaker, pointing out its advantages and disadvantages, and giving a diagram of connexions to the circuit.

5. What are the reasons for and against "earthing the neutral" in three-phase high-voltage systems of distributing power? Describe, with a diagram of connexions, the method you would consider best for a 6 600-volt three-phase system. Explain its working, and state your reasons for preferring the method described.

6. State in what positions "over-current" and "directional" circuit-breakers respectively should be placed.

7. Describe, with sketches, some good form of protective device for interconnectors, such as the Merz-Price system, and state its advantages.

8. Describe the split conductor system of protection as applied to a number of sub-stations supplied through a ring main. Explain why the switches should have split contacts.

9. Explain, with sketches of connexions, a current-balancing method of protecting a three-phase alternator against internal faults.

10. Why are reactance coils used in large generating systems? Describe the effects secured by placing them in various places.

Two 1 000 kilovolt-ampere, 6 000-volt, single-phase alternators are separately connected to two groups of feeders taking loads of 1 500 kilovolt-amperes and 500 kilovolt-amperes respectively, each load having a power-factor of 0·8. The two alternators are connected in parallel through a reactance whose impedance is 17 ohms, and whose power-factor is zero. Draw a vector diagram of the currents, and calculate the power-factor at which each alternator is working, assuming the load to be divided equally between the two machines.

CHAPTER XIV

SYMMETRICAL COMPONENTS

1. Symmetrical Components

Since it is much simpler to deal with a balanced three-phase load than with an unbalanced one, a method by which the latter can be replaced by two or more of the former has many advantages. This can be done for any three-phase load by the method of symmetrical components.

This consists in splitting the load up into three components in each of which the currents in the three phases are equal, and either in phase or 120° apart in phase. The three components are referred to by the names "positive sequence," "negative sequence," and "zero sequence."

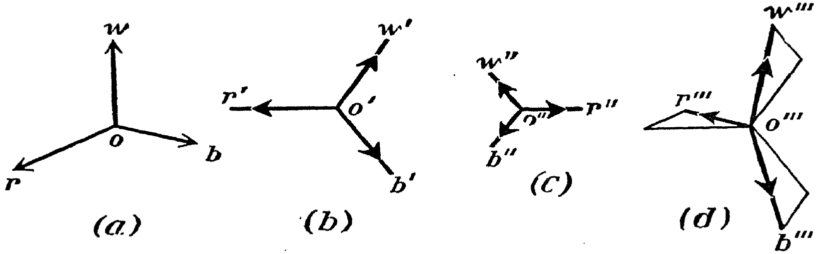


Fig. 14.01.—Positive and Negative Sequence Currents.

(*a*) Voltages. (*b*) + ve sequence currents. (*c*) — ve sequence currents. (*d*) Resultant currents.

Phase sequence means the order in which three lines or terminals reach their maximum potential. So the statement that the (positive) phase sequence is R, W, B means that the potential of R reaches its maximum before that of W, and the potential of B reaches its maximum after that of W; the time intervals being approximately ⅓rd of a period of any of these potentials. Thus the potentials of R, W, B may be obtained from three vectors *or*, *ow*, *ob* [Fig. 14.01 (*a*)] rotating about *o*, but not necessarily of equal length nor at 120°. If the voltages are balanced the vectors are then of equal length and at 120°.

The positive sequence component of the currents is an ordinary balanced load. That is to say it consists of three equal currents, ⅓rd period apart, in the same phase sequence as the voltages, and therefore lagging behind the corresponding voltages (if they are balanced) by equal angles [Fig. 14.01 (*b*)].

The negative sequence component of the currents is another set of three equal currents, $\frac{1}{3}$rd of a period apart, but in the opposite phase sequence to that of the voltages, viz. BWR [see Fig. 14.01 (*c*)].

If these two sets of currents are added, the current in line W is $o'''w'''$ [Fig. 14.01 (*d*)], the vector sum of $o'w'$ and $o''w''$; and the other two resultant currents are obtained in a similar way. Obviously the resultant currents are not balanced.

2. Zero Sequence Currents

In a 3-conductor line the vector sum of the currents is necessarily zero. But in a 4-wire system this condition need not hold for the currents in the three outers. The same is true of a 3-conductor line if earth leakage occurs. In these cases the vector sum of the three main currents is called the *residue*. This residue must return by the fourth wire, or by the earth in the case of leakage.

For the purpose of symmetrical components this residue is considered as being supplied equally by the three outers, *i.e.* three equal currents with no phase differences between them are supposed to flow in the three outers, and to join and form the complete fourth wire return current. They constitute the zero sequence component of any three currents. If these zero sequence currents are taken from the three line currents the vector sum of what is left will contain no residue. Thus to get the zero sequence component of three currents:—Find the vector sum and divide by 3. This gives both the magnitude and phase.

The voltages between three lines can have no residue. For (volts of R above W) + (volts of W above B) + (volts of B above R) must add to zero at every instant. But the voltages to the star-point may contain a residue.

3. Separation of Positive and Negative Sequence Components

It has been shown in Art. 1 that the combination of positive and negative sequence components gives a set of unbalanced currents. The converse problem of obtaining the components of a given set of unbalanced currents can be solved in several ways, of which the following is one. First find the zero sequence component (if any) and remove it* (see Art. 2). Let OA, OB, OC be the currents in the three lines, in positive phase sequence, after removal of the zero sequence component from each, if such exists originally. Draw

* This is not essential, for if zero sequence currents are present they will come 120° apart and so will not affect the results.

DE, equal and parallel to OB (see Fig. 14.02). Draw EF equal to OA (current in leading phase) in a direction 120° behind OA. Draw FG equal to OC (current in lagging phase) in a direction 120° ahead of OC. Join DG. Then ⅓rd of DG gives the positive sequence component in magnitude and phase. That is to say it gives the positive sequence current in phase B, which is accompanied in phase A by an equal current but 120° ahead, and in phase C by an equal current but 120° behind [see Fig. 14.02 (*c*)].

It is evident that if OA, OB, OC were balanced currents, DEFG would be a straight line; and ⅓rd of DG would be equal to OB.

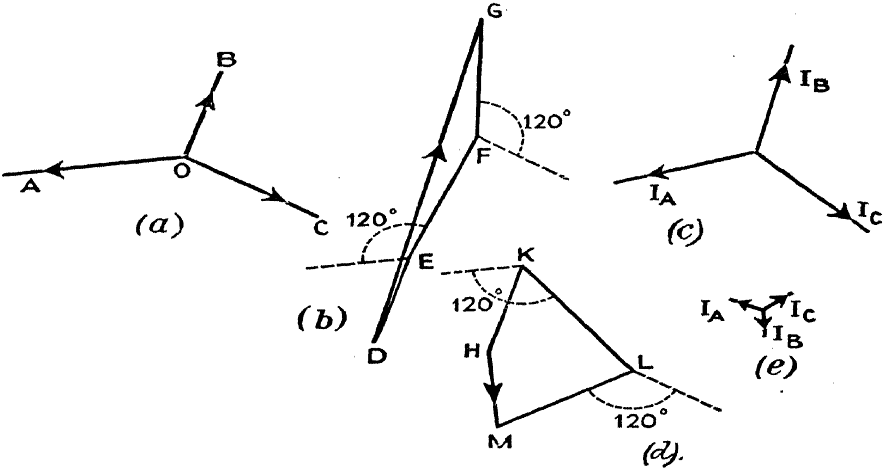


Fig. 14.02.—SEPARATION OF POSITIVE AND NEGATIVE SEQUENCE COMPONENTS.

So in that case the construction would result in obtaining the original currents at the finish. The proof that it applies to the unbalanced case is given later (in Art. 5).

To obtain the negative sequence component. Draw HK [Fig. 14.02 (*d*)] equal and parallel to OB. Draw KL equal to OA in a direction 120° ahead of OA. Draw LM equal to OC in a direction 120° behind OC. Join HM. Then ⅓rd of HM gives the negative sequence component. This consists of a current in phase B equal to HM/3 and parallel to it; accompanied in phase A by an equal current 120° *behind* this, and in phase C by an equal current 120° ahead of it [see Fig. 14.02 (*e*)].

4. The Three-Phase Operator

It is mathematically convenient in three-phase work to use a symbol (λ) to denote the operation of turning a vector through 120°. If this is done three times the vector is restored to its original position, so $\lambda^3 = 1$ (cf. $j^4 = 1$, Chapter I., Art. 2); $\therefore\ \lambda^3 - 1 = 0$; $\therefore\ (\lambda - 1)(\lambda^2 + \lambda + 1) = 0$; $\therefore\ \lambda = 1$, or $\lambda^2 + \lambda + 1 = 0$. The solution of this quadratic equation is

$$-\tfrac{1}{2} \pm \sqrt{-3} \quad i.e.\ -\tfrac{1}{2} \pm j\frac{\sqrt{3}}{2}.$$

If OA in Fig. 14.03 is unity, the other two roots of the equation are OB and OC, obtained by drawing ON, $= \frac{1}{2}$ OA, in opposite direction to OA, and NB and NC, each $= (\sqrt{3}/2)$ OA, perpendicular to OA. The angles AOB and AOC are each 120°. Thus the three cube roots of unity (λ) are three unit vectors, one in the "real" direction, and the others at 120° with this direction.

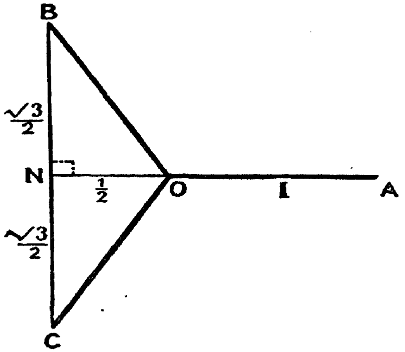


Fig. 14.03.
THREE-PHASE OPERATOR.

In what follows the meaning of λ will be confined to turning a vector through 120° in the positive direction. Thus λ will be equivalent to $-(1/2) + j(\sqrt{3}/2)$. Clearly $\lambda^{\frac{1}{2}}$, will represent turning a vector through 60° in the positive direction. The following relations will be seen to hold for λ in its restricted meaning:—

$$\lambda^3 = 1;\ \lambda^{\frac{3}{2}} = -1;\ 1 + \lambda + \lambda^2 = 0;\ 1 + \lambda = -\lambda^2 = \lambda^{\frac{1}{2}}.$$

The last equality means that turning a vector through 240° and then reversing it is the same as turning the vector through 60° or adding to the original an equal one turned through 120°.

5. Proof of Separation Method

The construction for the positive sequence component given in Art. 3 may be expressed in the following way by the use of the operator λ (see Art. 4).

Let I_P be the positive sequence component in phase B. Then $3\,I_P = DG$ [in Fig. 14.02 (*b*)] $= OB + \lambda^2.OA + \lambda.OC$ (since turning forward through 240° is the same as turning back 120°).

Similarly if I_N = negative sequence component in phase B.

$$3\,I_N = HM\ [\text{in Fig. 14.02 } (d)] = OB + \lambda.OA + \lambda^2.OC;$$

$$\therefore\ 3\,(I_P + I_N) = 2\,OB + (\lambda + \lambda^2)\,(OA + OC).$$

But since there is no residue, $OA + OC = -\,OB$, and $\lambda + \lambda^2 = -\,1$ (see Art. 4);

$$\therefore\ 3\,(I_P + I_N) = 3\,OB.$$

Thus the vector sum of I_P and I_N gives the original current OB.

Again the positive sequence component of phase A [I_A in Fig. 14.02 (*c*)] is $\lambda.I_P$: and its negative sequence component [I_A' in Fig. 14.02 (*e*)] is $\lambda^2.I_N$. And, using the values obtained above,

$$3\,(\lambda I_P + \lambda^2 I_N) = (\lambda + \lambda^2)\,OB + (\lambda^3 + \lambda^3)\,OA + (\lambda^2 + \lambda^4)\,OC;$$

$$\text{since } \lambda^4 = \lambda,\ \text{this} = 2\,OA + (\lambda + \lambda^2)\,(OB + OC) = 3\,OA.$$

Thus the vector sum of the positive and negative sequence components again gives the original current, OA.

In the same way it can be proved that the vector sum of the two components in phase C gives OC.

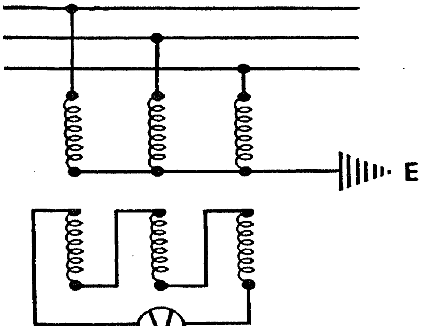


Fig. 14.04.
ZERO SEQUENCE VOLTAGE.

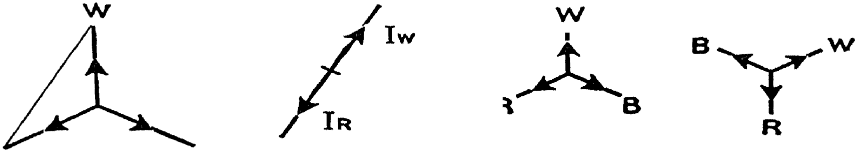
6. Measurement of Zero Sequence Component

The zero sequence component of the current in a 4-wire system can be obtained in magnitude by measuring the current in the fourth wire, and dividing this by 3. But if the fourth wire is not available, or if there is earth leakage from a 3-wire system the measurement can be made by connecting an ammeter in place of the leakage relay in Fig. 13.13.

Similarly the zero sequence component of the voltages to an earthed neutral can be found by connecting three similar potential transformers to a voltmeter as shown in Fig. 14.04. The voltage applied to the voltmeter is the resultant of the three line voltages to earth, altered in the ratio of the potential transformers.

7. Single-Phase Load

If a balanced 3-phase Y-connected load (*e.g.* a 3-phase motor) has one phase open-circuited current will continue to flow in the other two phases, the line voltage being applied to them in series.



Voltages. Currents. Positive sequence. Negative sequence

Fig. 14.05.—COMPONENTS OF 1-PHASE CURRENT.

Thus two lines carry equal but opposite currents, and the third has zero current.

Assuming that phase B has no current, and that the load is non-inductive, this condition is shown in the vector diagrams of Fig. 14.05.

If I_R is turned back through 120° (see Art. 3) and then added to I_W the result is $\sqrt{3}\,I$, 30° ahead of I_W. Therefore the positive sequence component is $I/\sqrt{3}$, in phase with the voltages to neutral.

Similarly if I_R is turned forward through 120° and added to I_W the result is $\sqrt{3}\,I$, 30° behind I_W. So the negative sequence component is $I/\sqrt{3}$, but lagging 60° behind voltage to neutral in phase W. This (see Fig. 14.05) makes the negative sequence current in phase R lead the positive one, and in phase B they are opposite in phase. It is easy to see that the addition of the two sets of currents gives the original ones correctly. If the load is inductive the currents remain related to one another in this way, but will all be displaced by the angle of lag relative to the voltages.

This resolution of a single-phase load into two equal balanced loads, having opposite phase sequence, resembles the resolution of an alternating field into two equal, oppositely rotating fields in the treatment of the single-phase motor (Chapter VII., Art. 2).

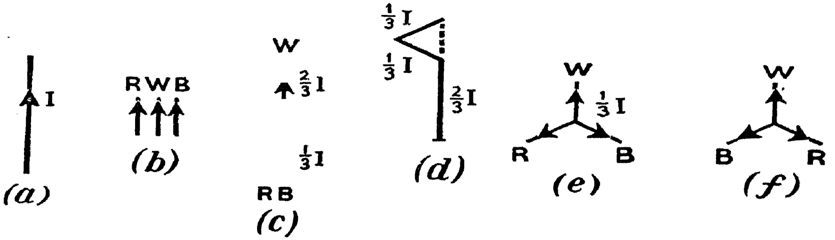


Fig. 14.06.—SINGLE-PHASE LOAD ON 4-WIRE SUPPLY.

8. Single-Phase Load on 3-Phase 4-Wire Mains

If a 4-wire supply has a load only between one outer and the fourth wire, the analysis into symmetrical components results as follows. If the load is I on phase W, the zero sequence component is I/3. Subtracting this from the original current in each phase gives what is shown in Fig. 14.06 (*c*), viz. $\frac{2}{3}$ I in phase W, and $\frac{1}{3}$ I returning by each of the other phases. The vector sum of I_W, plus I_R retarded 120°, plus I_B advanced 120° is I [see Fig. 14.06 (*d*)]. Hence the positive sequence component is I/3 [Fig. 14.06 (*e*)]. Similarly the negative sequence component is I/3 [Fig. 14.06 (*f*)].

The combination of the three separate sequence components gives the original single-phase current, since the R and B phase components cancel out.

9. Power with Symmetrical Components

Since the vector sum of positive sequence currents or voltages is zero, the association of zero sequence currents with positive sequence voltages, or of positive sequence currents with zero sequence voltages, gives zero total power. There will be power in at least two phases, but the positive power will be cancelled by equal negative power.

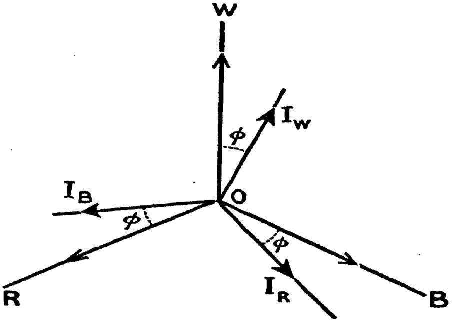


Fig. 14.07—VOLTAGES AND CURRENTS OF OPPOSITE SEQUENCE.

If positive sequence voltages and negative sequence currents are combined as in Fig. 14.07, again the total power will be zero. If V = phase voltage, power in phase W = VI cos ϕ; power in phase R = VI cos (120° − ϕ); power in phase B = VI cos (120° + ϕ). And I cos ϕ + I cos (120° − ϕ) + I cos (120° + ϕ) = 0, since this is the sum of the components of the three currents in phase with V_W.

The same is true of negative sequence voltages and positive sequence currents. Hence the total power in the general case is the sum of the powers due to the association of voltages and currents of like phase sequence. In the particular case of balanced voltages the whole power is that due to the positive sequence currents alone.

Applying this last result to the case analysed in Art. 8 gives, power = 3 . V . (I/3) . cos ϕ = VI cos ϕ. Similarly for the single-phase case of Art. 7, if V_L is the line voltage, power = $\sqrt{3}\, V_L\, (I/\sqrt{3})$ cos ϕ = V_L I cos ϕ.

ANSWERS

Questions on Chapter I. (*Page* 11.)

1. 4·03 kVA.; 500 watts; 0·124. 2. 2·68 ohms; 115·5 μF.

3. 9·90 ohms; 0·120 henry. 4. 37·5 amp.; 24·2 amp.; 18·1 amp.; 0·84.

5. 6 ohms; 86 millihenries. 6. 30·8, $\underline{|32{\cdot}4^\circ}$; 27·1, $\underline{|-16{\cdot}1^\circ}$.

7. (*a*) A, 222 amp., 0·356; B, 244 amp., 0·78; 6·46 volts. (*b*) A, 258 amp., 0·393; B, 208 amp., 0·81; 6·46 volts.

Questions on Chapter II. (*Pages* 45, 46.)

1. 1·146 I_m $(\sin\theta + \frac{1}{9}\sin 3\theta - \frac{1}{25}\sin 5\theta - \frac{1}{49}\sin 7\theta +$ etc.).

2. (*b*) Amplitudes:—fundamental, 96·5; 3rd harmonic, 13·7; 5th harmonic, 4·2. (*c*) 7th, 2·2; 9th, 0·7.

3. 5th, — 10·0; 7th, — 9·8; not including multiples. Fundamental, 200; 3rd, 14. 4. 21·6 amp.; 370 V.; 5·62 kW.; 0·703.

5. $0{\cdot}0314 \cos 314t + 0{\cdot}0157 \cos 1570t$.

6. 237 V. (*a*) 16·7 amp. (*b*) 2·80 amp. 7. 4·53 amp.

8. 31·7 amp.; 1440 V. 10. 0·86. 13. 1·69.

16. 1 to 0·940, or 1·064 to 1.

17. (*b*) $(1800/\pi^2)$ $(\sin\theta + \frac{1}{25}\sin 5\theta - \frac{1}{49}\sin 7\theta - \frac{1}{121}\sin 11\theta +$ etc.
(*c*) $(1800/\pi^2)$ $(\sin\theta - \frac{1}{25}\sin 5\theta + \frac{1}{49}\sin 7\theta - \frac{1}{121}\sin 11\theta +$ etc.

Questions on Chapter III. (*Pages* 80, 81.)

2. Coil No. 1:—0·098 amp., 7 % decrease. Coil No. 2:—0·140 amp., 33 % increase. 4. (*a*) 0·48 %; 0·32 %. (*b*) 1·20 %; 0·80 %.

5. (*a*) 0·27 %; 0·18 %. (*b*) 0·45 %; 0·31 %. In each case 16·4 amp. (at 220 V.). 6. 0·75 %; 1·7 %; 9·95 %.

7. 2·56 %; 4·59 %; 7·94 %.

9. (*a*) 15 watts; 8 watts. (*b*) 33 watts. (*c*) 48 watts; 41 watts.

11. 1·69 % and 2·44 % of the readings.

14. Power:— 200, 400, 825, 1670, and 2250 watts.
Power-factor:—0·36, 0·50, 0·62, 0·75, and 0·87.

20. (*a*) Slow by 1·7 % of meter reading. Increase recording by 0·3 %.
(*b*) Slow by 8·9 % of meter reading. 21. 48½°. Reduce by 33 %.

Questions on Chapter IV. (*Pages* 132, 133.)

2. 617 volts. 3. (*a*) 50 cyc.; (*b*) 0·837; (*c*) 6 680 volts.

4. (*a*) 3 860 volts; (*b*) 6 680 volts; (*c*) 578 kVA.; (*d*) 334 kVA.

5. 4 740 volts. 7. Disconnect conductors Nos. 1 and 50, 28 and 45, 6 and 23. Either Nos. 1, 28 and 6; or Nos. 50, 45 and 23.

8. Additional openings between Nos. 39 and 56, 17 and 34, 61 and 12. Connect Nos. 39 and 6, 17 and 50, 28 and 61. To star point, either Nos. 1, 45 and 23; or Nos. 34, 12 and 56.

11. 0·966; 0·911. 12. Reduced to 0·816 of previous value.

13. (*a*) 0·935. (*b*) 0·935. 17. 15 volts.

Questions on Chapter V. (*Pages* 159, 160.)

4. (*a*) Dia. = 333 amp.; (*b*) Dia. = 279 amp.; minimum p.f. 0·55; 91 amp.; 27·2 amp.; 1 046 V. 7. 520 V. between terminals.

11. 0·78 leading; 193 kVA.

12. 780 kVA. at 0·26 p.f. 262 kVA. at 0·76 p.f.

13. 0·13 μF per phase per kilowatt of load.

14. In £1000:—line, 12·85, 11·25, 10·00, 9·00; condensers, 0, 0·81, 1·61, 3·06; total, 12·85, 12·06, 11·61, 12·06. Most economical p.f. 0·943; line 9·55, condensers 2·00, total 11·55.

15. (*a*) 0·984. (*b*) £191. 16. 2160 kVA.; 0·56 leading.

Questions on Chapter VI. (*Pages* 216–218.)

2. 0·875. 9. 27 amp.; 0·84; 92 lb.-ft.; 1·95.

11. 2·47 ohms per phase, including winding resistances referred to the stator; 1·175 ohms per phase, including winding resistances referred to rotor.

12. (*a*) 4·62 amp.; (*b*) 3·77 amp.; (*c*) 2·14 amp.

16. 0·926, 0·86; 1·44; 2·65, 1·97.

17. (*a*) 0·85; 7·9 h.p.; 86 %; (*b*) 0·865; 12¾ h.p.; 90 %.

18. (*a*) 0·83; 6·4 h.p.; 82 %; (*b*) 0·88; 10·7 h.p.; 88 %.

23. Slot:—0·171 per slot (0·0725 per cm.); zig-zag:—0·250 per slot (0·106 per cm.): all in microhenries.

24. Yes, if the resisting torque is about 25·4 % of the maximum fundamental torque. 14·7 % of synchronous speed.

Questions on Chapter VII. (*Page* 241.)

2. 0·74; 4½ h.p.; 78 %.

5. (*a*) 60 kVA.; 80 kVA. (*b*) 142 kVA.; 114 kVA.

6. 1 200 r.p.m.; 600 r.p.m.; 400 r.p.m.

Questions on Chapter IX. (*Pages* 320, 321.)

1. 54·4 amp. (armature), 94·3 amp. (leads); 367 V. 2 320 watts; 1 465 watts; 2 035 watts; 45 kW. 2. (*a*) 54·8 amp.; (*b*) 23·7 amp.

3. (*a*) 21·9 amp.; (*b*) 32·3 amp. 8. 27·2 amp.

9. 54·4 amp.; 94·3 amp. 13. 65 and 56, neglecting losses.

14. 1st ring to conductors Nos. 1, 109, and 217; 2nd ring to Nos. 19, 127 and 235; 3rd ring to Nos. 37, etc.; 4th ring to Nos. 55, etc.; 5th ring to Nos. 73, etc.; 6th ring to Nos. 91, etc.

16. 120 kVA. 20. (*a*) 91·4 %; (*b*) 92·7 %.

Questions on Chapter XII. (*Pages* 417, 418.)

4. 221 kV. (R.M.S.) between lines.

5. 46·4 kV. to 39 kV. (max.) per cm., 32·8 kV. to 27·6 kV. (R.M.S.) per cm. for sinoidal waves; 48·5 cm. to 6·1 cm. 8. 2·89.

9. 54 %; 71 %. 15. (*a*) 400 ohms; (*b*) 182 ohms.

17. 173 kVA. 18. 346 V.; 103·8 kVA.

Questions on Chapter XIII. (*Page* 448.)

10. For equal kVA.; 0·93, 0·49. For equal kW.; 0·92, 0·65.

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